



General Certificate of Education
Advanced Level Examination
January 2010

Mathematics

MFP3

Unit Further Pure 3

Tuesday 19 January 2010 9.00 am to 10.30 am

For this paper you must have:

- a 12-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where

$$f(x, y) = x \ln(2x + y)$$

and

$$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. (3 marks)

(b) Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h f(x_r, y_r)$ and $k_2 = h f(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. (5 marks)

2 (a) Given that $y = \ln(4 + 3x)$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. (3 marks)

(b) Hence, by using Maclaurin's theorem, find the first three terms in the expansion, in ascending powers of x , of $\ln(4 + 3x)$. (2 marks)

(c) Write down the first three terms in the expansion, in ascending powers of x , of $\ln(4 - 3x)$. (1 mark)

(d) Show that, for small values of x ,

$$\ln\left(\frac{4 + 3x}{4 - 3x}\right) \approx \frac{3}{2}x \quad (2 \text{ marks})$$

- 3 (a) A differential equation is given by

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 3x$$

Show that the substitution

$$u = \frac{dy}{dx}$$

transforms this differential equation into

$$\frac{du}{dx} + \frac{2}{x}u = 3 \quad (2 \text{ marks})$$

- (b) Find the general solution of

$$\frac{du}{dx} + \frac{2}{x}u = 3$$

giving your answer in the form $u = f(x)$. (5 marks)

- (c) Hence find the general solution of the differential equation

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = 3x$$

giving your answer in the form $y = g(x)$. (2 marks)

- 4 (a) Write down the expansion of $\sin 3x$ in ascending powers of x up to and including the term in x^3 . (1 mark)

- (b) Find

$$\lim_{x \rightarrow 0} \left[\frac{3x \cos 2x - \sin 3x}{5x^3} \right] \quad (4 \text{ marks})$$

Turn over ►

5 It is given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^{-2x}$$

- (a) Find the value of the constant p for which $y = pxe^{-2x}$ is a particular integral of the given differential equation. *(4 marks)*
- (b) Solve the differential equation, expressing y in terms of x , given that $y = 2$ and $\frac{dy}{dx} = 0$ when $x = 0$. *(8 marks)*

6 (a) Explain why $\int_1^{\infty} \frac{\ln x^2}{x^3} dx$ is an improper integral. *(1 mark)*

(b) (i) Show that the substitution $y = \frac{1}{x}$ transforms $\int \frac{\ln x^2}{x^3} dx$ into $\int 2y \ln y dy$. *(2 marks)*

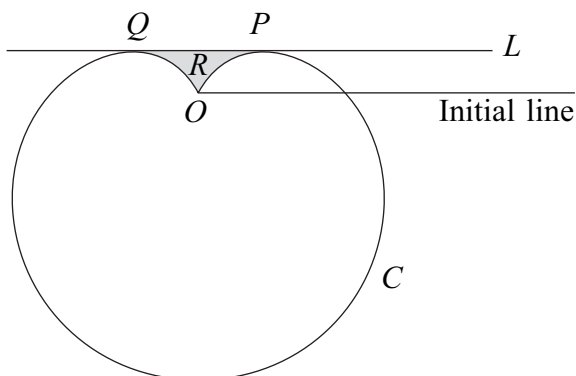
(ii) Evaluate $\int_0^1 2y \ln y dy$, showing the limiting process used. *(5 marks)*

(iii) Hence write down the value of $\int_1^{\infty} \frac{\ln x^2}{x^3} dx$. *(1 mark)*

7 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = 8x^2 + 9 \sin x$$
 (8 marks)

- 8 The diagram shows a sketch of a curve C and a line L , which is parallel to the initial line and touches the curve at the points P and Q .



The polar equation of the curve C is

$$r = 4(1 - \sin \theta), \quad 0 \leq \theta < 2\pi$$

and the polar equation of the line L is

$$r \sin \theta = 1$$

- (a) Show that the polar coordinates of P are $\left(2, \frac{\pi}{6}\right)$ and find the polar coordinates of Q .
(5 marks)
- (b) Find the area of the shaded region R bounded by the line L and the curve C . Give your answer in the form $m\sqrt{3} + n\pi$, where m and n are integers.
(11 marks)

END OF QUESTIONS

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