

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education
Advanced Level Examination
June 2013

Mathematics

MFP3

Unit Further Pure 3

Monday 10 June 2013 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J U N 1 3 M F P 3 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

1 It is given that $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = (x - y)\sqrt{x + y}$

and $y(2) = 1$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = hf(x_r, y_r)$ and $k_2 = hf(x_r + h, y_r + k_1)$ and $h = 0.2$, to obtain an approximation to $y(2.2)$, giving your answer to three decimal places. (5 marks)

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2 The Cartesian equation of a circle is $(x + 8)^2 + (y - 6)^2 = 100$.

Using the origin O as the pole and the positive x -axis as the initial line, find the polar equation of this circle, giving your answer in the form $r = p \sin \theta + q \cos \theta$.
(4 marks)

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4 Evaluate the improper integral

$$\int_0^{\infty} \left(\frac{2x}{x^2 + 4} - \frac{4}{2x + 3} \right) dx$$

showing the limiting process used and giving your answer in the form $\ln k$, where k is a constant. (6 marks)

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5 (a) Differentiate $\ln(\ln x)$ with respect to x . (1 mark)

(b) (i) Show that $\ln x$ is an integrating factor for the first-order differential equation

$$\frac{dy}{dx} + \frac{1}{x \ln x} y = 9x^2, \quad x > 1$$
(2 marks)

(ii) Hence find the solution of this differential equation, given that $y = 4e^3$ when $x = e$.
(6 marks)

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- 6** It is given that $y = (4 + \sin x)^{\frac{1}{2}}$.
- (a) Express $y \frac{dy}{dx}$ in terms of $\cos x$. *(2 marks)*
- (b) Find the value of $\frac{d^3y}{dx^3}$ when $x = 0$. *(5 marks)*
- (c) Hence, by using Maclaurin's theorem, find the first four terms in the expansion, in ascending powers of x , of $(4 + \sin x)^{\frac{1}{2}}$. *(2 marks)*

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7 A differential equation is given by

$$\sin^2 x \frac{d^2y}{dx^2} - 2 \sin x \cos x \frac{dy}{dx} + 2y = 2 \sin^4 x \cos x, \quad 0 < x < \pi$$

(a) Show that the substitution

$$y = u \sin x$$

where u is a function of x , transforms this differential equation into

$$\frac{d^2u}{dx^2} + u = \sin 2x \quad (5 \text{ marks})$$

(b) Hence find the general solution of the differential equation

$$\sin^2 x \frac{d^2y}{dx^2} - 2 \sin x \cos x \frac{dy}{dx} + 2y = 2 \sin^4 x \cos x$$

giving your answer in the form $y = f(x)$. (6 marks)

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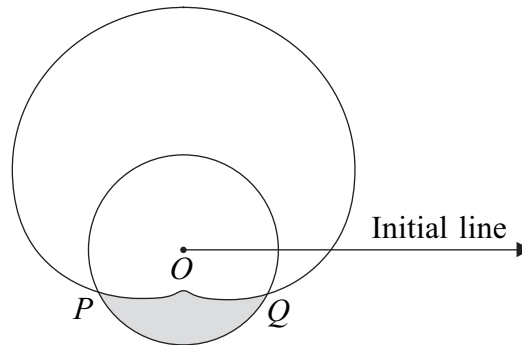
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- 8 The diagram shows a sketch of a curve and a circle.



The polar equation of the curve is

$$r = 3 + 2 \sin \theta, \quad 0 \leq \theta \leq 2\pi$$

The circle, whose polar equation is $r = 2$, intersects the curve at the points P and Q , as shown in the diagram.

- (a) Find the polar coordinates of P and the polar coordinates of Q . (3 marks)
- (b) A straight line, drawn from the point P through the pole O , intersects the curve again at the point A .
- (i) Find the polar coordinates of A . (2 marks)
- (ii) Find, in surd form, the length of AQ . (3 marks)
- (iii) Hence, or otherwise, explain why the line AQ is a tangent to the circle $r = 2$. (2 marks)
- (c) Find the area of the shaded region which lies inside the circle $r = 2$ but outside the curve $r = 3 + 2 \sin \theta$. Give your answer in the form $\frac{1}{6}(m\sqrt{3} + n\pi)$, where m and n are integers. (9 marks)

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END OF QUESTIONS



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ANSWER IN THE SPACES PROVIDED**

