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Pure Core 1

Report on the Examination

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General

Once again, the question paper seemed to provide a suitable challenge for able students, whilst at the same time allowing weaker students to demonstrate their understanding of differentiation, integration, factorising polynomials and rationalising the denominator of surds. When an answer is requested in a particular form, such as the equation of a straight line, students will not score full marks if their final answer is not in that specific form. Careful attention needs to be given to proofs when a printed answer is given, where once again the final line of a student's working should match the printed answer.

Algebraic manipulation continues to be a weakness; this was very evident when solving simultaneous equations, multiplying out brackets and factorising quadratic expressions. The number of arithmetic errors suggested that some students have become over-dependent on a calculator for simple arithmetic. In particular, students should be encouraged to simplify fractions by cancelling factors rather than simply multiplying out products in the numerator and denominator.

Weaker students might benefit from learning accurately a few basic formulae such as the general equation of a circle and the quadratic equation formula so as to avoid losing precious marks. Future students might appreciate the following advice:

- The straight line equation $y - y_1 = m(x - x_1)$ could sometimes be used with greater success than always trying to use $y = mx + c$
- The quadratic equation formula needs to be learnt accurately and values substituted correctly or no marks will be earned
- When solving a quadratic inequality, a sketch or a sign diagram showing when the quadratic function is positive or negative could be of great benefit
- The line of symmetry of the curve $y = (x - p)^2 + q$ has equation $x = p$
- For a curve with equation $y = \mathrm{f}(x)$, y is increasing when $\dfrac{\mathrm{d}y}{\mathrm{d}x} > 0$
- A concluding statement is expected whenever a question asks for a particular result to be proved or verified
- When asked to use the Factor Theorem, students are expected to make a statement such as 'therefore $(x + 1)$ is a factor of $\mathrm{p}(x)$' after showing that $\mathrm{p}(-1) = 0$
- The circle with equation $(x-a)^2 + (y-b)^2 = k$ has centre (a, b) and radius $\sqrt{k}$
- The tangent at the point A to a circle with centre C is perpendicular to the line AC.

Question 1

This surd question involving the rationalising of the denominator proved to be a good opener to the paper with many students offering a complete solution. Some used a grid method to evaluate the numerator and denominator, but with mixed success. The most common error occurred when multiplying $5\sqrt{3}$ by $2\sqrt{3}$ in the numerator. A few students who obtained $\dfrac{48-27\sqrt{3}}{3}$ wrote their final answer as $16-27\sqrt{3}$.

Question 2

In part (a)(i), most students were able to find the correct gradient, although some insisted on writing the gradient as $\frac{4}{3}x$. Unfortunately, quite a few were unable to make y the subject of the equation $4x - 3y = 7$ and, consequently, the incorrect value $-\frac{4}{3}$ was offered as the gradient by a significant number of students; a minority confused the gradient with the y-intercept and gave answers of $\pm\frac{7}{3}$.

In part (a)(ii), the most successful attempts at the equation of the line used an equation of the form $y - y_1 = m(x - x_1)$, as flagged above. Many who tried to use $y = mx + c$ lost marks because they made arithmetic errors when trying to find the value of c. Quite a large number did not appreciate that parallel lines have the same gradient and these usually found the negative reciprocal of their gradient from part (a)(i) to use for the gradient of the parallel line. Although most students found a correct equation, many mistakes were seen when trying to rearrange their equation into the required form with integer coefficients.

In part (b), most students made an attempt at the simultaneous equations, but it was alarming to see how few obtained the correct solution. Poor algebraic skills and carelessness were often evident here; elimination of one of the variables proved to be the most successful approach; those using substitution were usually unable to cope with the fractions and negative signs. Some students did not read the question carefully, and no credit was given to those who used the wrong pair of equations.

In part (c), once again, poor algebra was the main problem. Many of those students who successfully wrote $4(k-2)-3(2k-3)=7$ were unable to solve the equation for k correctly. Others scored no marks for substituting into the wrong equation.

Question 3

In part (a)(i), those students who used long division, perhaps misunderstanding what was meant by ‘The Factor Theorem’, scored no marks. Most realised the need to find the value of $p(x)$ when $x = -1$. However, it was also necessary, after showing that $p(-1) = 0$, to write a concluding statement regarding $x + 1$ being a factor; some failed to show explicitly that $-(-5) = +5$ in their verification and lost a mark; a few evaluated $p(1)$ and scored no marks.

In part (a)(ii), those who found the quadratic factor using inspection were the most successful; methods involving long division or equating coefficients usually contained algebraic errors. Once a candidate had the correct quadratic factor, it was good to see most expressing $p(x)$ as the product of three correct factors. Others found the remaining factors separately by repeated use of the Factor Theorem, but then sometimes forgot to write $p(x)$ as the product of factors. A few tried to write down the product of factors with no working at all and, although some were successful, most using this direct approach made at least one error in their factors and scored no marks at all.

In part (b), it was not acceptable to write $p(0) > p(1)$ first, simply followed by $-6 > -8$; the verification required the correct evaluation of $p(0)$ and $p(1)$ and a concluding statement such as ‘therefore $p(0) > p(1)$’. Some made errors in their calculations, usually when finding $p(1)$, but the major reason for losing marks was the omission of a suitable conclusion.

In part (c), it was pleasing to see many correct graphs, although some failed to see the connection with part (b) and showed their minimum point either to the left of or actually on the y-axis. Some caused difficulties for themselves by marking the values of the intercepts $x = -1$ and $x = 2$ equidistant from the origin; the minimum point was then almost invariably drawn in the wrong place.

Question 4

In part (a)(i), some students merely doubled the given equation and then divided by 2; this did not convince examiners and earned no marks. Students were expected to show the separate areas of the faces of the cuboid; the minimum working required was sight of $2xy + 6xy$ as well as $6x^2$, before combining these to give $6x^2 + 8xy = 32$ and hence the printed result. Some proved the result by considering half the surface area and, provided enough explanation was given, this also scored full marks.

In part (a)(ii), the majority earned a single mark for writing down a correct expression for the volume in terms of x and y. The algebra required to rearrange the equation from part (a)(i) to make y or xy the subject proved too difficult for many and once again highlighted students' algebraic weaknesses as they tried to reach the printed answer but violated various algebraic laws on the way.

In part (b), whilst the term $12x$ was almost invariably differentiated correctly, the same could not be said of the second term in the expression for V, and $-27x^2$ was a common incorrect answer.

In part (c)(i), although most students realised the need to substitute $x = \frac{4}{3}$ into their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$, not all showed the required working to prove that $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. It was not sufficient to write $12 - \dfrac{27\left(\frac{4}{3}\right)^2}{4} = 12 - 12 = 0$; evidence of correct manipulation of the fractions was needed. Once again, many students did not write a concluding statement with regard to the zero gradient implying that there was a stationary point.

In part (c)(ii), by far the most common error when finding $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ was the omission of the minus sign. The final mark was still available even if students made errors in finding the second derivative, provided the substitution of $x = \frac{4}{3}$ was done correctly and the appropriate conclusion was drawn regarding the nature of the stationary point.

Question 5

In part (a)(i), the coefficient of x, being an odd number, caused problems for some when completing the square; most students could not square 1.5 without a calculator; those who used fractions could not always simplify $5-\frac{9}{4}$,and for those using decimals $-1.25 + 5 = 3.75$ was commonly seen.

In part (a)(ii), the equation of the line of symmetry was not usually stated correctly, with several writing down the equation of a curve rather than a straight line; the incorrect equation $y=\frac{3}{2}$ was seen far too often, whereas others simply wrote down numbers such as $\frac{3}{2}$ or $\frac{11}{4}$; this topic was not well understood by many students.

In part (b)(i), most attempts earned the method mark for eliminating y and collecting like terms, but, surprisingly, not all were able to solve the resulting quadratic equation. A common incorrect solution to $x^2 - 4x = 0$ was $x = \pm 2$, whereas others obtained $y = x^2 - 4x$ from the two equations and made no progress.

Integration of polynomials is well drilled and part (b)(ii) earned full marks for weak and strong students alike. There were few errors seen but the most common involved the constant term; some kept the term as 5 and others omitted the final term possibly confusing differentiation with integration.

In part (b)(iii), most students realised the need to substitute appropriate limits but, without a calculator, arithmetic errors were abundant; 4^3 was often evaluated as 48 and the fractions and minus sign caused problems for many students who could not simplify $\frac{64}{3}-24+20$.

Those who were unable to complete part (b)(i) often chose the value 5 as their upper limit, possibly since the coordinates of A were $(0, 5)$, but no credit was given for this. However, those who had the solution $x = 2$ from their earlier work were given a method mark for using this value as their upper limit. A large number of students presented the value of the definite integral as their final answer to the area of the shaded region instead of considering the difference of the area of the appropriate trapezium and the value of the integral.

Question 6

In part (a), some weaker students did not seem familiar with the standard equation of a circle, with several writing the left hand side of the equation as $(x+5)^2+(y+8)^2$ or $(5-a)^2+(8-b)^2$. However, most errors were associated with the value of k; typical incorrect values seen on the right hand side of the equation were 5, 89 and $\sqrt{89}$.

In part (b)(i), the verification mark was rarely scored. Many, who had the correct circle equation, were able to verify that A was a point on the circle but then neglected to make a concluding statement. Those who had the wrong value of k in part (a) could not earn this mark unless they argued in terms of Pythagoras, stating the correct value of the radius.

In part (b)(ii), while most students found the gradient of AC correctly, many used the value of the gradient of AC when finding the equation of the tangent. It was pleasing to see a large number of students realise the need to find the negative reciprocal, but these students were sometimes unable to present their tangent equation in the required form.

In part (c)(i), most used the distance formula correctly and found that CM^2 was equal to 20 and then deduced that $CM = 2\sqrt{5}$. Occasionally the surd was not handled correctly, with some writing the final answer as $4\sqrt{5}$. No marks were awarded to students who simply wrote down the answer $2\sqrt{5}$ without supporting working.

Part (c)(ii) was not answered quite so well. Pythagoras was not always used correctly as the angle PCQ at the centre of the circle was often assumed to be a right angle or the length CM assumed to equal PM. Even those who found the correct value of PM either found the area of only half the triangle or were unable to deal with the product of the surds.

Question 7

In part (a)(i), the idea of y being increasing when $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ seemed unfamiliar to many students, who seemed content to substitute a few numerical values for x into the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ rather than tackling the three-line proof. Consequently, very few scored full marks on this part.

Many students ignored the inequality signs throughout part (a)(ii). It was pleasing to see many students attempting to factorise the quadratic, often successfully, rather than relying on the formula. Perhaps more students would be successful in solving quadratic inequalities if they were to draw a sign diagram or sketch a graph. There were many correct solutions, but students must realise that $x < 2,\ x > \frac{4}{3}$ is not the same as $\frac{4}{3} < x < 2$

In part (b)(i), most students attempted to evaluate $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 2$. Not all were able to cope with the evaluation of $-6\,(2)^2$, which they needed to do in order to convince examiners that $\frac{\mathrm{d}y}{\mathrm{d}x}$ was indeed equal to zero. Once more, some failed to write a concluding statement or referred to a stationary point rather than the fact that the tangent was parallel to the x-axis.

In part (b)(ii), it was pleasing to see some fully correct solutions. Most recognised that the equation of the tangent at P was $y = 3$, but it was disappointing to see a number of students write $y - 3 = 9(x - 2)$ followed by $y = x + 1$ as their tangent equation. In finding the equation of the normal at Q, some, having correctly found that $\frac{\mathrm{d}y}{\mathrm{d}x} = 10$ when $x = 3$, used the value 10 instead of $-\frac{1}{10}$ for the gradient of the normal. Arithmetic errors were very common by those who used $y = mx + c$ when finding the value of c; typically $y = \frac{x}{10} + \frac{7}{10}$ was seen instead of $y = \frac{x}{10} - \frac{13}{10}$. Curiously, quite a few students who had previously obtained $y = 3$ as the equation of the tangent, and consequently the correct y-coordinate of the point R, then substituted the correct value of x into their incorrect equation for the normal and found a different, and now incorrect value for the y-coordinate of R.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results statistics page of the AQA Website. UMS conversion calculator www.aqa.org.uk/umsconversion