Teacher Support Materials 2009

Maths GCE

MS03

Question 1

1 An analysis of a random sample of 150 urban dwellings for sale showed that 102 are semi-detached.

An analysis of an independent random sample of 80 rural dwellings for sale showed that 36 are semi-detached.

(a) Construct an approximate 99% confidence interval for the difference between the proportion of urban dwellings for sale that are semi-detached and the proportion of rural dwellings for sale that are semi-detached. *(6 marks)*

(b) Hence comment on the claim that there is no difference between these two proportions. *(2 marks)*

Student Response

1a) $\hat{p}_A = \frac{102}{150} = 0.68$ ✓ $\quad \hat{p}_B = \frac{36}{80} = 0.45$ ✓ B1

$z = \pm 2.5758$ ✓ B1

$CI \simeq (\hat{p}_A - \hat{p}_B) \pm z\sqrt{\frac{\hat{p}_A(1-\hat{p}_A)}{n_A} + \frac{\hat{p}_B(1-\hat{p}_B)}{n_B}}$ ✓ M1

$= (0.68 - 0.45) \pm 2.5758\sqrt{\frac{0.68 \times 0.32}{150} + \frac{0.45 \times 0.55}{80}}$ ✓ m1 A1

$= (0.0564, 0.404)$ ✓ A1

b) Zero doesn't lie within the considered interval ✓ so the claim is invalid ✓ *A bit strong!* B1 B1

8

Commentary

This illustrates a typical good response to this question with the candidate detailing the derivation in part (a); something that was sometimes partly lacking. It is pleasing to see that the final answer has taken note of the request to give final answers to three significant figures. In part (b), the final statement is too definitive but, given that it is the first question on the paper, this was not penalised. Similar definitive conclusions in later questions may well be penalised.

Mark scheme

1(a)	$\hat{p}_1 = \frac{102}{150} = 0.68$ $\hat{p}_2 = \frac{36}{80} = 0.45$	B1		Both CAO
	99% (0.99) $\Rightarrow$ z = 2.57 to 2.58	B1		AWFW (2.5758)
	CI for $(p_1 - p_2)$ is $(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	M1 m1		Use of $(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\text{attempted variance}}$ Use of correct expression for variance
	Thus $(0.68 - 0.45) \pm 2.5758 \times \sqrt{\frac{0.68 \times 0.32}{150} + \frac{0.45 \times 0.55}{80}}$	A1F		F on $\hat{p}_1$, $\hat{p}_2$ and z
	Hence 0.23 ± (0.173 to 0.174) **or** (0.056 to 0.057, 0.403 to 0.404)	A1	6	CAO & AWFW (accept 0.17) AWFW (accept 0.06 & 0.4)
				Note: Pooling of variances Maximum of B1 B1 M1
(b)	Whole of confidence interval is **above 0 or zero** so	B1F		F on (a) Or equivalent
	Disagree with claim / claim appears **doubtful**	B1F	2	F on (a) Or equivalent Dependent on previous B1F
	Total		**8**	

Question 2

2 A hotel chain has hotels in three types of location: city, coastal and country. The percentages of the chain's reservations for each of these locations are 30, 55 and 15 respectively.

Each of the chain's hotels offers three types of reservation: Bed & Breakfast, Half Board and Full Board.

The percentages of these types of reservation for **each** of the three types of location are shown in the table.

		Type of location		
		City	**Coastal**	**Country**
Type of reservation	**Bed & Breakfast**	80	10	30
	Half Board	15	65	50
	Full Board	5	25	20

For example, 80 per cent of reservations for hotels in city locations are for Bed & Breakfast.

(a) For a reservation selected at random:

(i) show that the probability that it is for Bed & Breakfast is 0.34 ; *(2 marks)*

(ii) calculate the probability that it is for Half Board in a hotel in a coastal location; *(2 marks)*

(iii) calculate the probability that it is for a hotel in a coastal location, given that it is for Half Board. *(4 marks)*

(b) A random sample of 3 reservations for Half Board is selected.

Calculate the probability that these 3 reservations are for hotels in different types of location. *(5 marks)*

Student response

Leave blank

2a) i) ~~BP(Bot~~

$P(B \& B) = (0.3 \times 0.8) + (0.55 \times 0.1) + (0.3 \times 0.15)$ ✓ ✓ ✓

$= \cancel{0.304}\ 0.24 + 0.055 + 0.045$

$\cancel{= 0.404} = 0.34$ ✓ 2.

ii) ~~P(HB~~

$P(\text{Coastal} \cap HB) = 0.55 \times 0.65$

$= 0.358$ ✓ 2

iii) $P(\text{Coastal} \mid HB) = \dfrac{P(\text{Coastal} \cap HB)}{P(HB)}$

$= \dfrac{0.358}{(0.3 \times 0.15) + (0.65 \times 0.55) + (0.5 \times 0.15)}$

$= \dfrac{0.358}{0.4775}$ ✓ 4

$= 0.749$

b)

Probability = $P(\text{City} \mid HB) \times P(\text{Coastal} \mid HB) \times P(\text{Country} \mid HB)$

$P(\text{City} \mid HB) = \dfrac{(0.3 \times 0.15)}{(0.3 \times 0.15) + (0.65 \times 0.55) + (0.5 \times 0.15)}$ M1

$= \dfrac{0.045}{0.4775} = 0.0942$ ✓

$P(\text{Country} \mid HB) = \dfrac{0.5 \times 0.15}{(0.3 \times 0.15) + (0.65 \times 0.55) + (0.5 \times 0.15)}$ M1

$= \dfrac{0.075}{0.4775}$ ✓ $= \cancel{0.168}$ 0.158 ✗ 0.157 M1

Probability $= 0.0942 \times 0.749 \times 0.158$ (×6) B0

$\cancel{= 5.40 \times 10^{-4}}$

$= 0.0111$ ✗ A0

11

Commentary

Again, this illustrates a typical response that did not gain full marks. In fact, the awarding of full marks was rare. In common with most candidates, there is a correct reasoning to show the given answer in part (a)(i). In part (a)(ii), again as was the norm, the use of the multiplication law for dependent events is shown correctly. Part (a)(iii) required the application of Bayes' Theorem and here, as was again fairly common, the correct application is demonstrated. Part (b) was not well attempted; the above illustrates one of the better attempts seen. Very few candidates realised that further applications of Bayes' Theorem were required; most simply calculated $0.3 \times 0.55 \times 0.15$ or multiplied this expression by 3. In the above, the candidate, save for a minor numerical slip, has done the difficult part but missed the need to take account of permutations by multiplying by 3! = 6.

Mark Scheme

2(a)(i)	P(B & B) = $(0.30 \times 0.80) + (0.55 \times 0.10) + (0.15 \times 0.30)$	M1		Use of **3 possibilities** each the product of **2 probabilities**
	= 0.24 + 0.055 + 0.045 = 0.34	A1	2	CAO; **AG**
(ii)	P(HB ∩ Coastal) = 0.55 × 0.65	M1		Can be implied by **correct** answer
	= 143/400 or 0.357 to 0.358	A1	2	CAO/AWFW (0.3575)
(iii)	P(Coastal | HB) = $\dfrac{P(\text{Coastal} \cap \text{HB})}{P(\text{HB})}$	M1 M1		$\dfrac{\text{answer to (ii)}}{\sum(3\times2)\text{ probabilities}}$
	$= \dfrac{0.3575}{(0.3\times0.15)+(0.3575)+(0.15\times0.5)}$	A1F		F on (ii)
	$= \dfrac{0.3575}{0.4775}$ = 143/191 or 0.747 to 0.75	A1	4	CAO/AWFW (0.74869)
(b)	P(City | HB) = $\dfrac{0.3\times0.15}{P(\text{HB})} = \dfrac{0.045}{0.4775} = \dfrac{90}{955}$	M1		
	P(Country | HB) = $\dfrac{0.15\times0.5}{P(\text{HB})} = \dfrac{0.075}{0.4775} = \dfrac{30}{191}$	M1		Or $\left(1 - (\text{a})(\text{iii}) - \dfrac{0.045}{0.4775}\right)$
	Thus Probability = $\dfrac{0.045}{P(\text{HB})} \times \dfrac{0.3575}{P(\text{HB})} \times \dfrac{0.075}{P(\text{HB})}$	M1		Multiplication of 3 different probabilities
	Multiplied by 3! = 6	B1		CAO
	$= 0.09424 \times 0.74869 \times 0.15707 \times 6$			
	= 0.063 to 0.068	A1	5	AWFW (0.06649)
	Total		**13**	

Question 3

3 The proportion, p, of an island's population with blood type A Rh^+ is believed to be approximately 0.35.

A medical organisation, requiring a more accurate estimate, specifies that a 98% confidence interval for p should have a width of at most 0.1.

Calculate, to the nearest 10, an estimate of the minimum sample size necessary in order to achieve the organisation's requirement. *(6 marks)*

Student Response

③. $p = 0.35$ 98% CI with width $= 0.1$

Width $= 2 \times z \times$ Standard error ✓ ~~∴ 0.1 = 2 × 2.3263 × 0.3~~ B1 B1

$\therefore 0.1 = 2 \times 2.3263 \times \sqrt{\dfrac{0.35(1-0.35)}{n}}$ ✓ M1 A1

$\therefore \sqrt{\dfrac{0.35(1-0.35)}{n}} = 0.021\ldots$ $\therefore \dfrac{0.35(1-0.35)}{n} = 4.61\ldots \times 10^{-4}$ m1

$\therefore n = \dfrac{0.35(1-0.35)}{4.61\ldots \times 10^{-4}}$ $\therefore n = 492$ (3s.f.)

$\therefore n = 500$ (to nearest 10) ✓ A1

⑥

Commentary

This question was answered correctly or almost so by all but the weakest candidates. Above is an illustration of a typically well-presented solution that shows all the necessary detail. Note that, 492 to the nearest 10, is 490 but here because phrase 'width of at most 0.1', an answer of 500 is acceptable, indeed preferable!

Mark Scheme

Q	Solution	Marks	Total	Comments
3	98% (0.98) CI $\Rightarrow z = 2.32$ to 2.33	B1		AWFW (2.3263)
	CI width is $2 \times z \times \sqrt{\frac{p(1-p)}{n}}$	M1		Used; allow $z \times \sqrt{\frac{p(1-p)}{n}}$
	$p = 0.35$ or 0.50	B1		
	Thus $2 \times 2.3263 \times \sqrt{\frac{0.35 \times 0.65}{n}} = 0.1$	A1F		Or equivalent F on z; allow no multiplier of 2 and/or $p = 0.50$
	Thus $\sqrt{n} = \frac{2 \times 2.3263}{0.1} \times \sqrt{0.35 \times 0.65}$	m1		Solving for $\sqrt{n}$ or n
	Thus $n = 492.5$ ($p = 0.35$) or $n = 541.2$ ($p = 0.50$)			
	Thus to nearest 10 $n = 500$ or 490	A1	6	Either
	Notes: No '$\times 2$' gives $n = 123.1$ No '$\times 2$' and $p = 0.50$ gives $n = 135.3$			
	Total		**6**	

Question 4

4 Holly, a horticultural researcher, believes that the mean height of stems on Tahiti daffodils exceeds that on Jetfire daffodils by more than 15 cm.

She measures the heights, x centimetres, of stems on a random sample of 65 Tahiti daffodils and finds that their mean, $\bar{x}$, is 40.7 and that their standard deviation, s_x, is 3.4.

She also measures the heights, y centimetres, of stems on a random sample of 75 Jetfire daffodils and finds that their mean, $\bar{y}$, is 24.4 and that their standard deviation, s_y, is 2.8.

Investigate, at the 1% level of significance, Holly's belief. *(8 marks)*

Student Response

Blank

4)

$H_0 : \bar{x} - \bar{y} = 15$ $\quad H_1 : \bar{x} - \bar{y} > 15$ Must use μs 0

one-tailed, SL $\alpha = 0.01 \Rightarrow z = 2.3263$ ✓ B1

$$z = \frac{u_x - u_y - (\bar{x} - \bar{y})}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}}$$

~~$u_x = \frac{40.7}{65} = 0.62615$~~

~~$u_y = \frac{24.4}{75} = 0.32533$~~

~~$\Rightarrow z = \frac{0.62615 - 0.32533 - 15}{\sqrt{\frac{3.4^2}{65} + \frac{2.8^2}{75}}} =$~~

$$\Rightarrow z = \frac{40.7 - 24.4 - 15}{\sqrt{\frac{3.4^2}{65} + \frac{2.8^2}{75}}} = 2.45$$ ✓ ✓ ✓ M1 A1 A1 A1

$\Rightarrow 2.45 > 2.3263$ so reject H_0 ✓

There is evidence, at the 1% level of significance, which supports Holly's belief. ✓ A1

6

Commentary

This solution is correct except for the statement of the two hypotheses; an error that resulted in many candidates losing 1 or 2 marks. **It is simply not acceptable to state similar hypotheses in terms of sample means or the word 'means'**. Hypotheses must involve μ

or the phrase ‘population mean’. Here $H_0: \mu_X - \mu_Y = 15$ and $H_1: \mu_X - \mu_Y > 15$ were expected for 2 marks; omission of ‘15’ resulted in 1 mark. The statement of the critical value and the calculation of the test statistic are correct and it is pleasing to see that the conclusion is in context and is qualified rather than being definitive.

4	H_0: $\mu_X - \mu_Y = 15$	B1		Or equivalent Accept H_0: $\mu_X - \mu_Y = 0$
	H_1: $\mu_X - \mu_Y > 15$	B1		Or equivalent
	SL α = 1% (0.01)			
	CV z = 2.32 to 2.33	B1		AWFW (2.3263) If H_1 involves '$\neq$' then accept 2.57 to 2.58 (2.5758)
	CV t = 2.35 to 2.36	(B1)		AWFW If H_1 involves '$\neq$' then accept 2.60 to 2.62
	$z = \dfrac{(\bar{x}-\bar{y})-15}{\sqrt{\dfrac{s_X^2}{n_X}+\dfrac{s_Y^2}{n_Y}}}$ **or** $z/t = \dfrac{(\bar{x}-\bar{y})-15}{\sqrt{s_p^2\left(\dfrac{1}{n_X}+\dfrac{1}{n_Y}\right)}}$	M1		Used Allow 'no –15'
	$s_p^2 = \dfrac{(64\times3.4^2)+(74\times2.8^2)}{65+75-2}$ $= \dfrac{1320}{138} = 9.56522$			$s_p = 3.09277$
	$z = \dfrac{(40.7-24.4)-15}{\sqrt{\dfrac{3.4^2}{65}+\dfrac{2.8^2}{75}}} = \dfrac{1.3}{\sqrt{0.28238}}$	A1 A1		Numerator; allow 'no –15' Denominator
	= 2.44 to 2.45	A1		AWFW (2.4464) 'no –15' gives z = 30.674
	OR			
	$z/t = \dfrac{(40.7-24.4)-15}{\sqrt{\dfrac{1320}{138}\left(\dfrac{1}{65}+\dfrac{1}{75}\right)}} = \dfrac{1.3}{\sqrt{0.27469}}$	(A1) (A1)		Numerator; allow 'no –15' Denominator
	= 2.48	(A1)		AWRT (2.4804) 'no –15' gives z = 31.100
	Thus evidence, at 1% level, to support Holly's belief	A1F	8	F on z and CV
	Total		**8**	

Question 5

5 The random variable X has a binomial distribution with parameters n and p.

(a) Given that

$$\mathrm{E}(X) = np \quad \text{and} \quad \mathrm{E}(X(X-1)) = n(n-1)p^2$$

find an expression for $\mathrm{Var}(X)$. *(3 marks)*

(b) Given that X has a mean of 36 and a standard deviation of 4.8 :

(i) find values for n and p ; *(3 marks)*

(ii) use a distributional approximation to estimate $\mathrm{P}(30 < X < 40)$. *(4 marks)*

Student Response

5. (a)

$E(x(x-1)) = n(n-1)p^2$ and $E(x(x-1)) = E(x^2) - E(x)$

$\therefore n(n-1)p^2 = E(x^2) - np$

$E(x^2) = n(n-1)p^2 + np$ ✓

$Var(x) = E(x^2) - [E(x)]^2$ ✓

$= n(n-1)p^2 + np - n^2p^2$

$= n^2p^2 - np^2 + np - n^2p^2$

$= np - np^2$

$= np(1-p)$ ✓ 3

b) (i) $np = 36$ ✓ $np(1-p) = 23.04$ ✓

$36(1-p) = 23.04$

$(1-p) = 0.64$

$\therefore p = 0.36$ ✓

$\therefore n(0.36) = 36$

$n = 100$ ✓ 3

(ii) $X \sim Bin(100, 0.36)$

$X \sim N(36, 4.8^2)$

$p(30 < x < 40)$

$= p(30.5 < x < 39.5)$ M1

~~$Z = \frac{30.5-36}{4.8/10}$~~ WR ~~$Z = \frac{39.5-36}{4.8/10}$~~ B1

$Z = \frac{30.5-36}{4.8}$ $Z = \frac{39.5-36}{4.8}$

$= -1.15$ ✓ $= 0.729$ ✓ M1

$p(-1.15 < Z < 0.73)$

$= 0.76730 - (1 - 0.87493)$ ✓ A1

$= 0.64223$ ✓

10

Commentary

An illustration of an answer scoring full marks as was often the case. The derivation of the expression for Var(*X*) in part (a) is well documented and does not contain any attempted hidden omissions! Similarly, in part (b)(i), the candidate has realised that $np(1-p)$ has to be equated to 4.8^2 and then used an efficient method to solve the pair of equations for *p* the *n*. In part (b)(ii), after noting correct continuity corrections, incorrect working is deleted and followed by a fully-correct solution.

Mark Scheme

5	$\underline{X \sim B(n,\ p)}$			
(a)	$\text{Var}(X) = E(X^2) - [E(X)]^2$	M1		Used; may be implied
	$= E[X(X-1)] + E(X) - [E(X)]^2$ $= n(n-1)p^2 + np - n^2p^2$	M1		Rearranging & substitution
	$= np - np^2 = np(1-p)$	A1		Or equivalent
	OR			
	$E[X(X-1)] = E(X^2) - E(X)$ $= n(n-1)p^2 = n^2p^2 - np^2$	(M1)		Expansion & substitution
	$\text{Var}(X) = E(X^2) - [E(X)]^2$	(M1)		Used; may be implied
	$= \{n^2p^2 - np^2 + E(X)\} - n^2p^2$			
	$= np - np^2 = np(1-p)$	(A1)	3	Or equivalent
(b)(i)	Mean $= np = 36$ SD $= \sqrt{np(1-p)} = 4.8$	B1		Both CAO
	Thus $36(1-p) = 4.8^2$	M1		Attempt to solve for p or n
	Thus $n = 100$ & $p = 0.36$	A1	3	Both CAO
(ii)	$P(30 < X < 40) =$			
	$P\left(Z < \dfrac{39.5-36}{4.8}\right) - P\left(Z < \dfrac{30.5-36}{4.8}\right) =$	M1 B1		Standardising (39.5, 40 or 40.5) or (29.5, 30 or 30.5) with 36 and 4.8 and/or $(36-x)$ Use of 39.5 & 30.5
	$P(Z < 0.73) - P(Z < -1.15) =$			
	$P(Z < 0.73) - [1 - P(Z < 1.15)] =$	m1		Area change
	$0.76730 - [1 - (0.87286 \text{ to } 0.87493)] =$			
	0.64 to 0.643	A1	4	AWFW (0.64112)
	Total		**10**	

Question 6

6 The table shows the probability distribution for the number of weekday (Monday to Friday) morning newspapers, X, purchased by the Reed household per week.

x	0	1	2	3	4	5
$P(X=x)$	0.16	0.15	0.25	0.25	0.15	0.04

(a) Find values for $E(X)$ and $Var(X)$. *(3 marks)*

(b) The number of weekday (Monday to Friday) evening newspapers, Y, purchased by the same household per week is such that

$$E(Y) = 2.0, \quad Var(Y) = 1.5 \quad \text{and} \quad Cov(X, Y) = -0.43$$

Find values for the mean and variance of:

(i) $S = X + Y$;

(ii) $D = X - Y$. *(5 marks)*

(c) The total cost per week, L, of the Reed household's weekday morning and evening newspapers may be assumed to be normally distributed with a mean of £2.31 and a standard deviation of £0.89.

The total cost per week, M, of the household's weekend (Saturday and Sunday) newspapers may be assumed to be independent of L and normally distributed with a mean of £2.04 and a standard deviation of £0.43.

Determine the probability that the total cost per week of the Reed household's newspapers is more than £5. *(5 marks)*

Student Response

Leave blank

6. a) $E(x) = (1\times0.15)+(2\times0.25)+(3\times0.25)+(4\times0.15)+(5\times0.04)$

$= 2.2$ ✓

$var(x) = E(x^2) - (E(x))^2$

$var(x) = (1\times0.15)+(4\times0.25)+(9\times0.25)+(16\times0.15)+(25\times0.04) - 2.2^2$

$= 6.8 - 4.84$ ✓

$= 1.96$

3

b) i) $S = X + Y$

$E(S) = 4.2$ ✓

$var(S) = var(X) + var(Y) + 2cov(X,Y)$

$= 1.96 + 1.5 + (2\times -0.43)$

$= 2.6$ ✓

ii) $D = X - Y$ ✓

$E(D) = 0.2$

$var(D) = var(X) + var(Y) - 2cov(X,Y)$

$= 1.96 + 1.5 - (2\times -0.43)$

$= 4.32$ ✓

5

c) $L \sim N(2.31, 0.89)$

$M \sim N(2.04, 0.43)$.

Mean B1

$P(L+M > 5)$

Variance B0

$(L+M) \sim N(4.35, 1.32)$ ✓ ✗

$\therefore P(L+M > 5) = 0.28578$?

$= 0.286$ (3sf). No working

0

9

Commentary

The candidate has remembered, from MS2B, how to calculate E(*X*) and Var(*X*) from a probability table for a discrete random variable; Var(*X*) = 6.8 was not unusual! In part (b), the candidate has clearly shown the correct use of the appropriate formulae, given in the formulae booklet under the heading 'Expectation algebra', to gain the full 5 marks. However, sadly in common with other candidates, most of the marks have been lost in part (c). Firstly it appears that the candidate has used SD(*X* + *Y*) = SD(*X*) + SD(*Y*)? Secondly, no subsequent working is shown that leads to the incorrect answer of 0.286. **Candidates are reminded** that unsubstantiated incorrect answers usually result in a loss of most, if not all, the marks available.

Mark Scheme

6(a)	$E(X) = \underline{\mathbf{2.2}}$	B1		CAO
	$Var(X) = E(X^2) - 2.2^2 =$	M1		Used; or equivalent
	$6.8 - 4.84 = 1.96$	A1	3	CAO
(b)(i)	$E(S) = E(X) + 2.0 = 4.2$	B1F		F on (a)
	$Var(S) = Var(X) + 1.5 + 2 \times (-0.43)$	M1		Used for S or D
	$= 2.6$	A1F		F on (a)
(ii)	$E(D) = E(X) - 2.0 = 0.2$	B1F		F on (a)
	$Var(D) = Var(X) + 1.5 - 2 \times (-0.43)$			
	$= 4.32$	A1F	5	F on (a)
(c)	$\underline{L \sim N(2.31,\ 0.89^2) \quad M \sim N(2.04,\ 0.43^2)}$			
	$T = L + M \sim N(\mathbf{4.35},\ \mathbf{0.977})$	B1 B1		Both CAO; SD = 0.98843
	$P(T > 5) = P\left(Z > \dfrac{5 - 4.35}{\sqrt{0.977}}\right)$	M1		Standardising 5 or 5.01 using C's mean & SD
	$= P(Z > 0.66) = 1 - P(Z < 0.66)$	m1		Area change
	0.25 to 0.26	A1	5	AWFW (0.25540)
	Total		**13**	

Question 7

7 The daily number of customers visiting a small arts and crafts shop may be modelled by a Poisson distribution with a mean of 24.

(a) Using a distributional approximation, estimate the probability that there was a total of at most 150 customers visiting the shop during a given 6-day period. *(5 marks)*

(b) The shop offers a picture framing service. The daily number of requests, Y, for this service may be assumed to have a Poisson distribution.

Prior to the shop advertising this service in the local free newspaper, the mean value of Y was 2. Following the advertisement, the shop received a total of 17 requests for the service during a period of 5 days.

(i) Using a Poisson distribution, carry out a test, at the 10% level of significance, to investigate the claim that the advertisement increased the mean daily number of requests for the shop's picture framing service. *(5 marks)*

(ii) Determine the critical value of Y for your test in part (b)(i). *(3 marks)*

(iii) Hence, assuming that the advertisement increased the mean value of Y to 3, determine the power of your test in part (b)(i). *(4 marks)*

Student Response

7. (a) $X \sim Po(24)$ per day $X \sim Po(144)$ 6-day ✓ B1

~~B~~ $\lambda > 15$ $X \sim N(144, 144)$. ✓ M1

$P(X \leq 150) = P(Z < \frac{150 - 144}{\sqrt{144}}) =$ B0

$= P(Z < 0.5) = 0.69146$ ✗ M1

A0

b) $Y \sim Po(2)$ a day $Y \sim Po(10)$ 5-day

i $H_0: \lambda_y = 10$ $H_1: \lambda_y > 10$ B1

~~TS~~ observed 17.

$P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.973$ M1

$= 0.027$ ✓ A1

10% level ————————— 0.1

Reject Ho : ~~The a~~ evidence that the advertisment increases the number of requests for the framing service ✓ M1 A1

ii If $P(X \geq A) \leq 0.1$ $1 - P(X \leq B) \leq 0.1$ M1

$P(X \leq B) \geq 0.9$ $B \geq 14$ M1

critical value of Y is $Y \leq 14$.

if $Y \leq 13$ then the advertise doesn't increase the requests for the service. A0

critical value ~~Y or 14~~ $Y \geq 14$

Question number

Leave blank

iii $P($~~Type II er~~ accept Ho when λ 5days) = 15) B1

Accept Ho when $Y \leq 13$. B1

~~$P(Y \leq 13) = 0.8645$~~ M1

$P(Y \leq 13$ when $\lambda = 15) = 0.3632$.

power $= 1 - 0.3632 = 0.6368$. A0

(13)

Commentary

The answer above illustrates one of the better attempts at this final question since many candidates scored poorly, or not at all, in part (b). Given the overall quality of the answer, it is somewhat surprising that this candidate lost 2 marks in part (a) for omitting the continuity correction; the value of 150 used should have been 150.5 . The fully correct answer to part (b)(i) is quite impressive and unusual as many candidates attempted an approximate z-test with $\lambda = 2$. In part (b)(ii) the candidate has missed the fact that $P(Y \geq CV) \leq 0.1$ equates to $P(Y \leq CV - 1) \geq 0.9$ and so has an answer of 14 rather than 15. Nevertheless, in part (b)(iii), the candidate has attempted correctly, using the CV of 14, to calculate the power and so only lost the final accuracy mark.

Mark Scheme

7	$\underline{X_D \sim \text{Po}(24)}$			
(a)	$T = X_{\Sigma D} \sim \text{Po}(\mathbf{144})$	B1		CAO
	Thus $T \sim$ approx $N(144,\ 144)$	M1		Normal with $\mu = \sigma^2$
	$P(T_{Po} \leq 150) \approx P(T_N < \mathbf{150.5})$	B1		CAO
	$= P\left(Z < \dfrac{150.5 - 144}{12}\right)$	M1		Standardising (149.5, 150 or 150.5) with $\mu > 24$ and $\sqrt{\mu}$
	$= P(Z < 0.54) = 0.705$ to 0.71	A1	5	AWFW (0.70598)
(b)(i)	H_0: λ (or mean) $= 2$ (or 10) H_1: λ (or mean) > 2 (or 10)	B1		Both; or equivalent
	$P(Y \geq 17) = 1 - P(Y \leq 16)$	M1		Accept $1 - P(Y \leq 17)$
	$= 1 - 0.0.9730 = 0.027$	A1		AWRT
	< 0.10 (10%) [$z = 2.05$ to $2.38 > 1.2816$]	M1		Comparison of probability with 0.1 Comparison of z with 1.2816 or 1.6449
	Thus evidence, at 10% level, of increase in mean daily number of requests	A1F	5	F on probability or on z
(ii)	CV of Y is such that $P(Y \geq CV) \leq 0.10$ (10%)	M1		Can be implied by 13, 14 or 15 Accept $P(Y = CV) = 0.10$
	Thus $P(Y \leq CV - 1) \geq 0.90$	M1		Can be implied by 13, 14 or 15 Accept $P(Y = CV) = 0.90$
	Thus CV = 15	A1	3	CAO
(iii)	Power $= 1 - P(\text{Type II error})$ $= 1 - P(\text{accept } H_0 \mid H_0 \text{ false})$ $= P(\text{accept } H_1 \mid H_1 \text{ true})$	B1		Or equivalent Stated or implied use
	$\lambda = 5 \times 3 = 15$	B1		Stated or implied use of Po(15)
	Thus power $= P(Y \geq CV)$	M1		Attempt at a probability based on C's CV from (ii) and Po(15)
	$= P(Y \geq 15) = 1 - P(Y \leq 14)$ $= 1 - 0.4657 = 0.53$ to 0.54	A1	4	AWFW (0.5343)
	Total		**17**	