General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MFP2

Unit Further Pure 2

Friday 15 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Use the definitions $\cosh x = \frac{1}{2}(\mathrm{e}^x + \mathrm{e}^{-x})$ and $\sinh x = \frac{1}{2}(\mathrm{e}^x - \mathrm{e}^{-x})$ to show that

$$\cosh^2 x - \sinh^2 x = 1$$

(3 marks)

(b) (i) Express

$$5\cosh^2 x + 3\sinh^2 x$$

in terms of $\cosh x$. *(1 mark)*

(ii) Sketch the curve $y = \cosh x$. *(1 mark)*

(iii) Hence solve the equation

$$5\cosh^2 x + 3\sinh^2 x = 9.5$$

giving your answers in logarithmic form. *(4 marks)*

2 (a) On the same Argand diagram, draw:

(i) the locus of points satisfying $|z - 4 + 2\mathrm{i}| = 4$; *(3 marks)*

(ii) the locus of points satisfying $|z| = |z - 2\mathrm{i}|$. *(3 marks)*

(b) Indicate on your sketch the set of points satisfying both

$$|z - 4 + 2\mathrm{i}| \leqslant 4$$

and $$|z| \geqslant |z - 2\mathrm{i}|$$ *(2 marks)*

3 The cubic equation

$$2z^3 + pz^2 + qz + 16 = 0$$

where p and q are real, has roots α, β and γ.

It is given that $\alpha = 2 + 2\sqrt{3}\,\mathrm{i}$.

(a) (i) Write down another root, β, of the equation. *(1 mark)*

(ii) Find the third root, γ. *(3 marks)*

(iii) Find the values of p and q. *(3 marks)*

(b) (i) Express α in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(2 marks)*

(ii) Show that

$$(2 + 2\sqrt{3}\,\mathrm{i})^n = 4^n\left(\cos\frac{n\pi}{3} + \mathrm{i}\sin\frac{n\pi}{3}\right)$$ *(2 marks)*

(iii) Show that

$$\alpha^n + \beta^n + \gamma^n = 2^{2n+1}\cos\frac{n\pi}{3} + \left(-\frac{1}{2}\right)^n$$

where n is an integer. *(3 marks)*

4 A curve C is given parametrically by the equations

$$x = \frac{1}{2}\cosh 2t, \qquad y = 2\sinh t$$

(a) Express

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$$

in terms of $\cosh t$. *(6 marks)*

(b) The arc of C from $t = 0$ to $t = 1$ is rotated through 2π radians about the x-axis.

(i) Show that S, the area of the curved surface generated, is given by

$$S = 8\pi\int_0^1 \sinh t\cosh^2 t\,\mathrm{d}t$$ *(2 marks)*

(ii) Find the exact value of S. *(2 marks)*

5 The sum to r terms, S_r, of a series is given by

$$S_r = r^2(r+1)(r+2)$$

Given that u_r is the rth term of the series whose sum is S_r, show that:

(a) (i) $u_1 = 6$; *(1 mark)*

(ii) $u_2 = 42$; *(1 mark)*

(iii) $u_n = n(n+1)(4n-1)$. *(3 marks)*

(b) Show that

$$\sum_{r=n+1}^{2n} u_r = 3n^2(n+1)(5n+2)$$ *(3 marks)*

6 (a) Show that the substitution $t = \tan\theta$ transforms the integral

$$\int \frac{\mathrm{d}\theta}{9\cos^2\theta + \sin^2\theta}$$

into

$$\int \frac{\mathrm{d}t}{9+t^2}$$ *(3 marks)*

(b) Hence show that

$$\int_0^{\frac{\pi}{3}} \frac{\mathrm{d}\theta}{9\cos^2\theta + \sin^2\theta} = \frac{\pi}{18}$$ *(3 marks)*

7 The sequence u_1, u_2, u_3,... is defined by

$$u_1 = 2, \quad u_{k+1} = 2u_k + 1$$

(a) Prove by induction that, for all $n \geqslant 1$,

$$u_n = 3 \times 2^{n-1} - 1$$ *(5 marks)*

(b) Show that

$$\sum_{r=1}^{n} u_r = u_{n+1} - (n+2)$$ *(3 marks)*

8 (a) (i) Show that $\omega = e^{\frac{2\pi i}{7}}$ is a root of the equation $z^7 = 1$. *(1 mark)*

(ii) Write down the five other non-real roots in terms of ω. *(2 marks)*

(b) Show that

$$1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = 0$$ *(2 marks)*

(c) Show that:

(i) $\omega^2 + \omega^5 = 2\cos\frac{4\pi}{7}$; *(3 marks)*

(ii) $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$. *(4 marks)*

END OF QUESTIONS

There are no questions printed on this page

There are no questions printed on this page

There are no questions printed on this page