Teacher Support Materials 2009

Maths GCE

Paper Reference MPC1

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Question 1

1 The line AB has equation $3x + 5y = 11$.

(a) (i) Find the gradient of AB. *(2 marks)*

(ii) The point A has coordinates $(2, 1)$. Find an equation of the line which passes through the point A and which is perpendicular to AB. *(3 marks)*

(b) The line AB intersects the line with equation $2x + 3y = 8$ at the point C. Find the coordinates of C. *(3 marks)*

Student Response

1

AB $3x + 5y = 11$

$5y = 11 - 3x$ $y = \frac{11}{5} - \frac{3}{5}x$

gradient $= -3/5$. ✓

A $(2, 1)$ perpendicular so gradient is the negative reciprocal. $-3/5 \times 5/3 = -1$.

gradient $= 5/3$ $y = mx + c$. $1 = 5/3 \times 2 + c$ $1 = 10/3 + c$

$c = 3/3 - 10/3$ $c = -7/3$ $y = 5/3\,x - 7/3$ ✓

$3y = 5x - 7$

b) $3x + 5y = 11$ $\times 2$ → $6x + 10y = 22$

$2x + 3y = 8$ $\times 3$. $6x + 9y = 24$ −

$1y = -2$

$y = -2$.

when $y = -2$

$3x + 5(-2) = 11$.

$3x - 10 = 11$

$3x = 21$ $\underline{(7, -2)}$ ✓

$x = 7$

Commentary

This was a very good solution to the first question.
(a) (i)The candidate showed all the steps clearly when making *y* the subject of the straight line equation. Often those who tried to do this work mentally made mistakes. It was pleasing to see this candidate making an actual statement about the gradient of *AB;* often a value appeared as if from thin air and this was often incorrect. Common wrong answers for the gradient were 3/5 and –3.
(ii)There is then a blank line before the next part of the question is attempted and a clear explanation was given as to how the gradient of the perpendicular line had been found. The equation *y=mx+c* was used here with all the relevant working and full marks would have been scored for the equation on the penultimate line. It was not necessary here to obtain an equation with integer coefficients.
(b) The correct two equations have been written down before multiplying by 2 and 3 respectively so as to form equations with the same coefficient of *x*. Full marks were scored for the correct values of *x* and *y* . The candidate then wrote down the correct coordinates and it would have been even better if a statement such as "the coordinates of C are (7,–2)" had been included.

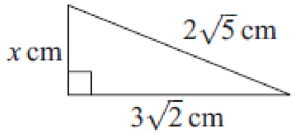
Mark scheme

Q	Solution	Marks	Total	Comments
1(a)(i)	$y=-\frac{3}{5}x+\frac{11}{5}$ Or correct expression for gradient using two correct points	M1		Attempt at $y = f(x)$ Or answer $=\frac{3}{5}$ or $-\frac{3}{5}x$ gets M1 But answer of $\frac{3}{5}x$ gets M0
	(Gradient of *AB* =) $-\frac{3}{5}$	A1	2	Correct answer scores 2 marks . Condone error in rearranging formula if answer for gradient is correct.
(ii)	$m_1m_2=-1$	M1		Used or stated
	Gradient of perpendicular = $\frac{5}{3}$	A1√		ft their answer from (a)(i) or correct
	$y-1=\frac{5}{3}(x-2)$ OE	A1	3	$5x-3y=7$; or $y=\frac{5}{3}x+c$, $c=-\frac{7}{3}$ etc CSO
(b)	Eliminating x or y but must use $3x+5y=11$ & $2x+3y=8$	M1		An equation in x only or y only
	$x = 7$	A1		
	$y=-2$	A1	3	Answer only of $(7,-2)$ scores 3 marks
	Total		8	

Question 2

2 (a) Express $\dfrac{5+\sqrt{7}}{3-\sqrt{7}}$ in the form $m+n\sqrt{7}$, where m and n are integers. *(4 marks)*

(b) The diagram shows a right-angled triangle.



The hypotenuse has length $2\sqrt{5}$ cm. The other two sides have lengths $3\sqrt{2}$ cm and x cm. Find the value of x. *(3 marks)*

Student Response

2a)

$\dfrac{5+\sqrt{7}}{3-\sqrt{7}} \times \dfrac{3+\sqrt{7}}{3+\sqrt{7}}$

Bottom → $9 - 7 = 2$

Top → $15 + 5\sqrt{7} + 3\sqrt{7} + 7 = 22 + 8\sqrt{7}$

$\dfrac{22+8\sqrt{7}}{2} = 11 + 4\sqrt{7}$ ✓

b)

$a^2 + b^2 = c^2$

$(2\sqrt{5})(2\sqrt{5})$ $(3\sqrt{2})(3\sqrt{2})$

$4 \times 5 = 20$ B1 $= 18$

$\sqrt{20} = \sqrt{18} + x$ M0

$x = \sqrt{2}$ FIW

Commentary

(a) The candidate correctly multiplied both the numerator and the denominator by the conjugate $3+\sqrt{7}$ and then evaluated these separately before combining the terms into a single fraction. The final answer was also correct and full marks were scored for this part.
(b) At first glance you might think that the candidate would have scored full marks for obtaining the correct value of *x*, but a double error has been made. Credit was given for finding the squares of the two surd expressions but a correct statement of Pythagoras's Theorem involving *x* is not seen. The candidate should have written "$20=18+x^2$" and it is a warning to candidates that simply getting the correct answer does not automatically result in full marks. The acronym FIW use by the marker flags up "from incorrect working".

Mark Scheme

2(a)	$\frac{5+\sqrt{7}}{3-\sqrt{7}}\times\frac{3+\sqrt{7}}{3+\sqrt{7}}$	M1		
	Numerator = $15+5\sqrt{7}+3\sqrt{7}+7$	m1		Condone one error or omission
	Denominator = $9-7$ $(=2)$	B1		Must be seen as the denominator
	(Answer =) $11+4\sqrt{7}$	A1	4	
(b)	$\left(2\sqrt{5}\right)^2=20$ *or* $\left(3\sqrt{2}\right)^2=18$	B1		Either correct
	their $\left(2\sqrt{5}\right)^2-\left(3\sqrt{2}\right)^2$	M1		Condone missing brackets and x^2
	$(x^2=20-18)$			$x^2=2 \Rightarrow$ B1, M1
	$(\Rightarrow x=)\ \sqrt{2}$	A1	3	$\pm\sqrt{2}$ scores A0 Answer only of 2 scores B0, M0 Answer only of $\sqrt{2}$ scores 3 marks
	Total		7	

Question 3

3 The curve with equation $y = x^5 + 20x^2 - 8$ passes through the point P, where $x = -2$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Verify that the point P is a stationary point of the curve. *(2 marks)*

(c) (i) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P. *(3 marks)*

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point. *(1 mark)*

(d) Find an equation of the tangent to the curve at the point where $x = 1$. *(4 marks)*

Student Response

Leave blank

$5x^4 + 40x$

$5(-2)^4 + 40(-2)$

$5 \times 16 - 80$. $\therefore$ when $x = -2$

$80 - 80 = 0$ there is a stationary point. ✓ 2

(c)(i) $\frac{d^2y}{dx^2} = 20x^3 + 40$. ✓

ii) $20(-2)^3 + 40$ 3

$= -160 + 40 = -120$ ✓ $< 0 \therefore$ maximum ✓ 1

d) $y = x^5 + 20x^2 - 8$.

$y = 1^5 + 20(1)^2 - 8$

$= 1 + 20 - 8 = 13$ ✓ B1

$5x^4 + 40x$

$y = mx + c$. M1 $5(1)^4 + 40(1)$

$5 + 40 = 45$.

$13 = 45 \times 1 + c$ 3

$c = 13 - 45$

$c = -38$

M1

$y = 45x - 38$. A0 12

Commentary

This solution was generally very good.
(a) On the previous page the candidate had scored full marks for the correct first derivative.
(b)The working shown here is a good example of the essential steps to include when verifying that a curve has a stationary point. If the candidate had simply written "=0" after the second line of working then this would not have scored full marks; also if the statement regarding a stationary point had not been included then this would have also denied the candidate full marks.
(c) The second derivative was correct and credit was given for answering parts(i) and (ii) together. Strictly speaking, the candidate has not really answered part(i) before attempting part (ii) since the request was to "find the value" of the second derivative at the point *P*.
(d) The candidate realised the need to find the gradient of the curve in order to find the gradient of the tangent. A mistake was made when trying to find the value of *c* in the equation *y=mx+c.* Had the candidate used an alternative form for the straight line and simply written *y*-13=45(*x*-1) , then full marks would have been scored for this part of the question. Examiners keep emphasising that perhaps candidates could benefit from learning more than one form for the equation of a straight line.

Mark Scheme

3(a)	$\frac{dy}{dx} = 5x^4 + 40x$	M1 A1 A1	3	One of these powers correct One of these terms correct All correct (no + c etc)
(b)	$x = -2 \quad \frac{dy}{dx} = 5\times(-2)^4 + (40\times -2)$	M1		Substitute $x = -2$ into their $\frac{dy}{dx}$
	$\frac{dy}{dx} = 5\times 16 + (40\times -2) = 0$			
	$\Rightarrow P$ is stationary point	A1		CSO Shown = 0 plus statement eg "st pt", "as required", "grad = 0"etc
	Or *their* $\frac{dy}{dx} = 0 \quad \Rightarrow x^n = k$	(M1)		
	$x^3 = -8 \quad \Rightarrow x = -2$	(A1)	2	CSO $x = 0$ need not be considered
(c)(i)	$\frac{d^2y}{dx^2} = 20x^3 + 40$	B1√		Correct ft their $\frac{dy}{dx}$
	$= 20\times(-2)^3 + 40$	M1		Subst $x = -2$ into their second derivative
	$(= -160 + 40) \qquad = -120$	A1	3	CSO
(ii)	Maximum (value) their c(i) answer must be < 0 Other valid methods acceptable provided "maximum" is the conclusion	E1√	1	Accept minimum if their *c(i) answer* > 0 and correctly interpreted Parts (i) and (ii) may be combined by candidate but −120 must be seen to award A1 in part (c)(i)
(d)	(When $x = 1$) $y = 13$	B1		
	When $x = 1$, $\frac{dy}{dx} = 5 + 40$	M1		Sub $x = 1$ into their $\frac{dy}{dx}$
	$y =$ (their 45)$x + k$ OE	m1		ft their $\frac{dy}{dx}$
	Tangent has equation $y - 13 = 45(x - 1)$	A1	4	CSO OE $y = 45x + c, \quad c = -32$
	Total		**13**	

Question 4

4 (a) The polynomial $p(x)$ is given by $p(x) = x^3 - x + 6$.

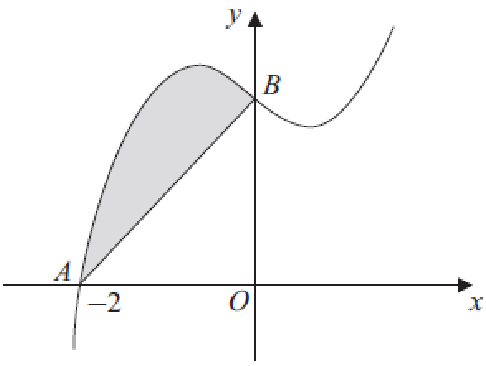
(i) Find the remainder when $p(x)$ is divided by $x - 3$. *(2 marks)*

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $p(x)$. *(2 marks)*

(iii) Express $p(x) = x^3 - x + 6$ in the form $(x+2)(x^2 + bx + c)$, where b and c are integers. *(2 marks)*

(iv) The equation $p(x) = 0$ has one root equal to -2. Show that the equation has no other real roots. *(2 marks)*

(b) The curve with equation $y = x^3 - x + 6$ is sketched below.



The curve cuts the x-axis at the point $A(-2, 0)$ and the y-axis at the point B.

(i) State the y-coordinate of the point B. *(1 mark)*

(ii) Find $\int_{-2}^{0} (x^3 - x + 6)\,dx$. *(5 marks)*

(iii) Hence find the area of the shaded region bounded by the curve $y = x^3 - x + 6$ and the line AB. *(3 marks)*

Student Response

4a) $P(x) = x^3 - x + 6$

i) $(-3)^3 - 3 + 6 = -27 - (3) + 6$ M0

$= -18$ **0**

ii) $(-2)^3 - (-2) + 6 = -8 + 2 + 6 = 0$ M1

$\therefore (x+2) =$ factor A1 **2**

iii) $x^3 - x + 6 = (x+2)(x^2 - 2x + 3)$ ✓ **2**

iv) $b^2 - 4ac = (-2)^2 - 4 \times -(2)^3 \times 6$ M0 **0**

$= 4 - 4 \times -8 \times 6 = -32 \times 6 \neq 0 \therefore$ no other real roots

b) i) $y = x^3 - x + 6$

$y = \underline{\underline{6}}$ at b ✓ B1

ii) $\int_{-2}^{0} (x^3 - x + 6)$

M1 A1 A1

$= \left[\frac{x^4}{4} - \frac{x^2}{2} + 6x \right]_{-2}^{0}$

$0 - \frac{(-2)^4}{4} - \frac{a(-2)^2}{2} + 6(-2)$ ✓ M1

$= \frac{16}{4} - 2 - 12$

A0

$= 4 - 14 = -10$ Area under curve $= \underline{\underline{10}}$

Commentary

(a)(i) The candidate should have used p(3) in order to find the remainder when p(x) is divided by x–3 and consequently no marks were scored for finding p(–3).
(ii) Here sufficient working was shown to demonstrate that p(–2) = 0 and a statement was made about x+2 being a factor and so both marks were earned.
(iii) Many candidates scored full marks for writing down the quadratic factor by inspection as seen here.
(iv) A common mistake was to use the coefficients of the cubic when calculating the discriminant b^2–4ac. In this case it is not exactly clear where the candidate has obtained the values since b = –2, a = –2 and c = 6 have been used. Credit was only given here for a correct discriminant with a correct conclusion about there being no real roots.
(b)(i) The correct y-coordinate of B was stated.
(ii) The individual terms were integrated correctly but the omission of brackets caused problems. The working clearly results in an incorrect answer of –10 and so, even though there was some attempt to rectify this, it could not earn full marks, despite the correct value for the integral actually being 10.

Mark Scheme

4(a)(i)	$\mathrm{p}(3)=27-3+6$ (Remainder) = 30 **Or** long division up to remainder Quotient= x^2+3x+8 and remainder = 30 clearly stated or indicated	M1 A1 (M1) (A1)	 2	p(3) attempted
(ii)	$\mathrm{p}(-2)=-8+2+6$ $\mathrm{p}(-2)=0 \Rightarrow x+2$ is factor Minimum statement required "factor"	M1 A1	 2	p(−2) attempted : **NOT** long division Shown = 0 plus statement May make statement *first* such as "x+2 is a factor if p(−2) = 0"
(iii)	$b=-2$ $c=3$ or long division/comparing coefficients $\mathrm{p}(x)=(x+2)(x^2-2x+3)$	B1 B1 (M1) (A1)	 2	No working required for B1 + B1 Try to mark first using B marks Award M1 if B0 earned and a clear method is used Must write final answer in this form if long division has been used to get A1
(iv)	$b^2-4ac=(-2)^2-4\times3$ $b^2-4ac=-8$ (or <0) $\Rightarrow$ no (other) real roots **Or** $(x-1)^2+2$ $(x-1)^2+2>0$ therefore no real roots Or $(x-1)^2=-2$ has no real roots	M1 A1 (M1) (A1)	 2	Discriminant correct from their quadratic M0 if $b=-1$, $c=6$ used (using cubic eqn) CSO All values must be correct plus statement Completion of square for their quadratic Shown to be positive plus statement regarding no real roots
(b)(i)	$(y_B=)\ 6$	B1	1	Condone (0, 6)
(ii)	$\frac{x^4}{4}-\frac{x^2}{2}+6x$ $\left[\ \right]_{-2}^{0}=0-(4-2-12)$ $=10$	M1 A1 A1 m1 A1	 5	One term correct Another term correct All correct (ignore + c or limits) F(−2) attempted CSO Clearly from F(0) − F(−2)
(iii)	Area of $\Delta=\frac{1}{2}\times2\times6$ $=6$ Shaded region area = 10 − 6 = 4	M1 A1 A1	 3	Condone − 2 and ft their y_B value **Or** $\int_{-2}^{0}(3x+6)\mathrm{d}x$ and attempt to integrate Must be positive allow −6 converted to +6 CSO 10 must come from correct working
	Total		**17**	

Question 5

5 A circle with centre C has equation

$$(x-5)^2+(y+12)^2=169$$

(a) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the circle passes through the origin O. *(1 mark)*

(ii) Given that the circle also passes through the points $(10, 0)$ and $(0, p)$, sketch the circle and find the value of p. *(3 marks)*

(c) The point $A(-7, -7)$ lies on the circle.

(i) Find the gradient of AC. *(2 marks)*

(ii) Hence find an equation of the tangent to the circle at the point A, giving your answer in the form $ax+by+c=0$, where a, b and c are integers. *(3 marks)*

Student Response

5 a) $(x-5)^2 + (y+12)^2 = 169$

(i) $C = (5, -12)$

(ii) $r = \sqrt{169} = 13$

b) (i) $(0-5)^2 + (0+12)^2 = 169$

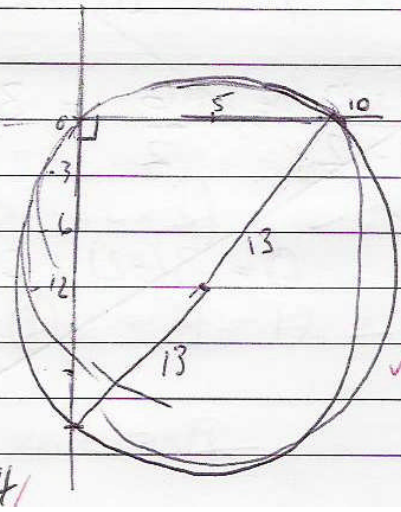
$(-5)^2 + (12)^2 = 169$

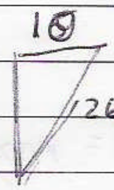
$25 + 144 = 169$

B0

(ii)

$26 \times 20 = 520$
$26 \times 6 = 156$
676





$26^2 = 10^2 + p^2$

$676 = 100 + p^2$

$p^2 = 576$

$p = \sqrt{576}$ $p = -24$

$20 \times 20 = 400$
$24 \times 4 = 96$

c) $A(-7,-7)$ $C(5,-12)$

(i)

$G = \frac{-12(-)7}{5--7} = \frac{-19}{12}$ M0

(ii)

$G = \frac{12}{19}$ B1 M1 $A(-7,-7)$

$y = mx + c$

$y = \frac{12}{19}x + c$

$-7 = \frac{12}{19}(-7) + c$

$-7 = -\frac{84}{19} + c$

$\frac{-133}{19} + \frac{84}{19} = c$ $c = \frac{-49}{19}$

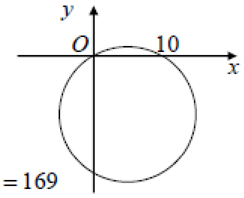
$y = \frac{12}{19}x - \frac{49}{19}$ $19y = 12x - 49$

A0

$12x - 19y - 49 = 0$

Commentary

(a) The correct coordinates of the centre were stated and the radius was also correct.
(b)(i) The mark for verifying that the circle passed through *O* was only awarded when candidates wrote a suitable conclusion after correct working; in this case no concluding statement was written by the candidate.
(ii)The circle (after a couple of attempts) was drawn through the origin and cut the *x*-axis and the *y*-axis as required. The candidate then used the geometry of the circle to produce a right angled triangle in order to find where the circle crossed the *y*-axis. Having written $p^2 = 576$, the candidate realised that *p* must be negative and so full marks were given for a correct final answer of $p = -24$.
(c) A careless arithmetic slip with a minus sign prevented the candidate from finding the correct gradient of *AC*. Nevertheless, credit was given for finding the negative reciprocal in order to obtain an equation for the tangent to the circle.

5(a)(i)	$C\ (5, -12)$	B1	1	
(ii)	Radius = 13 (or $\sqrt{169}$)	B1	1	$\pm\sqrt{169}$ or ± 13 as final answer scores B0
(b)(i)	$(-5)^2 + 12^2$ *or* $25 + 144$			
	$= 169 \Rightarrow$ circle passes through O	B1	1	Correct arithmetic plus statement Eg "O lies on circle", "as required" etc
(ii)	Sketch 	B1		Freehand circle through origin and cutting positive x-axis with centre in 4th quadrant Condone value 10 missing or incorrect
	$25 + (p + 12)^2 = 169$	M1		Or doubling their y_C-coordinate
	$(p + 12) = \pm 12 \qquad p = -24$	A1	3	Condone use of y instead of p SC B2 for correct value of p stated or marked on diagram
(c)(i)	grad $AC = \dfrac{-12+7}{5+7}$	M1		correct expression, but ft their C
	$= -\dfrac{5}{12}$	A1	2	Condone $\dfrac{5}{-12}$
(ii)	grad tangent $= \dfrac{12}{5}$	B1 √		$\dfrac{-1}{\text{their grad } AC}$
	$y + 7 = \dfrac{12}{5}(x + 7)$	M1		ft "their $\dfrac{12}{5}$" must be **tangent** and not AC
	$\Rightarrow 12x - 5y + 49 = 0$	A1	3	OE with **integer** coefficients with all terms on one side of the equation
	Total		**11**	

Question 6

6 (a) (i) Express $x^2 - 8x + 17$ in the form $(x-p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence write down the minimum value of $x^2 - 8x + 17$. *(1 mark)*

(iii) State the value of x for which the minimum value of $x^2 - 8x + 17$ occurs. *(1 mark)*

(b) The point A has coordinates $(5, 4)$ and the point B has coordinates $(x, 7-x)$.

(i) Expand $(x-5)^2$. *(1 mark)*

(ii) Show that $AB^2 = 2(x^2 - 8x + 17)$. *(3 marks)*

(iii) Use your results from part (a) to find the minimum value of the distance AB as x varies. *(2 marks)*

Student Response

Q6.

a)

i- $(x-4)^2 + \cancel{\ }1$ ✓ 2

ii- $\cancel{\ } -1$ ✗ 0

iii-

~~$x^2 - 8x + 17 = -33$~~

~~$x^2 - 8x = -40$~~

~~$x(x-8) = -40$~~

~~$x = -40$ or $x = -2$~~

$x = \cancel{\ } + 4$ ✓ 1

b)

i- $(x-5)(x-5) = x^2 - 10x + 25$ ✓ 1

Q6

b)

ii-

~~$(7-x)-4$~~

~~$(7-x-4)^2 = 49 + x^2 + 16$~~

~~$= x^2 + 65$~~

$(7-x-4) = 3-x$

$(3-x)^2 = (3-x)(3-x) = 9-6x+x^2$ ✓ M1

$9-6x+x^2+x^2-10x+25$ A1

$= 2x^2-16x+34$ ✓

$\downarrow$

$2(x^2-8x+17)$ A0

iii-

$AB^2 = 2(4^2-8(4)+17)$

$= 2(16-32+17)$

$= 2(1)$

$= 2$ ✓

$AB = \sqrt{2}$ ✓

Commentary

(a) The candidate completed the square correctly but wrote the minimum value of the expression as –1 instead of 1. A common mistake when completing the square was to add 16 to 17 instead of subtracting 16 from 17. The candidate clearly did this initially and then went back to delete 33 and replace it with 1. It would seem this value of 33 was still in the candidate's mind when attempting part(iii) and then used a rather unorthodox method to try to solve the quadratic equation. At any rate, this aberration was spotted and the candidate recovered to find the correct value of *x*.
(b)(i) The expansion was done correctly.
(ii)This candidate made good progress but at no stage was " AB^2 = ... " written down and so the final accuracy mark was not earned. When candidates are asked to prove a given result, they should make sure their steps are valid mathematical statements culminating in the exact form of any printed answer.
(iii) This part was answered well by this candidate who gave a clear distinction between the expression for AB^2 and the answer involving *AB.* It was rare to see this part answered correctly. This candidate was able to use the correct value of x found in part (a)(iii); others simply stated that the minimum value of AB^2 was 2, when they had obtained the correct answer for part(a)(ii).

Mark Scheme

6(a)(i)	$(x-4)^2$ *or* $p=4$ $+1$ *or* $q=1$	B1 B1	 2	ISW for $p=-4$ if $(x-4)^2$ seen
(ii)	(Minimum value is) 1	B1√	1	Correct or FT "their q" (NOT coords)
(iii)	(Minimum occurs when $x=$) 4	B1√	1	Correct or FT "their p" – may use calculus Condone (p, **) for this B1 mark
(b)(i)	$(x-5)^2 = x^2-10x+25$	B1	1	
(ii)	$(x-5)^2+(7-x-4)^2$ $=(x-5)^2+(3-x)^2$ $=x^2-10x+25+9-6x+x^2$ $AB^2=2x^2-16x+34$ $=2(x^2-8x+17)$	M1 A1 A1	 3	Condone one slip in one bracket May be seen under $\sqrt{\ }$ sign From a fully correct expression AG CSO
(iii)	Minimum $AB^2 = 2\times$ "their (a)(ii)" Minimum $AB=\sqrt{2}$	M1 A1	 2	Or use of their $x=4$ in expression Or use of their $B(4, 3)$ and $A(5, 4)$ in distance formula M0 if calculus used Answer only of $2\times$ "their (a)(ii)" scores M1, A0
	Total		**10**	

Question 7

7 The curve C has equation $y = k(x^2 + 3)$, where k is a constant.

The line L has equation $y = 2x + 2$.

(a) Show that the x-coordinates of any points of intersection of the curve C with the line L satisfy the equation

$$kx^2 - 2x + 3k - 2 = 0$$

(1 mark)

(b) The curve C and the line L intersect in two distinct points.

(i) Show that

$$3k^2 - 2k - 1 < 0$$

(4 marks)

(ii) Hence find the possible values of k. *(4 marks)*

Student Response

7. $y = k(x^2+3)$ $\quad y = 2x+2$

a) $k(x^2+3)$

$= kx^2+3k = 2x+2$

$= kx^2+3k-2x-2=0$ ✓

$\therefore kx^2-2x+3k-2=0$ ✓

b(i) $b^2-4ac<0$ ✗ B0

$4-(4k(3k-2))$ ✓ M1

$4-(12k^2-8k)$ A0

$= 12k^2-8k-4<0 \quad (\div 4)$

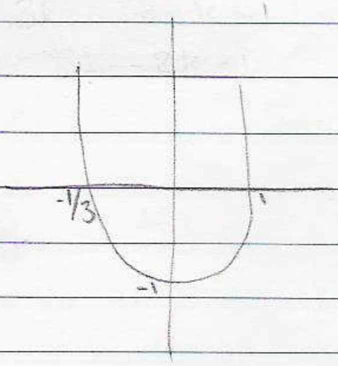
$3k^2-2k-1<0$

(ii) ~~$(3k-1)(k+1)$~~

$(3k+1)(k-1)$

$k=-1/3 \quad k=1$

$-1/3 \leq k \leq 1$

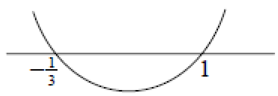
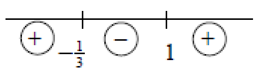


(4 − 1)

Commentary

(a) Apart from a couple of trailing equals signs, this part was answered well.
(b)(i) The 'less than' sign in the answer caused many candidates to write down an incorrect statement involving the discriminant. It would appear that this candidate was working backwards from the printed answer and so only a single mark for the discriminant was earned. Had the brackets been removed correctly then a further accuracy mark would have been earned.
(ii) The quadratic was factorised correctly and the correct critical values were written down. The candidate used a sketch to good effect but then failed to give the final answer as a strict inequality and so lost the final mark. Candidates are strongly urged to draw a sketch or sign diagram when solving quadratic inequalities.

Mark Scheme

7(a)	$k\left(x^2+3\right)=2x+2$ $\Rightarrow kx^2-2x+3k-2=0$	B1	1	AG OE all terms on one side and = 0
(b)(i)	Discriminant = $(-2)^2-4k(3k-2)$	M1		Condone one slip (including x is one slip) Condone 2^2 or 4 as first term
	$=4-12k^2+8k$	A1		condone recovery from missing brackets
	Two distinct real roots $\Rightarrow b^2-4ac>0$ $4-12k^2+8k>0$	B1√		"their discriminant in terms of k" >0 Not simply the statement $b^2-4ac>0$
	$\Rightarrow 12k^2-8k-4<0$			Change from >0 to <0 and divide by 4
	$\Rightarrow 3k^2-2k-1<0$	A1	4	AG CSO
(ii)	$(3k+1)(k-1)$	M1		Correct factors or correct use of formula
	Critical values 1 and $-\frac{1}{3}$	A1		May score M1, A1 for correct critical values seen as part of incorrect final answer with or without working
	Use of sign diagram or sketch  	M1		If previous A1 earned, sign diagram or sketch must be correct for M1 Otherwise, M1 may be earned for an attempt at the sketch or sign diagram using their critical values.
	$\Rightarrow -\frac{1}{3}<k<1$ or $1>k>-\frac{1}{3}$ condone $-\frac{1}{3}<k$ AND $k<1$ for full marks but not OR or "," instead of AND	A1	4	Full marks for correct final answer with or without working $\leqslant$ loses final A mark *Answer only* of $1<k<-\frac{1}{3}$ or $k<-\frac{1}{3};k<1$ *etc* scores M1,A1,M0 since the correct critical values are evident *Answer only* of $\frac{1}{3}<k<1$ etc where critical values are not both correct gets M0,M0
	Total		9	
	TOTAL		75	