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General Certificate of Education (A-level)
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Mathematics MPC4

(Specification 6360)

Pure Core 4

Report on the Examination

General

This paper proved to be quite challenging for most students, with relatively few high marks seen, although there were some excellent scripts, where students showed a thorough knowledge of the specification and an ability to apply it to the questions set. At the other end of the spectrum, the paper was generally accessible and the proportion of low marks was about as expected.

Presentation of solutions was generally good, with students making it clear when they had deleted an attempt. Most students indicated the question part reference with their solution to it, although it tended to be the weaker students who did not always make it clear which part of a question they were answering. There were some misreads and students often did not give an answer in the form requested, which suggests that they had not read the question carefully. No one question stood out as particularly easy or difficult. A full range of marks was seen on all questions, but some questions parts can be identified as being done well or poorly: done relatively well were Q1(a), (b)(i), Q3(a), (b)(i), Q4 (a)(i) and (a)(ii), and Q7 (a) and (b); done poorly or with little or no attempt were Q1 (b)(ii), Q5(b), Q7(c) and Q8(a). Most students did attempt all the questions.

Question 1

In part (a)(i), the partial fractions were found correctly by most students, substituting $x = 0$ and $x = 3$ being preferred over setting up simultaneous equations or a mix of both.

Most students went on to integrate the partial fraction form correctly, although some omitted the brackets on $\ln(x-3)$, but this was condoned. Some students were confused by the integral of $\frac{1}{x}$, with some trying to make this become x^0. Very few students produced complete nonsense for both integrals, although some differentiated.

Students attempted part (b)(i) by long division or equating coefficients and were usually successful in correctly finding all three coefficients. Those who chose equating coefficients often did far more work than was necessary, as they multiplied out and regrouped the whole expression and were more likely to make an error. Relatively few used $x = -\frac{1}{2}$ to find *r* first, and those who did rarely progressed beyond a value for *r*.

In part (b)(ii), although most students attempted to use their result from part (b)(i), most of these failed to realise that *r* was not multiplied by $(2x-1)$, and simply subtracted r from q to give a relatively simple polynomial integral, which gained no credit. Those who did use algebra correctly to set up the integral, usually gave the integral as a log term, but often with an error in the coefficient. Here too, some students differentiated rather than integrated.

Question 2

In part (a), whilst most students found the value of R correctly and left it as $\sqrt{10}$, the same was not true of the angle α. Some students seemingly failed to notice that α was given as an acute angle and found $\tan\alpha$ to be -3; some confused sine and cosine and had $\tan\alpha = \pm\frac{1}{3}$. Some students progressed to $\tan\alpha = 3$ from $\sin\alpha = 3$, $\cos\alpha = 1$ which was penalised for both marks.

For part (b), most students rearranged the equation correctly using their result from part (a), although many then dropped the negative sign when finding the inverse sine. Many students seemingly could not cope with the result of $-39.23°$ that their calculator gave them, when all that was required was to add $71.6°$ (ie α) to get the first solution. Some students found the third quadrant solution only, whereas some found more than two solutions. There were many students who ignored the request to round to the nearest degree in their final answer; this was condoned if their answers were correct to the greater number of decimal places they gave. Many students did give a fully correct solution, often by using a sketch graph to help them decide where the solutions lay.

Question 3

In part (a), most students got the first two terms of the expansion correct, but often there was a mistake in the x^2 term, with $4x$ becoming just x being the common error. Some students made arithmetic errors with 4^2, by failing to actually square the 4, and others failed to simplify the binomial coefficient correctly.

For part (b)(i), most students took out 4 as a factor correctly, although some followed this with an error in interpreting $4^{-\frac{1}{2}}$. Others left x, rather than $\frac{x}{4}$, as the term in the bracket. Those who did have the correct bracket at this stage usually expanded correctly, although some made sign errors, particularly $\frac{x}{4}$ instead of $\left(-\frac{x}{4}\right)$, whilst again some could not simplify the binomial coefficients correctly. A common error was with the sign on the x-term, which lost the final mark. Some lost the final mark for not multiplying their expansion by $\frac{1}{2}$. Expansions using the result from the formula book were rare, but usually were done correctly when attempted.

Relatively few students wrote down a correct inequality in part (b)(ii). Common errors were $x < 4$, $x > 4$, $\frac{x}{4} < 1$ or $x \neq 4$. Some students used the modulus symbol correctly, whereas others showed no understanding, writing expressions such as $|x| < -4$.

Many students understood what was required in part (c) and attempted to multiply their two expansions, some doing this efficiently and correctly, but others doing more work than necessary, as only terms up to x^2 were required. Many algebraic errors were seen. Some students just multiplied terms of the same power. Some wrote down one expansion divided by another and divided each term in the numerator by the corresponding term in the denominator, but often a quadratic expression just appeared with little or no working seen. Some students added or subtracted their expressions with no apparent attempt to multiply them. Some put a square root sign around their final answer, showing little understanding of what the question was asking.

Question 4

Most students did part (a)(i) correctly, with very few failing to round to the requested nearest £10. Some confused P and V, so the value of the investment actually went down.

For part (a)(ii), most students showed that they understood the information given and wrote down a correct opening expression, although there was uncertainty over which way the inequality should go. Some then simplified and solved using logarithms efficiently to get the correct answer. Some resorted to trial and improvement, often getting to the correct answer, whilst some students tried a hybrid of both logs and trial and error. Some students failed to give their answer in complete years, thus losing the final mark.

In part (b) also, most students showed they understood the information given and wrote down a correct opening expression. Some made errors in simplifying, but many students who attempted a solution by logarithms, took logarithms incorrectly and their unknown (T or n) cancelled out of their equation. A common error was to combine 1.5 from $\left(\frac{1500}{1000}\right)$ with $\left(1+\frac{1.5}{100}\right)$ to get a term $n\log 1.5\left(1+\frac{1.5}{100}\right)$. Relatively few students got to the correct answer using logarithms. Some realised they had made an error but found the correct answer of 28 from somewhere; it was not always clear what they had done. However, many abandoned their attempt at this point. Many, though, did get a correct answer through trial and improvement, which was accepted for full marks if done correctly, but was worth no marks otherwise.

Question 5

For part (a)(i), most students knew that they needed to differentiate the two parametric equations and use the chain rule which many did correctly, with very few having it the wrong way up. Errors were relatively rare but included sign errors, coefficient errors or dropping the 2 from the double angle. Some chose to expand $\sin 2\theta$ and use the product rule, again mostly successfully, but some made sign errors here too.

The manipulation to the required form was often confused as some students lost their way, many of these immediately replacing the $\frac{1}{\sin\theta}$ term from differentiation by $\operatorname{cosec}\theta$, and abandoning, or fudging, as they could not get to the required form. Those who immediately replaced $\cos 2\theta$ with $1-2\sin^2\theta$ usually went on to be successful. The common error was to omit the 2 in the expansion. Those who went via the $\cos^2\theta-\sin^2\theta$ form were sometimes successful, although many made a sign or coefficient error in attempting to simplify to the required form.

Part (a)(ii) was essentially testing a student's ability to use $\theta = \frac{\pi}{6}$ correctly in their version of $a\sin\theta + b\operatorname{cosec}\theta$ in finding the gradient of a tangent and the associated normal. Many did it correctly, although a significant number only found the gradient of the tangent whereas some unnecessarily found the equation of the normal. Some students used their chain rule expression, which was perfectly acceptable. The relationship between the gradients was generally very well known.

Many students made little or no progress with part (b), especially if they tried to start from inverse sines and cosines from the parametric equations. Those who expanded $\sin 2\theta$ and then squared the whole equation usually progressed to trying to use the parametric equations to eliminate θ, but relatively few could then manipulate correctly to the required form. Few scored high marks here, but some very insightful and clever derivations were seen, based on $\sin^2\theta + \cos^2\theta = 1$.

Question 6

Although this question had no structure to it, most students realised it involved implicit differentiation and attempted to find an expression for $\frac{dy}{dx}$, with most not realising this was in fact unnecessary, as to make progress in solving the problem they only needed to put their $\frac{dy}{dx}$ terms equal to zero. Many students got the implicit differentiation fully correct, some using a spurious $\frac{dy}{dx}$ on the left hand side, but only using it as an aide memoire, which was condoned on this occasion, whereas a few brought it into their expression and this was penalised. Errors were seen in all the terms but commonly the 3 did not go to zero, $\frac{dy}{dx}$ was missing from the y-term or there was a coefficient error. Sign errors were also made in using the product rule, which appeared to be related to a failure to use brackets, and some students gave this derivative as a single term..

Many did solve their derivatives expression correctly for $\frac{dy}{dx}$, but when equated to zero some did not continue any further, some equated both numerator and denominator to zero, and some multiplied across, treating the zero as if it were 1. Some just guessed values for x and y at this stage, making the numerator come to zero. Credit was only given for equating the numerator to zero, and obtaining y in terms of x, or vice versa, so that a solution could be continued if students realised they now needed to substitute back into the original equation. Some did this correctly, and some fully correct solutions were seen, but others pursued equations obtained through incorrect algebra, and gained no further credit. Some students who had progressed correctly found only the positive value of x, so could not find two stationary points. Some others who had correctly found two values of x, substituted into the original equation, rather than the equation from the derivatives, and found a spurious value of y.

Question 7

In part (a), most students set up the simultaneous equations required correctly, with many of these spotting the solution $\mu = -1$ immediately and thus finding the value of λ easily. Other students used conventional simultaneous equations methods, which took rather longer but could still be done successfully. Most then went on to use the third equation to show that $q = 4$, with relatively few assuming $q = 4$ and showing that it works. Most found the coordinates of the point P correctly, with the occasional coefficient error, although most gave their solution as a column vector rather than coordinates, which was condoned this time.

In part (b), most students knew that they were to show a scalar product came to zero, and most correctly choose the direction vectors and demonstrated this was the case. There was no credit for just stating the result, and students who failed to state the conclusion to their scalar product $= 0$ were also penalised.

Many students misunderstood the notation in part (c)(i), not realising they were being asked to find the square of the length AP. Some students squared their coordinates of point P, before subtracting those of point A, and leaving it as their answer, whereas some multiplied the corresponding coordinates. Some students showed some understanding in getting an answer of $(16, 0, 4)$ but left the answer as a column vector and not a length squared, so got no credit. Some students who had misunderstood part (c)(i) found the length of AP correctly, when attempting part (c)(ii).

In order to make any progress with part (c)(ii), students needed to express the vector $\overrightarrow{BP}$ or $\overrightarrow{AB}$ in terms of a parameter, find the length of the vector and use the properties of the isosceles triangle to set up an equation. There were some good attempts at this stage although rather fewer went on to complete fully successfully. Errors were made in expanding expressions such as $(a + bp)^2$ or in the collection of terms, although many did get to, and attempted to solve, a quadratic equation as required and gained credit for using a correct method. Some students attempted to use the scalar product $\overrightarrow{AP} \bullet \overrightarrow{BP} = 0$ but this just reduces to $0 = 0$ if done correctly. There is an alternative method avoiding the use of a parameter, based on equating vector $\overrightarrow{BP}$ to a multiple of the direction vector of line l_2, but this was rarely seen.

Question 8

For part (a), some student wrote down the correct differential equation apparently fully understanding all the information given and interpreting it correctly. However, all sorts of errors abounded in other attempts, some not even involving a derivative, and some with derivatives in x and y. Some attempts bore little or no relation to the information given, although credit was given if $\pm(2-h)$ appeared in an otherwise incorrect expression. Many had a spurious t and/or h, either as a multiple or power, and the k appeared in a variety of places. Some students did not even form an equation, leaving a proportionality sign in their answer.

In part (b), most students knew they were expected to separate the variables and did it correctly, although there were some notation errors in the positioning of $\mathrm{d}x$, at the front rather than the rear of the integrand. Those who failed to separate the variables, just produced nonsense.

Most students integrated $k\ \mathrm{d}t$ successfully, but relatively few made good progress with the integral in x. No guidance was given, but students split fairly evenly between attempts by parts and by substitution, with similar success, or lack of it, via either method. Many students just integrated each term in the product; presumably this was an attempt at parts. Those who showed the parts and how the integral might be achieved scored a method mark, but many could not integrate $\sqrt{(2x-1)}$ correctly, making an error in the required coefficient. Those who used substitution usually chose to put $u = 2x-1$ but did not always change their whole integral to be in terms of u; a missing $\mathrm{d}u$ was penalised for the method mark. Some with a correct first stage then did not multiply out to obtain an integrable form, but just integrated each term in the product. All students, no matter what their attempt at the integral, could obtain a method mark if they included a constant and tried to find it using the given initial conditions. Those few who did integrate and find the constant correctly, usually went on to calculate the final requested value of t correctly.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results statistics page of the AQA Website. UMS conversion calculator www.aqa.org.uk/umsconversion