# Teacher Support Materials 2009

# Maths GCE

# Paper Reference MPC2



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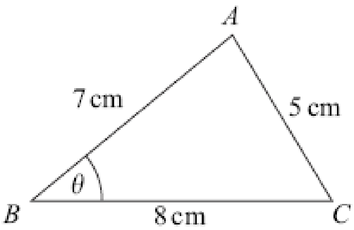
### *Question 1*

**1** The triangle $ABC$, shown in the diagram, is such that $AB = 7\text{ cm}$, $AC = 5\text{ cm}$, $BC = 8\text{ cm}$ and angle $ABC = \theta$.




(a) Show that $\theta = 38.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle $ABC$, giving your answer, in $\text{cm}^2$, to three significant figures. *(2 marks)*

### *Student Response*

Question number | | Leave blank

1(a) $a^2 = b^2 + c^2 - 2bc \cos A$

$5^2 = 7^2 + 8^2 - 2\times7\times8 \cos A$ M1

$25 = 49 + 64 - 2\times7\times8 \cos A$

$\theta = 38.2$ A0 to the nearest 0.1

1

(b) $\frac{1}{2}ab\sin C$

$\frac{1}{2}\times 7\times 8\times \sin 38.2$

$= 17.3\text{ cm}^2$

2

3

**AB08**

### *Commentary*

The candidate realised that the cosine rule was required in part (a) and applied good examination technique by quoting its general form and then substituting the values for the lengths for the award of 1 mark. No rearrangement to find the value of cos *A* (the candidate's cos $\theta$) to an acceptable degree of accuracy has been offered and so no further marks can be awarded for just quoting the value of $\theta$ as printed in the question.

In part (b) the candidate stated the formula for the area of the triangle in a general form, substituted the relevant values and evaluated the expression correctly and gave the final answer to the required degree of accuracy to score the 2 marks.

## *Mark scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** | $5^2 = 7^2 + 8^2 - 2 \times 7 \times 8\cos\theta$ | M1 | | Use of the cosine rule – must be correct (PI by the correct line below) |
| | $\cos\theta = \dfrac{7^2+8^2-5^2}{2\times7\times8}$ $(= \dfrac{88}{112} = 0.7857\ldots)$ | m1 | | Rearrangement |
| | $\theta = 38.21\ldots. = 38.2°$ (to nearest 0.1°) | A1 | 3 | CSO (Must see either exact value for $\cos\theta$ or at least 4sf value for either $\cos\theta$ or $\theta$ before the printed answer 38.2°) AG |
| **(b)** | Area $= \dfrac{1}{2}\times7\times8\sin\theta$ | M1 | | OE eg Area $= \sqrt{10(10-5)(10-8)(10-7)}$ $(= \sqrt{300})$ |
| | $= 17.3$ {cm$^2$} to 3sf | A1 | 2 | Condone 17.31 to 17.33 inclusive |
| | **Total** | | **5** | |

### *Question 2*

**2** (a) Write down the value of $n$ given that $\frac{1}{x^4} = x^n$. *(1 mark)*

(b) Expand $\left(1 + \frac{3}{x^2}\right)^2$. *(2 marks)*

(c) Hence find $\int \left(1 + \frac{3}{x^2}\right)^2 \mathrm{d}x$. *(3 marks)*

(d) Hence find the exact value of $\int_1^3 \left(1 + \frac{3}{x^2}\right)^2 \mathrm{d}x$. *(2 marks)*

### *Student Response*

2a) $\frac{1}{x^4} = x^n \rightarrow n = -4$ — 1

b) $\left(1 + \frac{3}{x^2}\right)^2 \Rightarrow (1 + 3x^{-2})(1 + 3x^{-2})$

$= 1 + 3x^{-2} + 3x^{-2} + 9x^{-4}$

$= 1 + 6x^{-2} + 9x^{-4}$ — 2

c) $\int \left(1 + \frac{3}{x^2}\right)^2 dx \Rightarrow \int 1 + 6x^{-2} + 9x^{-4} \; dx$

$= x + \frac{6x^{-1}}{-1} + \frac{9x^{-3}}{-3}$

$= x - 6x^{-1} - 3x^{-3} + c$ — 3

d) $\int_1^3 \left(1 + \frac{3}{x^2}\right) dx \Rightarrow \left[x - 6x^{-1} - 3x^{-3}\right]_1^3$

$= \left[3 - 6(3)^{-1} - 3(3)^{-3}\right] - \left[1 - 6(1)^{-1} - 3(1)^{-3}\right]$

$= \left[3 - 2 - \frac{1}{9}\right] - \left[1 - 6 - 3\right] = \frac{8}{9} - 1 + 6 + 3$

$= 8\frac{8}{9} = \frac{80}{9}$ — 2

8

### *Commentary*

| |
|---|
| The exemplar shows a correct solution with sufficient details shown. In part (a) the candidate wrote down the correct value of *n* as instructed. In part (b) the candidate changed $\frac{3}{x^2}$ to $3x^{-2}$, in readiness for later integration, and wrote the given expression as a 'product of two brackets' before expanding. Good examination technique was shown by writing down the four resulting terms then simplifying by collecting like terms. In part (c) the candidate correctly integrated the expansion from (b), simplified the coefficients and added the constant of integration. In part (d) the candidate used brackets to emphasise the substitution of 3 and 1 for *x* and the relevant subtraction before doing any further calculations. In the final two lines the calculations are carried out correctly and the candidate recognised the need to give the final answer in an exact form. A common error was to write $\frac{1}{9}$ as 0.111 which resulted in a final answer which was a non exact decimal approximation to $\frac{80}{9}$. |

### *Mark Scheme*

| | | | | |
|---|---|---|---|---|
| **2(a)** | $(n =) -4$ | B1 | 1 | Accept $x^{-4}$ |
| **(b)** | $\left(1+\frac{3}{x^2}\right)^2 = 1+\frac{6}{x^2}+\frac{9}{x^4}$ | B2,1,0 | 2 | Apply ISW after B2 stage<br>(B1 if correct but unsimplified seen) |
| **(c)** | $\int \left(1+\frac{3}{x^2}\right)^2 \mathrm{d}x = x - 6x^{-1} - 3x^{-3} + c$ | M1 | | At least one power of $x$ correctly obtained in the integration of an expansion |
| | | A2,1,0 | 3 | A2 terms correct **and** '$+ c$'<br>(A1F two terms in $x$ correct ft on expansion provided integrating $x$ to a negative power) |
| **(d)** | $\int_1^3 \left(1+\frac{3}{x^2}\right)^2 \mathrm{d}x = \left[x - \frac{6}{x} - \frac{3}{x^3}\right]_1^3$ | | | |
| | $= \left(3 - \frac{6}{3} - \frac{3}{27}\right) - (1-6-3)$ | M1 | | Dealing correctly with limits; F(3) − F(1)<br>(must have attempted integration to get F) |
| | $= 8\frac{8}{9}$ | A1 | 2 | CSO;<br>OE provided value is **exact**, eg $\frac{80}{9}$, $\frac{240}{27}$;<br>ISW dec value after exact value seen<br>NMS scores 0/2 |
| | **Total** | | **8** | |

## *Question 3*

3 The $n$th term of a sequence is $u_n$.

The sequence is defined by

$$u_{n+1} = ku_n + 12$$

where $k$ is a constant.

The first two terms of the sequence are given by

$$u_1 = 16 \qquad u_2 = 24$$

(a) Show that $k = 0.75$. *(2 marks)*

(b) Find the value of $u_3$ and the value of $u_4$. *(2 marks)*

(c) The limit of $u_n$ as $n$ tends to infinity is $L$.

(i) Write down an equation for $L$. *(1 mark)*

(ii) Hence find the value of $L$. *(2 marks)*

## *Student Response*

Leave blank

3) a)

$24 = K(16) + 12$ ✓

$12 = 16K$ ✓

$3/4 = K = 0.75$ ✓ 2

b) $U_3 = 3/4(24) + 12 = 30$ ✓

$U_4 = 3/4(30) + 12 = 34.5$ ✓ 2

c) (i) $\boxed{L = \frac{a}{1-r}}$ ✗ M0 0

(ii) $L = \frac{16}{0.25} = \boxed{64}$ ✗ M0. 0

4

### *Commentary*

The exemplar illustrates a solution which was frequently seen.
In part (a) the candidate substituted the correct values into the formula $u_2 = ku_1 + 12$ and showed sufficient detail in solving the resulting equation to obtain the printed value for *k*. In part (b) the candidate applied the formula $u_{n+1} = \frac{3}{4}u_n + 12$ for *n*=2 and *n*=3 to find the correct values for $u_3$ and $u_4$ respectively. The candidate's solution for part (c) shows a common error. The candidate, by writing $\frac{a}{1-r}$, had incorrectly assumed that the terms in the sequence form an infinite geometric series and that its sum to infinity gives the limiting value of $u_n$. To form an equation for *L*, candidates were expected to replace both $u_n$ and $u_{n+1}$ by *L* in the formula $u_{n+1} = ku_n + 12$ to obtain the equation $L = \frac{3}{4}L + 12$ and then in part (c)(ii) to solve this equation to show that *L*=48. As the wording for part (c)(ii) started with 'Hence', no marks could be awarded for the value of *L* unless the equation for *L* had first been written down.

### *Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)** | $24 = 16k + 12$ | M1 | | Condone with 0.75 (OE) subst for $k$ |
| | $k = 12 \div 16 = 0.75$ | A1 | 2 | AG; OE fraction; if verification must explicitly state the conclusion |
| **(b)** | $u_3 = 30$ | B1 | | |
| | $u_4 = 34.5$ | B1F | 2 | ft on 0.75 × cand's $u_3$ + 12 |
| **(c)(i)** | $L = 0.75L + 12$ | M1 | 1 | Replacing $u_{n+1}$ and $u_n$ by $L$ |
| **(ii)** | $L = \frac{12}{1-k} = \frac{12}{1-0.75}$ | m1 | | PI, but previous M must be scored |
| | $L = 48$ | A1 | 2 | SC: (c)(i) incorrect and then in (c)(ii) $L = 0.75L + 12$ leading to $L = 48$ scores B2 |
| | **Total** | | 7 | |

***Question 4***

4 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 \sqrt{x^3+1}\,dx$, giving your answer to four significant figures. *(4 marks)*

(b) The curve with equation $y = \sqrt{x^3+1}$ is stretched parallel to the $x$-axis with scale factor $\frac{1}{2}$ to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(2 marks)*

***Student Response***

4) a) $\int_0^6 \sqrt{x^3+1}\,dx \approx 1\{(1+\sqrt{217})+2(3+\sqrt{65})\}$

$\approx 37.86$ (4s.f).

$a=0$ $\quad x_0=0 \quad y_0=1$

$b=6 \quad x_1=2 \quad y_1=3$

$n=3 \quad x_2=4 \quad y_2=\sqrt{65}$

$h=\frac{6-0}{3}=2 \quad x_3=6 \quad y_3=\sqrt{217}$

4

b). $y=\sqrt{x^3+1} \quad y=(x^3+1)^{1/2}$

sf ½, x-direction = $y=f(2x)$

$y=\sqrt{(2x)^3+1}$ 2

$y=\sqrt{8x^3+1}$ 6

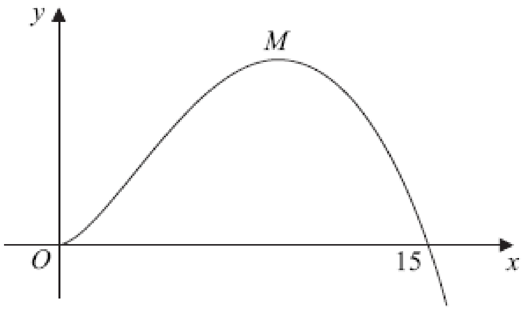
***Commentary***

In lines 3 to 6 the candidate set out all the relevant values for use in the trapezium rule as given on page 8 in the formulae booklet supplied for use in the examination. In the 1st line the candidate substituted these values into the trapezium rule and then evaluated the resulting numerical expression to obtain the correct 4sf value as required. In part (b) the candidate has used brackets appropriately to obtain the correct expression for f($x$). Either form, $\sqrt{(2x)^3+1}$ or $\sqrt{8x^3+1}$ was awarded the 2 marks. If the candidate had left the answer as $y$=f(2$x$), shown in line 2 of part (b), no marks would have been awarded since f($x$) has been defined in a different context within the question. The most common wrong answer was $\sqrt{2x^3+1}$, which was crossed out in line 3 of the candidate's solution. Lack of brackets resulting in $\sqrt{2x^3+1}$ for f($x$) scored 1 mark.

| | | | | |
|---|---|---|---|---|
| **4(a)** | $h = 2$ | B1 | | PI |
| | $g(x) = \sqrt{x^3 + 1}$<br>I $\approx$ $h/2\{\ldots\}$<br>$\{\ldots\} = g(0) + g(6) + 2[g(2) + g(4)]$ | M1 | | OE summing of areas of the 'trapezia'..<br>Can award even if MR expression for $g(x)$ but must be using from 0 to 6 |
| | $\{\ldots\} = 1 + \sqrt{217} + 2(3 + \sqrt{65})$<br>$1 + 14.73... + 2(3 + 8.06\ldots)$ | A1 | | OE Accept 2dp evidence for surds |
| | (I $\approx$) 37.8554… = 37.86 (to 4sf) | A1 | 4 | Must be 37.86 |
| **(b)** | $f(x) = \sqrt{(2x)^3 + 1} = \sqrt{8x^3 + 1}$ | M1 | | $\sqrt{kx^3 + 1}$, $k \neq 1$ or 0 or $f(x) = g(2x)$ |
| | | A1 | 2 | Either form acceptable |
| | **Total** | | **6** | |

***Question 5***

**5** The diagram shows part of a curve with a maximum point $M$.




The equation of the curve is

$$y = 15x^{\frac{3}{2}} - x^{\frac{5}{2}}$$

(a) Find $\frac{dy}{dx}$. *(3 marks)*

(b) Hence find the coordinates of the maximum point $M$. *(4 marks)*

(c) The point $P(1, 14)$ lies on the curve. Show that the equation of the tangent to the curve at $P$ is $y = 20x - 6$. *(3 marks)*

(d) The tangents to the curve at the points $P$ and $M$ intersect at the point $R$. Find the length of $RM$. *(3 marks)*

*Student Response*

Leave blank

5.

a) $y = 15x^{\frac{3}{2}} - x^{\frac{5}{2}}$

$\frac{dy}{dx} = \frac{3}{2} \times 15x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$

$= \frac{45x^{\frac{1}{2}}}{2} - \frac{5x^{\frac{3}{2}}}{2}$ ✓

3

b) When $\frac{dy}{dx} = 0$ then $\frac{45x^{\frac{1}{2}}}{2} - \frac{5x^{\frac{3}{2}}}{2} = 0$ ✓ M1

$45x^{\frac{1}{2}} - 5x^{\frac{3}{2}} = 0$

$9x^{\frac{1}{2}} - x^{\frac{3}{2}} = 0$ ✓

$9x - x^3 = 0$ X M0 $\rightarrow -x^3 + 9x = 0$

~~$(-x^2+3)(x+3)$~~ $x(x+3)(-x+3) = 0$

$x = 0$ OR $x = 3$ ~~or x=~~ $x > 0$ $\therefore \underline{\underline{x = 3}}$

When $x = 3$ $y = 15 \times 3^{\frac{3}{2}} - 3^{\frac{5}{2}} = 62.354$

1

$M = (3, 62.4)$ X

c) At P $x = 1$ When $x = 1$, $\frac{dy}{dx} = \frac{45 \times 1^{\frac{1}{2}}}{2} - \frac{5 \times 1^{\frac{3}{2}}}{2}$ ✓

$= 20$ ✓ $\therefore$ Grad ~~of~~ tangent $= 20$ ✓

PTO

Leave blank

At P, Eq$^{n}$ of tangent $\Rightarrow y-14=20(x-1)$

$\therefore y=20x-6$

3

d) Eq$^{n}$ of tangent at M $\Rightarrow y-62.4=0(x-3)$

$\therefore y=62.4$ B1F

$y=20x-6 \rightarrow 62.4=20x-6$
$y=62.4$

$\therefore 20x=68.4$ M1
$x=3.42$

When $x=3.42$ $y=20\times3.42-6=62.4$

$R(3.42, 62.35)$

$RM=\sqrt{(3.42-3)^2+(62.35-62.35)^2}$

$=0.42$ A1F

3

10

### *Commentary*

The exemplar illustrates the common error in part (b) as well as showing several features of good examination technique.

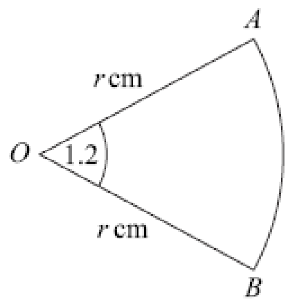
In part (a) the candidate obtained the correct expression for $\frac{dy}{dx}$, showing the process in line 2 before completing the simplification in line 3. In line 1 of part (b) the candidate set up the correct equation to find the *x*-coordinate of the maximum point *M* by equating $\frac{dy}{dx}$ to zero but in line 4 the candidate has either incorrectly squared $9x^{\frac{1}{2}} - x^{\frac{3}{2}}$ (obtaining two terms instead of three) or has incorrectly simplified $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ to get $x^3$ instead of $x^2$. In line 1 of part (c) the candidate showed the method for finding the gradient of the tangent to the curve at *P* and went on to present a convincing solution to obtain the printed equation for the tangent. Even though the candidate has used incorrect coordinates for *M* from part (b), full marks have been awarded for correct follow through work in part (d). The use of the approximation 62.4 for 62.35.. at various stages has been condoned as the candidate's solution shows an appreciation that the *y*-coordinates of *R* and *M* are the same in the relevant formulae. Perhaps the candidate's solution could have been shortened slightly if the candidate had drawn a sketch showing the horizontal tangent at *M* which would have led to the length of *RM* just being the difference in the *x*-coordinates of *R* and *M*.

### *Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 5(a) | $\frac{dy}{dx} = \frac{45}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ | M1<br>A2,1,0 | 3 | One power correctly obtained<br>A1 for each term on the RHS coeffs simplified |
| (b) | $\frac{45}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} = 0$ | M1 | | cand's (a) = 0 |
| | $\frac{5}{2}x^{\frac{1}{2}}(9 - x) = 0$ | m1 | | Must be solving eqn of form $ax^m + bx^n = 0$, $m$ and $n$ non-zero, with at least one of $m$ and $n$ non-integer and reaching a stage from which the non-zero value of $x$ can be stated PI. Must deal with powers of $x$ correctly and any squaring of $kx^p$ terms or expressions must be correct. |
| | At $M$, $x = 9$ | A1 | | |
| | $y_M = 162$ | A1 | 4 | M1 must be scored, else 0/4 |
| (c) | At $P(1, 14)$, $\frac{dy}{dx} = \frac{45}{2} - \frac{5}{2} = 20$ | M1 | | Attempt to find $y'(1)$ |
| | Tangent at $P$: $y - 14 = m(x - 1)$ | m1 | | $m$ = cand's value of $y'(1)$ |
| | $y - 14 = 20x - 20$; $y = 20x - 6$ | A1 | 3 | CSO; AG |
| (d) | Tangent at $M$: $y = 162$ | B1F | | ft $y$ = cand's $y_M$ |
| | At $R$, $162 = 20x - 6$; $x = 8.4$ | M1 | | Solving cand's numerical $y_M = 20x - 6$ to find a value for $x$ |
| | Distance $RM = \lvert x_M - x_R \rvert = 9 - 8.4 = 0.6$ | A1F | 3 | ft on coordinates of $M$ |
| | **Total** | | **13** | |

***Question 6***

6 The diagram shows a sector $OAB$ of a circle with centre $O$ and radius $r$ cm.




The angle $AOB$ is 1.2 radians. The area of the sector is $33.75\text{ cm}^2$.

Find the perimeter of the sector. *(6 marks)*

***Student Response***

6) $\theta = 1\cdot2$
$A = 33\cdot75$

$A = \frac{1}{2}r^2\theta.$ ✓
$33\cdot75 = 0\cdot5 \times r^2 \times 1\cdot2.$ ✓
$33\cdot75 = 0\cdot6r^2$
$r^2 = \frac{33\cdot75}{0\cdot6}$ ✓
$r^2\ 56\cdot25$
$r = 7\cdot5\text{cm}.$ ✓

$s = \theta r$ ✓ $\Rightarrow 1\cdot2 \times 7\cdot5 = 9.$ ✓
$7\cdot5 + 7\cdot5 + 9 = 24\text{cm}.$ ✓
$r + r + s$

6
6

***Commentary***

The exemplar illustrates a correct solution which was frequently seen.

The candidate states the correct general formulae $A = \frac{1}{2}r^2\theta$ and $s = r\theta$ and used the first of these to find an equation in $r^2$ which was then rearranged correctly and the square root taken to find the correct value for $r$. The candidate then used $s = r\theta$ to find the arc length and added twice the radius to obtain the correct value for the perimeter of the sector.

| | | | | |
|---|---|---|---|---|
| **6** | {Area of sector =} $\frac{1}{2}r^2\theta$ | M1 | | $\frac{1}{2}r^2\theta$ seen or used for the area; PI |
| | $r^2 = \frac{33.75}{\frac{1}{2}\theta}$ (= 56.25) | m1 | | Correct rearrangement to $r^2 = \ldots$ or $r = \ldots$ |
| | $r = 7.5$ | A1 | | PI eg by a correct arc length |
| | {Arc =} $r\theta$ | M1 | | $r\theta$ seen or used for the arc length |
| | …. = 9 | A1F | | ft on 1.2 × cand's $r$ provided the **two** M's scored; if not explicit, PI by ft on 3.2 × cand's $r$ for perimeter |
| | {Perimeter =} 24 {cm} | A1 | 6 | CAO |
| | **Total** | | **6** | |

## *Question 7*

7 A geometric series has second term 375 and fifth term 81.

(a) (i) Show that the common ratio of the series is 0.6. *(3 marks)*

(ii) Find the first term of the series. *(2 marks)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) The $n$th term of the series is $u_n$. Find the value of $\sum_{n=6}^{\infty} u_n$. *(4 marks)*

## ***Student Response***

Leave blank

7ai $U_2 = 375$

$U_5 = 81$

$U_n = ar^{n-1}$

$U_2 = ar = 375$ ✓

$U_5 = ar^4 = 81$ ✓

$ar = 375$ 1

$ar^4 = 81$ 2

$r^3 = \cancel{\phantom{00}} 0.216$ ✓ $2 \div 1$ ✓ 3

$r = 0.6$ ✓

7aii $U_1 = U_2 \div r$ ✓

$U_1 = \underline{625} = a$ ✓ $U_1 = a$ 2

7b $S_\infty = \dfrac{a}{1-r}$

$S_\infty = \dfrac{625}{1-0.6}$ ✓

~~$S_\infty$~~ $S_\infty = 1562.5$ ✓ 2

c $\displaystyle\sum_{n=6}^{\infty} U_n = S_\infty - S_6$ X M0 $= \underline{72.9}$

$S_\infty = 1562.5$ A0

$S_6 = \dfrac{625(1-0.6^6)}{1-0.6}$ M1 $= 1489.6$ 1

8

***Commentary***

The exemplar illustrates the common error in part (c).
In part (a)(i) the candidate stated the general formula for the *n*th term of the geometric series as given on page 4 of the formulae booklet and applied it for *n*=2 and *n*=5. The candidate labelled the resulting equations as 1 and 2 and indicated ‘2÷1’ to show how *a* was eliminated and obtained the printed answer for *r*. In part (a)(ii) the candidate obtained the correct value, 625, for the 1st term and in part (b) the candidate showed good examination technique by quoting the general formula for the sum to infinity of the geometric series, as given in the formulae booklet, before substituting the relevant values to obtain the correct value for $S_\infty$. In part (c) the candidate incorrectly stated that $\sum_{n=6}^{\infty} u_n = S_\infty - S_6$. Perhaps if the candidate had written $\sum_{n=6}^{\infty} u_n = u_6 + u_7 + \ldots + u_\infty = (u_1 + u_2 + \ldots + u_5 + u_6 + u_7 + \ldots + u_\infty) - (u_1 + u_2 + \ldots + u_5)$, or something similar, the correct result, $\sum_{n=6}^{\infty} u_n = S_\infty - S_5$, would have been used. The candidate was awarded 1 mark for correctly applying the formula for $S_n$ in the case *n* = 6 but no further marks were available.

## *Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 7(a)(i) | $ar = 375$; $ar^4 = 81$ | B1 | | For either OE or PI by next line |
| | $\Rightarrow 375r^3 = 81$ | M1 | | Elimination of $a$ OE |
| | $r^3 = \frac{81}{375} = \frac{27}{125} = 0.216 \Rightarrow r = 0.6$ | A1 | 3 | CSO AG Full valid completion<br>SC: Clear explicit verification, with statement max B1 out of 3. (If considers uniqueness then 3 is possible) |
| (ii) | $0.6a = 375$<br>$a = 625$ | M1<br>A1 | 2 | OE; PI |
| (b) | $\frac{a}{1-r} = \frac{a}{1-0.6}$ | M1 | | $\frac{a}{1-r}$ **used** with $\lvert$value of $r\rvert < 1$ |
| | $S_\infty = \frac{625}{0.4} = 1562.5$ | A1F | 2 | ft on cand's value for $a$…. ie $2.5 \times a$ |
| (c) | $\sum_{n=6}^{\infty} u_n = \sum_{n=1}^{\infty} u_n - \sum_{n=1}^{5} u_n$ | M1 | | |
| | $u_3 = 0.6u_2$ (= 225) and $u_4 = 0.6^2 u_2$ (= 135) | M1 | | Valid method to either find $u_3$ and $u_4$ or use of $\{S_n =\}\frac{a(1-r^n)}{1-r}$ for either $n = 5$ or $n = 6$ |
| | $\sum_{n=1}^{5} u_n = 625+375+225+135+81$ (= 1441) | A1 | | |
| | $\sum_{n=6}^{\infty} u_n = 1562.5 - 1441 = 121.5$ | A1 | 4 | |
| | **Alternative for (c):** | | | |
| | Recognise that the sum to infinity with first term $u_6$ is required | (M1) | | |
| | Valid method to find $u_6$ $(= 0.6u_5)$ | (M1) | | |
| | $\sum_{n=6}^{\infty} u_n = \frac{81 \times 0.6}{1-0.6}$ | (A1) | | |
| | $= 121.5$ | (A1) | | |
| | **Total** | | **11** | |

***Question 8***

**8** (a) Given that $\dfrac{\sin\theta - \cos\theta}{\cos\theta} = 4$, prove that $\tan\theta = 5$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$2\cos^2 x - \sin x = 1$$

can be written as

$$2\sin^2 x + \sin x - 1 = 0$$ *(2 marks)*

(ii) Hence solve the equation

$$2\cos^2 x - \sin x = 1$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$. *(5 marks)*

***Student Response***

Leave blank

8A) $\frac{\sin\theta - \cos\theta}{\cos\theta} = 4$

$\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} = 4$ $\qquad$ $\frac{\sin}{\cos} = \tan$

$\tan\theta - 1 = 4$ $\qquad$ $\tan\theta = 5$ as req$^d$ $\qquad$ 2

8i) $2\cos^2 x - \sin x = 1$ $\qquad$ $\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$2(1 - \sin^2 x) - \sin x = 1$

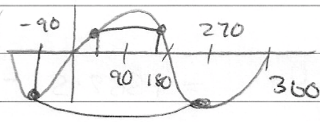
$2 - 2\sin^2 x - \sin x = 1$

$-2\sin^2 x - \sin x = -1$ $\qquad$ $-2\sin^2 x = \sin x - 1$ $\qquad$ 2

$2\sin^2 x + \sin x - 1 = 0$ as req$^d$

ii) $a = 2,\ b = 1,\ c = -1$ $\qquad$ $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad$ $0 \leq x \leq 360$

$\frac{-1 \pm \sqrt{1 + 8}}{4} = \frac{1}{2}$ and $-1$




$\sin x = \frac{1}{2}$, $x = 30, 150$

$\sin x = -1$, $x = -90, 270$

$x = 30°, 150°, 270°$ $\qquad$ 5

9

***Commentary***

The exemplar illustrates a correct solution showing sufficient working to justify the printed results in parts (a) and (b)(i).
In part (a) the candidate split the left-hand-side, stated the identity for tan and used it correctly to convincingly obtain the printed result $\tan\theta = 5$. In part (b)(i) the candidate stated and used the identity $\cos^2 x + \sin^2 x = 1$ in a convincing manner by starting with the given equation $2\cos^2 x - \sin x = 1$, using appropriate brackets when replacing $\cos^2 x$ by $1 - \sin^2 x$ and subsequently inserting sufficient steps to convince the examiner that the given equation could be written as $2\sin^2 x + \sin x - 1 = 0$. In part (b)(ii) the candidate recognised that the solutions of the equation $2\cos^2 x - \sin x = 1$ could be found by solving the quadratic equation $2\sin^2 x + \sin x - 1 = 0$. The candidate quoted and used the general quadratic formula to obtain the two values for sin $x$ from which the correct three solutions in the interval $0^\circ \le x \le 360^\circ$ were found.

***Mark Scheme***

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 8(a) | $\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} = 4$ | | | |
| | $\tan\theta - 1 = 4$ | M1 | | $\tan\theta = \frac{\sin\theta}{\cos\theta}$ stated or used |
| | $\tan\theta = 5$ | A1 | 2 | AG; CSO |
| (b)(i) | $2\cos^2 x - \sin x = 1$<br>$2(1 - \sin^2 x) - \sin x = 1$ | M1 | | Use of $\cos^2 x + \sin^2 x = 1$ |
| | $2 - 2\sin^2 x - \sin x = 1$<br>$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$ | A1 | 2 | AG; CSO |
| (ii) | $(\sin x + 1)(2\sin x - 1) = 0$ | M1 | | Factorisation or use of formula; PI by **both** correct values for $\sin x$ |
| | $\sin x = -1,\quad \sin x = 0.5$ | A1 | | Need both |
| | $(\sin x = -1)$ so $x = 270°$ | B1 | | |
| | $(\sin x = 0.5)$ so $x = 30°$ | A1 | | 30° as the only acute angle |
| | $x = 180 - 30 = 150°$ | B1F | 5 | ft for 2nd angle from c's $\sin x$ = non-integer |
| | | | | Ignore values outside interval 0°–360° but extras inside interval lose the corresp. B, A or B1F mark. If using rads, accepting either equivalent exact vals (in terms of pi) or 2dp values instead of degrees, penalise max of 1 mark from any of the final three marks (B1A1B1F) awarded NMS: 270° (B1); 30°, 150° (B1) [max 2/5] |
| | **Total** | | **9** | |

### *Question 9*

**9** (a) (i) Find the value of $p$ for which $\sqrt{125} = 5^p$. *(2 marks)*

(ii) Hence solve the equation $5^{2x} = \sqrt{125}$. *(1 mark)*

(b) Use logarithms to solve the equation $3^{2x-1} = 0.05$, giving your value of $x$ to four decimal places. *(3 marks)*

(c) It is given that

$$\log_a x = 2(\log_a 3 + \log_a 2) - 1$$

Express $x$ in terms of $a$, giving your answer in a form not involving logarithms. *(4 marks)*

***Student Response***

9Ai) $\sqrt{125} = \sqrt{5^3} = 5^{3/2}$ M1 A1

ii) $5^{2x} = 5^{3/2}$ $2x = \frac{3}{2}$ $x = \frac{3}{4}$ 1

B) $3^{2x-1} = 0.05$

$\log 3^{2x-1} = \log 0.05$

$(2x-1)\log 3 = \log 0.05$

$2x-1 = \frac{\log 0.05}{\log 3} = -2.726833028$

$2x = -1.726833028$ $x = -0.8634165139$ 3

$x = -0.8634$ (4dp)

c) $\log_a x = 2(\log_a 3 + \log_a 2) - 1$

$\log_a x = 2(\log_a 6) - 1$

$\log_a x = \log_a 6^2 - 1$ $\log_a x = (\log_a 36) - 1$

$\cancel{a^{\log}}$ $\log_a x = \log_a 36 - \log_a a$

$\log_a x = \log_a \frac{36}{a}$ 4

$x = \frac{36}{a}$

9

### *Commentary*

In part (a)(i) the candidate correctly wrote $\sqrt{125}$ in the form $5^p$ but did not explicitly find the value of *p*. Examiners expected candidates to write '$p = 1.5$' or '$p = \frac{3}{2}$' for their answer to part (a)(i). In part (a)(ii) the candidate used the work from part (a)(i) to find the correct value for *x*. In part (b) the candidate clearly used logarithms to solve the given equation and applied good examination technique by showing values in the intermediate working which were to a greater degree of accuracy than that requested for the final value of *x*. In part (c) the candidate in lines 2 and 3 applied two laws of logarithms correctly and in line 4 goes beyond the stage reached by most candidates, replacing −1 by $-\log_a a$. The candidate in line 5 used the remaining law of logarithms correctly, writing $\log_a 36 - \log_a a$ as $\log_a \frac{36}{a}$, and finally expressed *x* correctly as $\frac{36}{a}$.

### *Mark Scheme*

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 9(a)(i) | $\sqrt{125} = \sqrt{25\times5} = 5\sqrt{5}$ | M1 | | OE eg $\sqrt{125} = \sqrt{5^3}$ or $5^{1.5}$ seen |
| | $5^p = \sqrt{125} \Rightarrow p = 1.5$ | A1 | 2 | Correct value of *p* must be explicitly stated |
| | **Alternative for (a)(i):** | | | |
| | $p\log 5 = \frac{1}{2}\log 125$ | (M1) | | OE eg $p\log 5 = \log 11.18$<br>or eg $p = \log_5 \sqrt{125}$ |
| | $p\log 5 = \frac{3}{2}\log 5 \Rightarrow p = \frac{3}{2}$ | (A1) | | Correct value of *p* must be explicitly stated |
| (ii) | $5^{2x} = \sqrt{125} = 5^p \Rightarrow x = 0.5p = 0.75$ | B1F | 1 | Must be 0.5 × c's value of *p*<br>SC: $x = 0.75$ with c's ans (a)(i) $5^{1.5}$ scores B1F |
| (b) | $3^{2x-1} = 0.05$<br>$(2x-1)\log 3 = \log 0.05$ | M1 | | Take logs of both sides and use 3rd law of logs. PI eg by $2x - 1 = \log_3 0.05$ seen |
| | $x = \frac{\log_{10} 0.05}{2\log_{10} 3} + \frac{1}{2}$ | m1 | | Correct rearrangement to $x = \ldots$ PI |
| | $= -0.8634(165\ldots) = -0.8634$ to 4dp | A1 | 3 | Condone > 4dp. Must see logs clearly used in solution, so NMS scores 0/3 |
| (c) | $\log_a x = 2(\log_a 3 + \log_a 2) - 1$ | | | |
| | $= 2\log_a(3\times2) - 1$ | M1 | | A valid law of logs used |
| | $= \log_a(6^2) - 1$ | M1 | | Another valid law of logs used |
| | $= \log_a 36 - \log_a a$ | B1 | | $\log_a a = 1$ quoted or used<br>or $\log_a \frac{x}{k} = -1 \Rightarrow \frac{x}{k} = a^{-1}$ OE |
| | $\log_a x = \log_a\left(\frac{36}{a}\right) \Rightarrow x = \frac{36}{a}$ | A1 | 4 | CSO Must be $x = \frac{36}{a}$ or $x = 36a^{-1}$ |
| | **Total** | | **10** | |