Teacher Support Materials 2009

Maths GCE

Paper Reference MM2B

Question 1

1 A particle moves under the action of a force, **F** newtons. At time t seconds, the velocity, $\mathbf{v}\ \mathrm{m\,s^{-1}}$, of the particle is given by

$$\mathbf{v} = (t^3 - 15t - 5)\mathbf{i} + (6t - t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(3 marks)*

(b) The mass of the particle is 4 kg.

(i) Show that, at time t,

$$\mathbf{F} = (12t^2 - 60)\mathbf{i} + (24 - 8t)\mathbf{j}$$ *(2 marks)*

(ii) Find the magnitude of **F** when $t = 2$. *(4 marks)*

Student Response

1 a) $v = (t^3 - 15t - 5)\underline{i} + (6t - t^2)\underline{j}$

$\frac{dv}{dt}$

$= a \quad (3t^2 - 15)\underline{i} + (6 - 2t)\underline{j}$ ✓

b) i) $F = m \cdot a$

$F = \left((3t^2 - 15)\underline{i} + (6 - 2t)\underline{j}\right) \times 4$

$= \sout{12t^2 ? 15n}$ ✓

$(12t^2 - 60)\underline{i} + (24 - 8t)\underline{j}$

ii) when $t = 2$

$F = (12(2)^2 - 60)\underline{i} + (24 - 8(2))\underline{j}$

$= (48 - 60)\underline{i} + (24 - 16)\underline{j}$

$= -12\underline{i} + 8\underline{j}$ ✓

A

Commentary

Virtually all candidates completed this question well. A few, as shown in this example, found the force as a vector in part (c) but forgot to find the magnitude as required.

Mark scheme

1(a)	$\mathbf{a} = \frac{d\mathbf{v}}{dt} = (3t^2 - 15)\mathbf{i} + (6 - 2t)\mathbf{j}$	M1A1 A1	3	A1 (**i** terms) A1 (**j** terms)
(b)(i)	Using $\mathbf{F} = m\mathbf{a}$: Force $= 4 \times \{(3t^2 - 15)\mathbf{i} + (6 - 2t)\mathbf{j}\}$ $= (12t^2 - 60)\mathbf{i} + (24 - 8t)\mathbf{j}$	 M1 A1	 2	 AG
(ii)	When $t = 2$, force $= -12\mathbf{i} + 8\mathbf{j}$ Magnitude of force $= \sqrt{12^2 + 8^2}$ N $= 14.4$ (N)	M1A1 M1 A1	 4	
	Total		**9**	

Question 2

2 A slide at a water park may be modelled as a smooth plane of length 20 metres inclined at 30° to the vertical. Anne, who has a mass of 55 kg, slides down the slide. At the top of the slide, she has an initial velocity of $3\ \text{m s}^{-1}$ down the slide.

(a) Calculate Anne's initial kinetic energy. *(2 marks)*

(b) By using conservation of energy, find the kinetic energy and the speed of Anne after she has travelled the 20 metres. *(6 marks)*

(c) State one modelling assumption which you have made. *(1 mark)*

Student response

2. a) the initial K.E. $= \frac{1}{2}mv^2$

$= \frac{1}{2}(55)(3)^2$

$= 248\text{J}$ ✓

b) $55(9.8)(20\sin 30°) + 248 = \frac{1}{2}(55)v^2$

MR $\quad v = 14.3\ \text{ms}^{-1}$ No KE

c) Anne is a particle. ✓

Commentary

Part (a) of this question was well answered. In part (b) many candidates assumed that the slope was inclined at 30 to the horizontal rather than 30 to the vertical as given in the question. As this example shows candidates also failed to read the question carefully and only gave one of the two answers required; in this case only Anne's final speed and not the final kinetic energy.

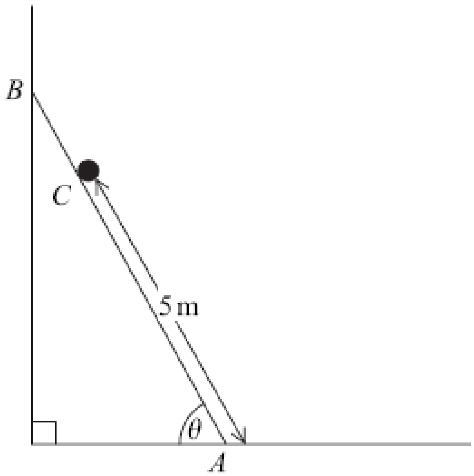
Mark Scheme

2(a)	KE $= \frac{1}{2} \times 55 \times 3^2$	M1		
	$= 247.5$ J	A1	2	
(b)	Change in PE as slides down:			
	$mgh = 55 \times 9.8 \times 20 \cos 30$	M1		Need cos 30 or sin 30
	$= 9335.7\ldots$	A1		
	Using Conservation of Energy:			
	KE at end of slide $= 247.5 + 9335.7$	m1		'a' + '9335.7'
	$= 9580$ J	A1		accept 9583
	Speed of Anne is $\sqrt{\dfrac{9583}{\frac{1}{2} \times 55}}$	m1		
	$= 18.7 \text{ m s}^{-1}$	A1	6	
(c)	Anne is a particle; no air resistance	E1	1	
	Total		**9**	

Question 3

3 A uniform ladder, of length 6 metres and mass 22 kg, rests with its foot, A, on a rough horizontal floor and its top, B, leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall, and the angle between the ladder and the floor is θ.

A man, of mass 90 kg, is standing at point C on the ladder so that the distance AC is 5 metres. With the man in this position, the ladder is on the point of slipping. The coefficient of friction between the ladder and the horizontal floor is 0.6. The man may be modelled as a particle at C.

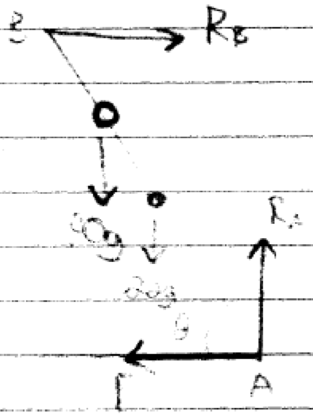


(a) Show that the magnitude of the frictional force between the ladder and the horizontal floor is 659 N, correct to three significant figures. *(4 marks)*

(b) Find the angle θ. *(5 marks)*

Student Response

3.



A- $F_r = \mu R$

$F_r = \mu R_A$

$F_r = 0.6 \times [(90 \times 9.8) + (22 \times 9.8)]$

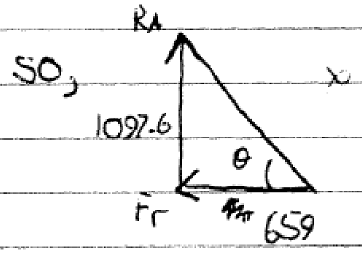
$F_r = 0.6 \times 1097.6$

$F_r = 658.56$

$= 659N$ (3sf) as req ✓

B- At the point of slipping $F_r = R_B$

so,



$\tan\theta = \frac{1097.6}{659}$ ✗

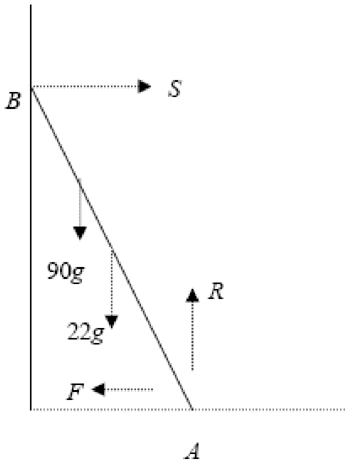
$\tan\theta = 1.67$

$\theta = 59.0°$ ✗

Commentary

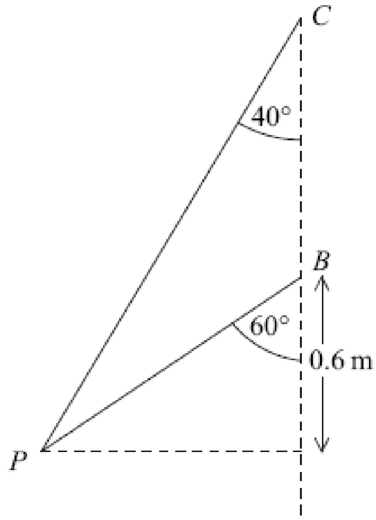
Many candidates found the value of the friction, giving their answer to more than the three figure answer required so that they could ‘show’ that they had indeed shown that the frictional force was 659 N to three significant figures.
In part (b) many candidates, as shown in this example, appeared to try to find the direction of the resultant force at the foot of the ladder rather than take moments to find the angle of inclination of the ladder.

Mark Scheme

3(a)				
	Resolve vertically: $R = 22g + 90g$ $= 112g$	B1		
	Using $F = \mu R$: $F = 0.6R$ $F = 0.6 \times 112g$	M1 A1		[Needs $0.6\times112g$ or 0.6×1097.6 NOT 0.6×1097 unless 658.56 seen]
	$= 67.2g$ or 658.56 $F = 659$ N	A1	4	AG (659 must be shown from correct working)
(b)	Resolve horizontally: $S = F$	B1		
	Moments about A: $90g\times5\times\cos\theta + 22g\times3\times\cos\theta$	M1A1		M1 (one term, force × distance × cos or sin)
	$= 67.2g\times6\times\sin\theta$ $450g + 66g = 403.2g\tan\theta$ $\tan\theta = \dfrac{516}{403.2}$	A1		
	$\theta = 52.0°$	A1	5	accept 52 **Alternative**: or moments about B: M1 A2, 1 or 0 for four-term moment equation + M1 for rearranging etc (dep on 4 term) + A1 for answer
	Total		**9**	

Question 4

4 Two light inextensible strings each have one end attached to a particle, P, of mass 6 kg. The other ends of the strings are attached to the fixed points B and C. The point C is vertically above the point B. The particle moves, at constant speed, in a horizontal circle, with centre 0.6 m below point B, with the strings inclined at 40° and 60° to the vertical, as shown in the diagram. Both strings are taut.



(a) As the particle moves in the horizontal circle, the tensions in the two strings are equal.

Show that the tension in the strings is 46.4 N, correct to three significant figures.

(4 marks)

(b) Find the speed of the particle. *(4 marks)*

Student Response

4. a) For vertical equilibrium

$6g = T\cos 40 + T\cos 60$

~~$6g$~~ ~~$12g = 2T\cos 40 + T$~~

~~$\Rightarrow 3T$~~

$6g = 0.766T + 0.5T$

$\Rightarrow T = \frac{6g}{1.266}$ ✓

$= 46.4 \text{ N } (3s.f.)$

b) Using $F = m\frac{v^2}{r}$

$46.4 = \frac{6v^2}{0.6}$ ✗

$\Rightarrow v^2 = \frac{46.4 \times 0.6}{6}$

$v^2 = \frac{116}{25}$ ✗

$\Rightarrow v = 2.15 \text{ ms}^{-1}$

Commentary

The fact that the particle was being supported by two strings caused candidates some problems. As the answer to part (a) was given many were able to resolve vertically and obtain the value of the tension. In part (b) the common error, as shown in this example, was to revert back to a more usual one string problem and consider only the horizontal force and the centipedal acceleration.

Mark Scheme

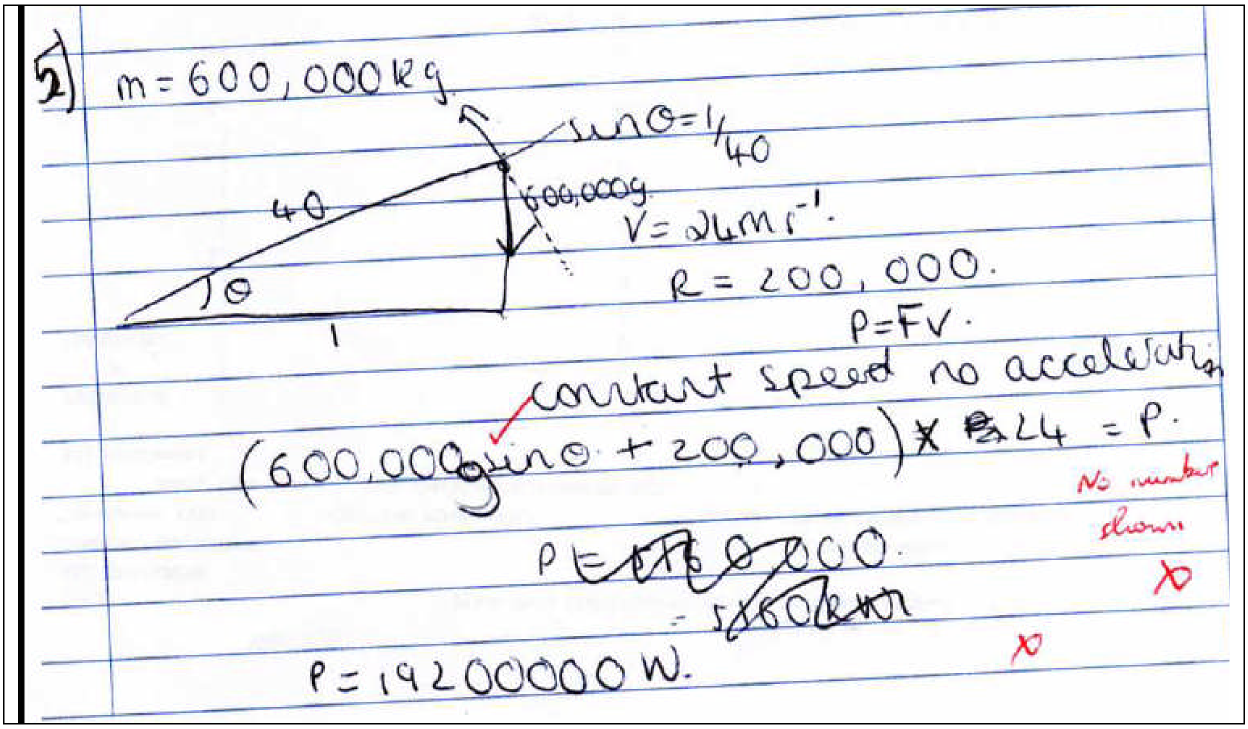
4(a)	Resolving vertically: $T\cos 60 + T\cos 40 = mg$ $1.266\,T = 6g$ $T = 46.4$ N	 M1A1 M1 A1	 4	 AG no marks if g deleted
(b)	Radius of circle is 0.6 tan 60 Horizontally: $\frac{mv^2}{r} = T\cos 50 + T\cos 30$ $\frac{6v^2}{1.039} = 46.4\cos 50 + 46.4\cos 30$ or 70.01 $v^2 = 12.123$ Speed is 3.48 m s^{-1}	B1 M1 A1 A1	 4	$r = 1.039$ or 1.04 Accept sin instead of cos for M1
	Total		**8**	

Question 5

5 A train, of mass 600 tonnes, travels at constant speed up a slope inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{40}$. The speed of the train is $24\,\mathrm{m\,s^{-1}}$ and it experiences total resistance forces of 200 000 N.

Find the power produced by the train, giving your answer in kilowatts. *(6 marks)*

Student Response



Commentary

This problem caused many candidates problems, often in attempting to use a value for the angle rather than simply using the gravitational component to be mg sin α with sin α to be $\frac{1}{40}$ as given in the question. This candidate showed all the correct steps in her working but since she obtained the incorrect answer she could not obtain any of the accuracy marks potentially available in this question.

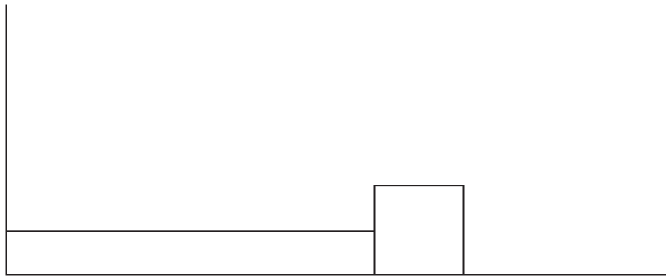
Mark Scheme

5	Force acting against gravity is $mg\sin\theta$	M1		Or 147000
	Force acting against gravity and resistance is $mg\sin\theta + 200000$	m1		200 000 + 'mg sinθ'
	$= 600000g\sin\theta + 200000$			
	$= 347000$	A1		
	Using power = force × velocity	M1		
	$= 347000 \times 24$	A1F		
	$= 8330$ kW	A1	6	
	Total		**6**	

Question 6

6 A block, of mass 5 kg, is attached to one end of a length of elastic string. The other end of the string is fixed to a vertical wall. The block is placed on a horizontal surface.

The elastic string has natural length 1.2 m and modulus of elasticity 180 N. The block is pulled so that it is 2 m from the wall and is then released from rest. Whilst taut, the string remains horizontal. It may be assumed that, after the string becomes slack, it does not interfere with the movement of the block.



(a) Calculate the elastic potential energy when the block is 2 m from the wall. *(2 marks)*

(b) If the horizontal surface is smooth, find the speed of the block when it hits the wall. *(3 marks)*

(c) The surface is in fact rough and the coefficient of friction between the block and the surface is μ.

Find μ if the block comes to rest just as it reaches the wall. *(7 marks)*

Student Response

6) a) $e.p.e = \frac{\lambda x^2}{2l} = \frac{180 \times 0.8^2}{2 \times 1.2} = ~~48~~ 48\,J$

b) $e.p.e = Ke + pe$

$48 = \frac{1}{2} m v^2 + 0$

$48 = \frac{1}{2} \times 5 v^2$

$v^2 = 48 / (0.5 \times 5)$

$v = \sqrt{19.2}$

$v = 4.38\,ms^{-1}$

c) ~~Ke~~ v when elastic becomes slack $= 4.38\,ms^{-1}$

$R = mg = 5g$

~~Ke~~

~~Ke~~ work done by friction = change in Ke

work done ~~Ke~~ $= \frac{1}{2} m u^2 - \frac{1}{2} m v^2$

$= \frac{1}{2} \times 5 \times 4.38^2 - 0 = 47.96\,N$

work done = FS

$F_r = 47.96 / 1.2\,m = 39.97\,N$

$F_r = \mu R$

$\mu = F_r / R = 39.97 / 5g = 0.82$

$\mu = 0.82$

Commentary

Parts (a) and (b) of this question were, as shown in this example, answered well. In pat (c) this example shows a typical response whereby the change of [kinetic] energy was found by using velocities which wasted time and, usually, brought in inaccuracies. This candidate found the change in energy to be 47.96 J whereas it was exactly 48J. To find the work done by the force this candidate used ‘work done equals force *x* distance’ but unfortunately used the common error of 1.2 metres to be the distance moved instead of 2 m.

Mark Scheme

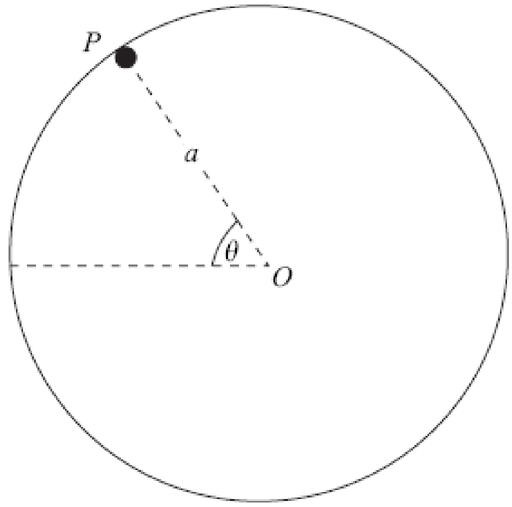
6(a)	$\text{EPE} = \frac{\lambda x^2}{2l}$			
	$= \frac{180 \times 0.8^2}{2 \times 1.2}$	M1		
	$= 48$ J	A1	2	
(b)	Using initial EPE = KE when string becomes slack:	M1		
	$48 = \frac{1}{2} \times 5 \times v^2$	A1F		
	$v = \sqrt{\frac{96}{5}}$			
	$= 4.38$ m s^{-1}	A1F	3	ft $\sqrt{\frac{'a'}{2.5}}$
(c)	Normal reaction is $5g$ or 49	M1		
	Frictional force is $5g \times \mu$	m1A1		
	Work done by frictional force is $5\mu g \times 2$	m1		
	$= 10\mu g$	A1		
	Stops at wall $\Rightarrow 10\mu g = 48$	m1		m1 $10\mu g = 'a'$
	$\mu = 0.490$	A1	7	accept $\frac{24}{49}$ OE
	Total		**12**	

Question 7

7 In crazy golf, a golf ball is hit so that it starts to move in a vertical circle on the inside of a smooth cylinder.

Model the golf ball as a particle, P, of mass m. The circular path of the golf ball has radius a and centre O. At time t, the angle between OP and the horizontal is θ, as shown in the diagram.

The golf ball has speed u at the lowest point of its circular path.



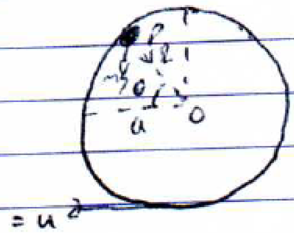
(a) Show that, while the golf ball is in contact with the cylinder, the reaction of the cylinder on the golf ball is

$$\frac{mu^2}{a} - 3mg\sin\theta - 2mg$$ *(6 marks)*

(b) Given that $u = \sqrt{3ag}$, the golf ball will not complete a vertical circle inside the cylinder. Find the angle which OP makes with the horizontal when the golf ball leaves the surface of the cylinder. *(4 marks)*

Student Response

(7)



mass = m   r = a

a)

$\boxed{R + \frac{mv^2}{r} + mg\sin\theta = 0}$ ✗

E_k at start = GPE gained + E_k at P.

$\frac{1}{2}mu^2 = mg(2a - a\sin\theta) + \frac{1}{2}mv^2$ ✗

$u^2 = 2g(a + a\sin\theta) + v^2$

$v^2 = 2ag + 2ag\sin\theta - u^2$

$R = -m\frac{(2ag - 2ag\sin\theta - u^2)}{a} - mg\sin\theta$

$\therefore R = -2mg - 2mg\sin\theta + \frac{mu^2}{a} - mg\sin\theta$

$= \frac{mu^2}{a} - 3mg\sin\theta - 2mg$ as req'd.

✗

Commentary

In part (a) many candidates showed that they knew that they had to use an energy equation and a force equation. Signs were frequently incorrect and this example shows a common situation where the candidate wrote down two suitable equations, found that these gave an incorrect result and thus changed both equations to attempt to obtain the printed result. In such cases it is essential that the candidate corrects all the steps shown in the working.

Mark Scheme

7(a)	By conservation of energy to point where QP makes an angle θ with upward vertical:			
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1+\sin\theta)$	M1 A1		for 3 terms, 2 KE and 1 PE $mga\ (1+\sin\theta)$ term
	$v^2 = u^2 - 2ag(1+\sin\theta)$	A1		
	Resolve radially			
	$R = \frac{mv^2}{a} - mg\sin\theta$	M1A1		M1 for 3 terms, include $\sin\theta$ or $\cos\theta$
	$= \frac{mu^2}{a} - 3mg\sin\theta - 2mg$	A1	6	AG
(b)	When particle leaves the track, $R = 0$	M1		
	$0 = 3mg - 3mg\sin\theta - 2mg$	A1		
	$\sin\theta = \frac{1}{3}$	M1		SC3 $\sin^{-1}\frac{1}{3}$
	$\theta = 19.5°$	A1	4	accept 19.4° or $\theta = 0.340^c$
	Total		**10**	

Question 8

8 A stone, of mass m, is moving in a straight line along smooth horizontal ground.

At time t, the stone has speed v. As the stone moves, it experiences a total resistance force of magnitude $\lambda m v^{\frac{3}{2}}$, where λ is a constant. No other horizontal force acts on the stone.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v^{\frac{3}{2}}$$

(2 marks)

(b) The initial speed of the stone is $9\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \frac{36}{(2 + 3\lambda t)^2}$$

(7 marks)

(c) Find, in terms of λ, the time taken for the speed of the stone to drop to $4\,\mathrm{m\,s^{-1}}$.

(3 marks)

Student Response

8\. a) Using $F = ma$

$$-\lambda m v^{3/2} = m.\frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{-\lambda m v^{3/2}}{m}$$ ✓

$$\frac{dv}{dt} = -\lambda v^{3/2}$$

b) $\int v^{-3/2}\, dv = -\lambda \int dt$

$$-2v^{-1/2} = -\lambda t + c$$

when $t = 0$, $v = 9$

$$\Rightarrow c = -2/3$$ ✓

$$-2v^{-1/2} = -\lambda t - 2/3$$

$$\frac{1}{\sqrt{v}} = \frac{\lambda t}{2} + \frac{1}{3}$$ ✓

$$\sqrt{v} = \frac{2}{\lambda t} + \frac{1}{3}$$ ✗

$$\sqrt{v} = \frac{6}{2+3\lambda t}$$

$$\Rightarrow v = \frac{36}{(2+3\lambda t)^2}$$ ✗?

c) $4 = \frac{36}{(2+3\lambda t)^2}$

$$4(2+3\lambda t)^2 = 36$$

$$2+3\lambda t = \sqrt{9}$$

$$3\lambda t = \pm 1$$

$$t = \frac{1}{3\lambda}$$ ✓

Commentary

Many candidates found the majority of this question straightforward. As shown in this example , parts (a) and (b) were usually correct but, in part (c) when square rooting the equation the candidate forgot that $\sqrt{9} = \pm 3$ and hence one root had to be shown to be not appropriate.

Mark Scheme

8(a)	Using $F = ma$:			
	$-\lambda m v^{\frac{3}{2}} = m\frac{dv}{dt}$	M1		
	$\therefore \frac{dv}{dt} = -\lambda v^{\frac{3}{2}}$	A1	2	AG
(b)	$\int \frac{dv}{v^{\frac{3}{2}}} = -\lambda \int dt$	M1		
	$-\frac{2}{v^{\frac{1}{2}}} = -\lambda t + c$	A1		Condone no '+c'
	When $t = 0$, $v = 9 \Rightarrow c = -\frac{2}{3}$	M1 A1		Dep. on correct integration (accept sign or '$\frac{1}{2}$' error)
	$\frac{2}{\sqrt{v}} = \lambda t + \frac{2}{3}$	A1		
	$\frac{\sqrt{v}}{2} = \frac{1}{\lambda t + \frac{2}{3}}$			
	$v = \left(\frac{6}{2 + 3\lambda t}\right)^2$	m1		Needs correct algebra
	$v = \frac{36}{(2 + 3\lambda t)^2}$	A1	7	AG
(c)	When $v = 4$,			**or** $\frac{36}{(2+3\lambda t)^2} = 4$ M1
	$\frac{2}{\sqrt{v}} = \lambda t + \frac{2}{3} \Rightarrow 1 = \lambda t + \frac{2}{3}$	M1A1		$(2 + 3\lambda t)^2 = 9$ A1
	$t = \frac{1}{3\lambda}$	A1	3	$t = \frac{1}{3\lambda}$ A1 needs statement why $2+3\lambda t \neq -3$
	Total		**12**	