

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
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2	
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TOTAL	



General Certificate of Education  
Advanced Subsidiary Examination  
June 2013

# Mathematics

# MFP1

## Unit Further Pure 1

Friday 17 May 2013 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J U N 1 3 M F P 1 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

**1** The equation

$$x^3 - x^2 + 4x - 900 = 0$$

has exactly one real root,  $\alpha$ .

Taking  $x_1 = 10$  as a first approximation to  $\alpha$ , use the Newton–Raphson method to find a second approximation,  $x_2$ , to  $\alpha$ . Give your answer to four significant figures.

*(3 marks)*

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**2** The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} p & 2 \\ 4 & p \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$$

(a) Find, in terms of  $p$ , the matrices:

(i)  $\mathbf{A} - \mathbf{B}$ ; (1 mark)

(ii)  $\mathbf{AB}$ . (2 marks)

(b) Show that there is a value of  $p$  for which  $\mathbf{A} - \mathbf{B} + \mathbf{AB} = k\mathbf{I}$ , where  $k$  is an integer and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix, and state the corresponding value of  $k$ . (4 marks)

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3 (a) Find the general solution, in degrees, of the equation

$\cos(5x + 40^\circ) = \cos 65^\circ$  (5 marks)

(b) Given that

$\sin \frac{\pi}{12} = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

express  $\sin \frac{\pi}{12}$  in the form  $\left(\cos \frac{\pi}{4}\right) \left(\cos(a\pi) + \cos(b\pi)\right)$ , where  $a$  and  $b$  are rational. (3 marks)

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4 (a) It is given that  $z = x + yi$ , where  $x$  and  $y$  are real numbers.

(i) Write down, in terms of  $x$  and  $y$ , an expression for  $(z - 2i)^*$ . (1 mark)

(ii) Solve the equation

$$(z - 2i)^* = 4iz + 3$$

giving your answer in the form  $a + bi$ . (5 marks)

(b) It is given that  $p + qi$ , where  $p$  and  $q$  are real numbers, is a root of the equation  $z^2 + 10iz - 29 = 0$ .

Without finding the values of  $p$  and  $q$ , **state** why  $p - qi$  is **not** a root of the equation  $z^2 + 10iz - 29 = 0$ . (1 mark)

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**5 (a)** A curve has equation  $y = 2x^2 - 5x$ .

The point  $P$  on the curve has coordinates  $(1, -3)$ .

The point  $Q$  on the curve has  $x$ -coordinate  $1 + h$ .

**(i)** Show that the gradient of the line  $PQ$  is  $2h - 1$ . *(3 marks)*

**(ii)** Explain how the result of part **(a)(i)** can be used to show that the tangent to the curve at the point  $P$  is parallel to the line  $x + y = 0$ . *(2 marks)*

**(b)** For the improper integral  $\int_1^{\infty} x^{-4}(2x^2 - 5x) dx$ , either show that the integral has a finite value and state its value, or explain why the integral does not have a finite value. *(3 marks)*

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6 The equation

$$2x^2 + 3x - 6 = 0$$

has roots  $\alpha$  and  $\beta$ .

- (a) Write down the value of  $\alpha + \beta$  and the value of  $\alpha\beta$ . (2 marks)
- (b) Hence show that  $\alpha^3 + \beta^3 = -\frac{135}{8}$ . (3 marks)
- (c) Find a quadratic equation, with integer coefficients, whose roots are  $\alpha + \frac{\alpha}{\beta^2}$  and  $\beta + \frac{\beta}{\alpha^2}$ . (6 marks)

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- 7 (a)** Show that the equation  $4x^3 - x - 540\,000 = 0$  has a root,  $\alpha$ , in the interval  $51 < \alpha < 52$ . (2 marks)
- (b)** It is given that  $S_n = \sum_{r=1}^n (2r - 1)^2$ .
- (i)** Use the formulae for  $\sum_{r=1}^n r^2$  and  $\sum_{r=1}^n r$  to show that  $S_n = \frac{n}{3}(kn^2 - 1)$ , where  $k$  is an integer to be found. (5 marks)
- (ii)** Hence show that  $6S_n$  can be written as the product of three consecutive integers. (2 marks)
- (c)** Find the smallest value of  $N$  for which the sum of the squares of the first  $N$  odd numbers is greater than 180 000. (2 marks)

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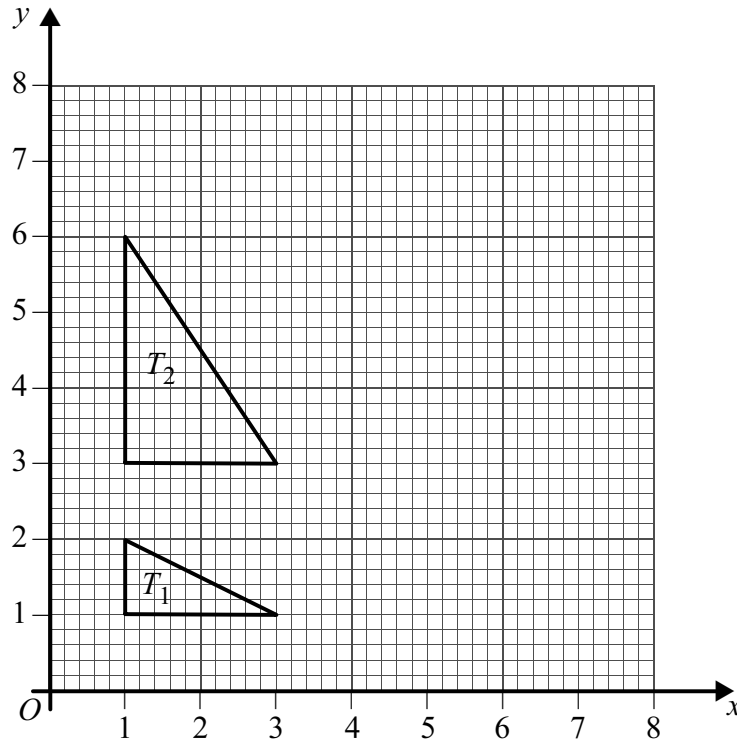
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8 The diagram shows two triangles,  $T_1$  and  $T_2$ .



- (a) Find the matrix which represents the stretch that maps triangle  $T_1$  onto triangle  $T_2$ . (2 marks)
- (b) The triangle  $T_2$  is reflected in the line  $y = \sqrt{3}x$  to give a third triangle,  $T_3$ . Find, using surd forms where appropriate:
  - (i) the matrix which represents the reflection that maps triangle  $T_2$  onto triangle  $T_3$ ; (2 marks)
  - (ii) the matrix which represents the combined transformation that maps triangle  $T_1$  onto triangle  $T_3$ . (2 marks)

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**9** A curve has equation

$$y = \frac{x^2 - 2x + 1}{x^2 - 2x - 3}$$

**(a)** Find the equations of the three asymptotes of the curve. *(3 marks)*

**(b) (i)** Show that if the line  $y = k$  intersects the curve then

$$(k - 1)x^2 - 2(k - 1)x - (3k + 1) = 0 \quad (1 \text{ mark})$$

**(ii)** Given that the equation  $(k - 1)x^2 - 2(k - 1)x - (3k + 1) = 0$  has real roots, show that

$$k^2 - k \geq 0 \quad (3 \text{ marks})$$

**(iii)** Hence show that the curve has only one stationary point and find its coordinates.

(No credit will be given for solutions based on differentiation.) *(4 marks)*

**(c)** Sketch the curve and its asymptotes. *(3 marks)*

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