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General Certificate of Education (A-level)
June 2012

Mathematics MFP3

(Specification 6360)

Further Pure 3

Report on the Examination

General

Students appeared to be well prepared for this examination and they were able to tackle all that they could do without there being any apparent evidence of shortage of time. Students answered questions in the required spaces, with supplementary paper being used by a few students where this proved necessary, rather than space for other questions, as instructed by the rubric on the front of the paper – this practice should continue to be encouraged by those preparing students for the examination.

Teachers may wish to emphasise the following points to their students in preparation for future examinations in this unit:

- When asked to show a printed result all lines of working should be checked thoroughly to ensure that there are no errors and the final line of working should match the printed result.
- Final answers should be checked to see that they match the request in the question. For example, if asked to find an obtuse angle in radians, 3.76 radians would not be a correct answer.
- To investigate an improper integral which has an infinite upper limit, the upper limit should first be replaced by 'a' for example, the resulting integral evaluated and then the limit as $a \to \infty$ investigated, ensuring that before the limit is taken all necessary substitution and manipulation are carried out. The limits as $a \to \infty$ of each of the terms of the form $a^k e^{-a}$ should be given special attention.

Question 1

This question on numerical solutions of first order differential equations proved a welcome opener again, with the majority of students scoring full marks. Almost all students gave their final answer to the specified degree of accuracy. It is advisable for students to show full working with clear substitutions into relevant formulae before carrying out the evaluations. For example, a number of students did not show $k_2 = 0.25 \times \left(\sqrt{2 \times 2.25} + \sqrt{9 + 1.25}\right)$ and instead wrote a similar statement but not with both 4.5 and 10.25 under the square root signs. Without evidence of a correct method, such solutions were heavily penalised.

Question 2

This question on limits was generally answered well. Most students used the expansion of $\sin x$ from the formulae book and replaced x with $2x$ correctly to find the expansion of $\sin 2x$. In part (b) most students used the correct expansion of $\ln(1+kx)$. The majority used their expansion from part (a) in the numerator, although some made an algebraic error in the subtraction of $\sin 2x$. Most then correctly divided numerator and denominator by x^3 before taking the limit. Some errors were seen in rearranging the equation $\frac{4}{3k} = 16$ to find the value of k. Not all students showed sufficient terms in the numerator and/or denominator for full marks to be scored.

Question 3

Most students were able to make good progress with this question which tested finding an area involving a curve whose polar equation was given. The most common error was the use of incorrect limits and this was frequently followed by further errors so as to match the printed answer. A number of students lost the final accuracy mark because their solution involved an incorrect line of working, the most common one being substitution of limits for θ into the mixed expression $\left(\theta + \ln \sec x\right)$.

Question 4

It was pleasing to see that once again most students were able to show that they knew how to find and use a correct integrating factor to solve a first order differential equation. The more common errors were failure to multiply the right-hand side by the integrating factor before integrating, forgetting to divide the constant of integration by the integrating factor when rearranging the general solution into the form y=f(x) or even forgetting to include a constant of integration. Students who answered part (a) correctly were generally successful in part (b).

Question 5

Part (a) was, as expected, generally answered well, with students clearly showing the method of integration by parts. In part (b), a higher proportion of students than in previous papers replaced the upper limit with a constant (for example, a) indicated that $a \to \infty$, and then found $\mathrm{F}(a) - \mathrm{F}(0)$. However, a significant number of students were not explicit in indicating that the limit as $a \to \infty$, $a^k e^{-a} = 0$ and hence they lost the final two marks. This is illustrated by the common error which was just to state that as $a \to \infty$, $e^{-a} \to 0$ so given integral = 2.

Question 6

Most students used the chain rule correctly to obtain the correct answer for part (a) and then the quotient rule to find a correct expression for the second derivative, but not all of these students provided a convincing conversion to the required printed form. In part (c) a smaller proportion of students were completely successful. Students who used the expected method of keeping the right-hand side in terms of e^{-y}, were able to find the third derivative but a significant minority did not then use the chain rule alongside the product rule to find the fourth derivative which frequently led to the incorrect answer $\frac{d^4y}{dx^4} = -e^{-y}\left(\frac{dy}{dx}\right) - (e^{-y})^2$. It was more surprising to see students who had correctly differentiated but then replaced the $-e^{-y}\left(\frac{dy}{dx}\right)^2$ term by $-e^{-y}\left(\frac{d^2y}{dx^2}\right)$ to reach $\frac{d^4y}{dx^4} = 0$. Students who found the third and fourth derivatives in terms of x gained credit but few could then write the fourth derivative in the required form. In part (d), most students displayed good knowledge of Maclaurin's theorem, but a significant number of students failed to score the final mark due to earlier errors.

Question 7

Part (a), on using the given substitution to transform one differential equation into the other, had less structure than in previous sessions. It was pleasing to see that a significant number of students were generally able to present a convincing solution, although once again, it appeared that some students mistakenly believe that the square of the first derivative is the same as the second derivative. The double errors involved are illustrated by the following:

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}t}\ \text{(M1A1)};\quad \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}t}+\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}x}\ \text{(M0)}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}t}+\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}t}\frac{\mathrm{d}y}{\mathrm{d}t}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{x^2}\frac{\mathrm{d}y}{\mathrm{d}t}+\frac{1}{x^2}\frac{\mathrm{d}^2y}{\mathrm{d}t^2}$$

which in most of these cases was carried on to reach the printed answer. Part (b), as expected, was generally answered more successfully than part (a) as students showed that they understood the method of using a complementary function and particular integral to solve a second order differential equation. Errors seen in solutions included incorrect factorising of the auxiliary equation, writing the complementary function in terms of x instead of t, and missing the constant term in the particular integral.

Question 8

The first parts of this question on polar coordinates were generally answered well. In part (a) most students were able to correctly substitute $r\cos\theta$ for x and $r\sin\theta$ for y and then use the relevant double angle formula to convincingly reach the printed equation. In parts (b)(i) and (b)(ii) the coordinates of N, P and Q were frequently found correctly, usually with the polar coordinates given in the correct form. Part (b)(iii) proved, as expected, to be more challenging. However, it was pleasing to see a number of excellent solutions being submitted by students who used a variety of correct strategies. The most common successful method was to find the lengths PQ, PN and QN and then to use the cosine rule to find α. One method did lead to a common error. Students who found PN and then angle ONP using the sine rule did not always realise that angle ONP had to be obtuse (ambiguous case of the sine rule) so it was not uncommon to see $(2\pi - 2\angle ONP)$ evaluated as 3.76, thus giving α as a reflex angle.

Mark Ranges and Award of Grades

Grade boundaries and cumulative percentage grades are available on the Results statistics page of the AQA Website. UMS conversion calculator www.aqa.org.uk/umsconversion