Teacher Support Materials 2009

Maths GCE

Paper Reference MM1A/W

Question 1

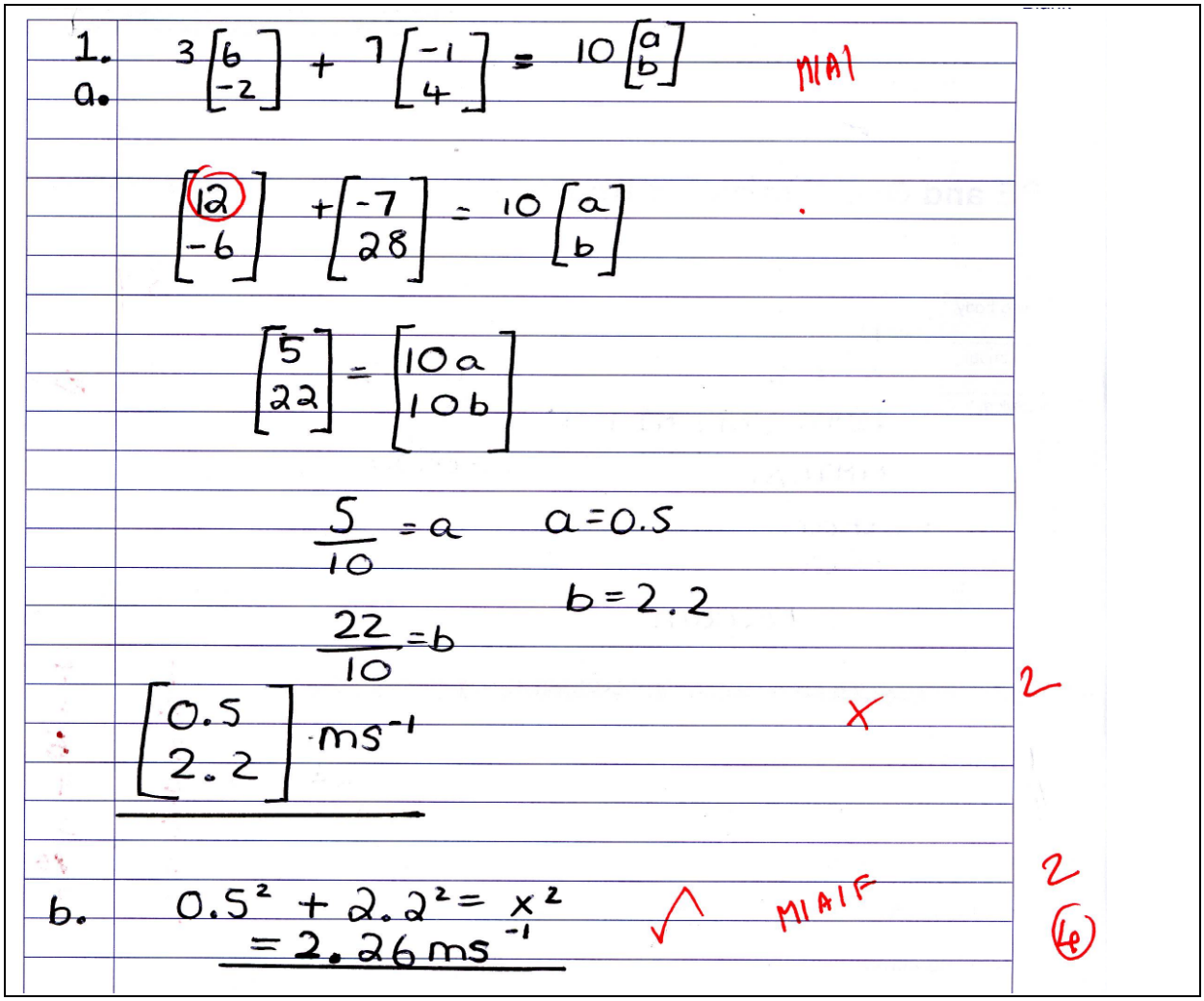
1 Two particles, A and B, are moving on a smooth horizontal surface when they collide. During the collision, the two particles coalesce to form a single combined particle. Particle A has mass 3 kg and particle B has mass 7 kg.

Before the collision, the velocity of A is $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$ m s^{-1} and the velocity of B is $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$ m s^{-1}.

(a) Find the velocity of the combined particle after the collision. *(3 marks)*

(b) Find the speed of the combined particle after the collision. *(2 marks)*

Student Response



Commentary

This candidate has written down a correct statement at the start of the answer, which secures the first two marks for the question. However in the second line of working, a simple arithmetic error, circled by the examiner causes the loss of the final accuracy mark for part (a).

The answer to part (b) is now clearly incorrect, but as the candidates has used the correct method and obtained the correct speed for the velocity found in part (a). The script shows how the examiner has awarded follow through marks in this case.

This example illustrates how candidates can lose marks through minor errors and how the follow through marks can be awarded. It also shows the importance of a clear statement or equation at the start of the question to ensure that partial marks are awarded.

Mark scheme

1(a)	$3\begin{bmatrix}6\\-2\end{bmatrix}+7\begin{bmatrix}-1\\4\end{bmatrix}=10\mathbf{v}$	M1		M1: Forming three term equation for conservation of momentum, but condone incorrect signs. Must see combined mass of 10.
		A1		A1: Correct equation with correct signs. Accept $3\begin{bmatrix}6\\-2\end{bmatrix}+7\begin{bmatrix}-1\\4\end{bmatrix}=3\mathbf{v}+7\mathbf{v}$ oe
	$\mathbf{v}=\frac{1}{10}\begin{bmatrix}11\\22\end{bmatrix}=\begin{bmatrix}1.1\\2.2\end{bmatrix}$	A1	3	A1: Correct velocity Consistent use of mg instead of m throughout deduct 1 mark
(b)	$v=\sqrt{1.1^2+2.2^2}$	M1		M1: Finding speed. Must be + inside square root.
	$v=2.46\ \text{ms}^{-1}$	A1F	2	A1F: Correct speed for their velocity Accept $1.1\sqrt{5}$ or $\frac{11\sqrt{5}}{10}$ or 2.45 or AWRT 2.46
	Total		**5**	

Question 2

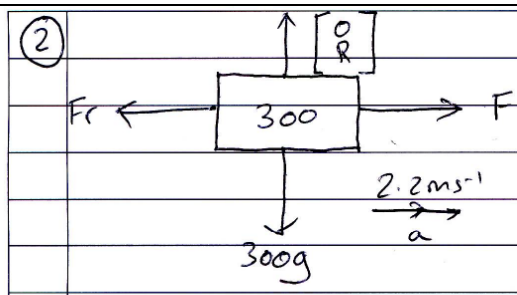
2 A motorcycle and rider, of total mass 300 kg, are accelerating in a straight line along a horizontal road at $2.2\,\text{m s}^{-2}$.

(a) Show that the magnitude of the resultant force acting on the motorcycle is 660 N. *(1 mark)*

(b) A forward driving force of P newtons together with a resistance force of magnitude 400 newtons act on the motorcycle. Find P. *(2 marks)*

(c) Find the time that it would take for the speed of the motorcycle to increase from $12\,\text{m s}^{-1}$ to $23\,\text{m s}^{-1}$. *(3 marks)*

Student response



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a) $F = ma$

$F = 300 \times 2{\cdot}2$

$F = 660\,N$ ✓ 1

b) $P = 660 - 400$ M1

$P = 260\,N$ ✗ 1

c) $u = 12 \quad v = 23 \quad a = 2{\cdot}2 \quad t = ?$

$v = u + at$

$23 = 12 + 2{\cdot}2t$

$11 = 2{\cdot}2t$

$t = \frac{11}{2{\cdot}2}$ ✓

$t = 5$ seconds ✓ 3

(5)

Commentary

In this example the candidates gains full marks in both parts (a) and (c) with good clear solutions.

However in part (b), the candidate only gains one mark. This mark is awarded because the candidate creates an equation which contains the correct terms, but not with the correct signs. This type of answer is often seen in examination solutions. The candidates would often produce better results if they used "Resultant Force = Mass x Acceleration" and simplified this rather than linking together the terms that they think should be in the equation. In this example the candidate knows that the 660 and 400 need to be used, but has not been able to link them correctly. Note that there is no attempt to use the method suggested above.

This example illustrates the difficulty that some students have in applying Newton's Second Law.

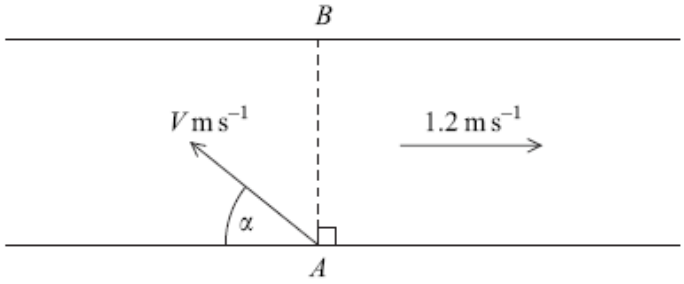
Mark Scheme

2(a)	Resultant Force = 300×2.2 $= 660$ N AG	B1	1	B1: Correct value from correct multiplication.
(b)	$P - 400 = 660$ $P = 1060$	M1 A1	2	M1: Three term equation of motion A1: Correct value for P
(c)	$23 = 12 + 2.2t$	M1		M1: Use of a constant acceleration equation to find t.
		A1		A1: Correct equation
	$t = \frac{23-12}{2.2} = 5$ s	A1	3	A1: Correct time
	Total		**6**	

Question 3

3 A river has parallel banks which are 16 metres apart. The water in the river flows at $1.2\,\text{m s}^{-1}$ parallel to the banks. A boat sets off from one bank at the point A and travels perpendicular to the bank so that it reaches the point B, which is directly opposite the point A. It takes the boat 10 seconds to cross the river.

The velocity of the boat relative to the water has magnitude $V\,\text{m s}^{-1}$ and is at an angle α to the bank, as shown in the diagram.

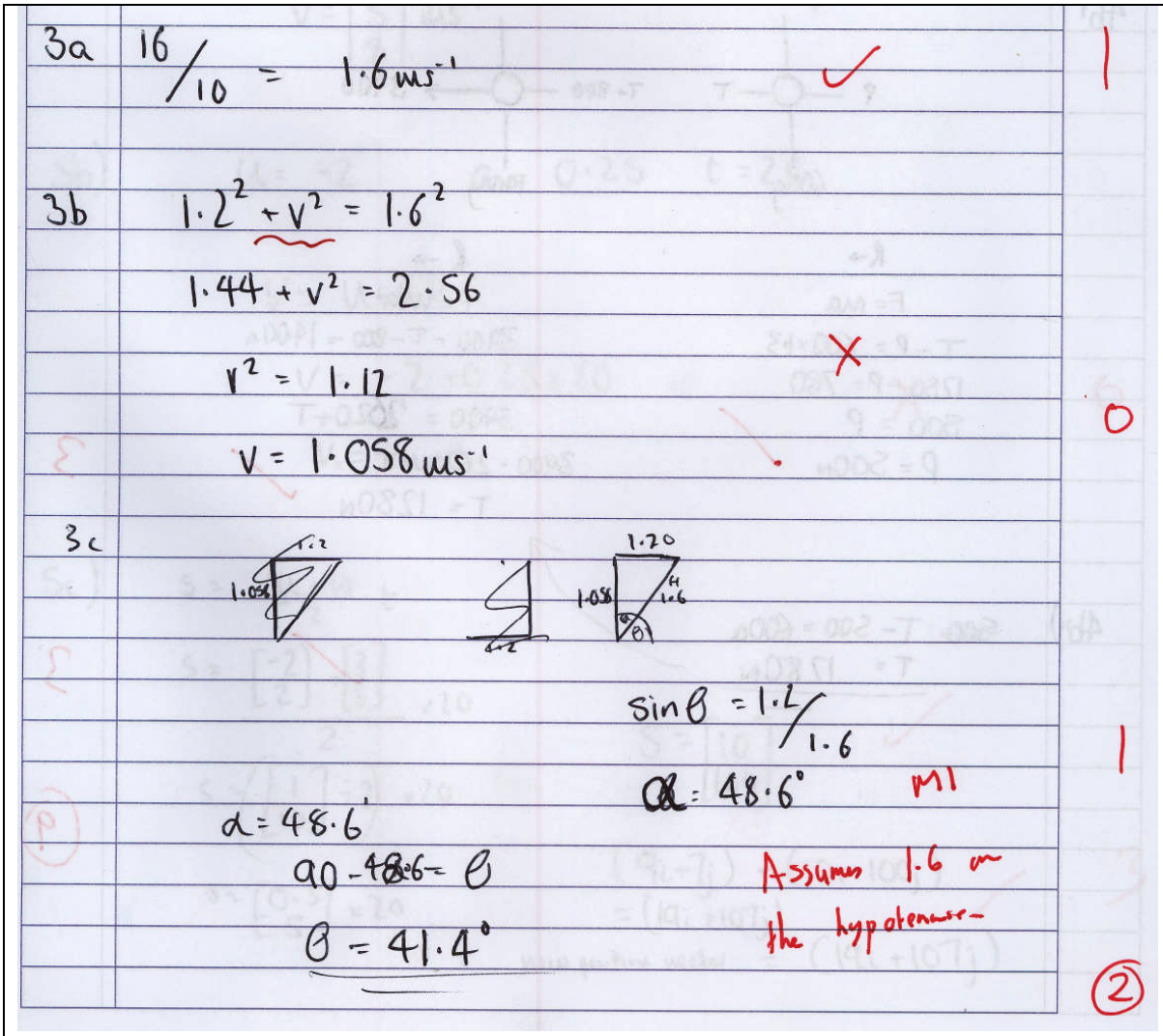


(a) Show that the magnitude of the resultant velocity of the boat is $1.6\,\text{m s}^{-1}$. *(1 mark)*

(b) Find V. *(3 marks)*

(c) Find α. *(2 marks)*

Student Response



Commentary

In this example the candidate shows clearly the division that is required in part (a). However in part (b) the candidate does not realise that the orientation of the velocities that are being used and treats 1.6 as if it were the hypoteneuse of a velocity triangle. It is interesting that the candidates did not draw a diagram, which may have helped avoid this confusion.

In part (c), the candidate does draw a diagram, but has the velocities in the wrong places. The candidate does get a method mark, as the hypoteneuse from the diagram on the script is used as the denominator. No follow through accuracy mark can be awarded here because the 1.6 should not be on the hypoteneuse of the triangle of the denominator of the sine ratio.

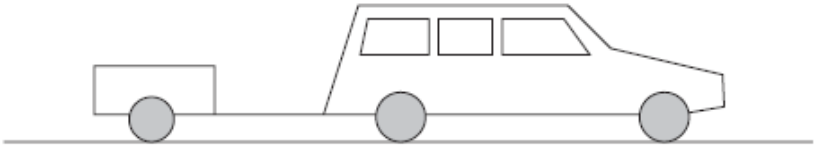
This example illustrates how marks can be lost if an accurate diagram is not drawn at the start of the question.

Mark Scheme

3(a)	$v=\frac{16}{10}=1.6\ \text{ms}^{-1}$ AG	B1	1	B1: Printed result obtained from correct division. Must see 16 divided by 10.
(b)	$V^2=1.6^2+1.2^2$ $V=\sqrt{4}=2\ \text{ms}^{-1}$	M1A1 A1	3	M1: Equation to find V based on Pythagoras. Must involve addition of the squares of two components. A1: Correct equation A1: Correct V
(c)	$\sin\alpha=\frac{1.6}{2}$ or $\frac{1.2}{2}$ $\alpha=53.1°$ OR $\cos\alpha=\frac{1.2}{2}$ or $\frac{1.6}{2}$ $\alpha=53.1°$ OR $\tan\alpha=\frac{1.6}{1.2}$ or $\frac{1.2}{1.6}$ $\alpha=53.1°$	M1 A1F (M1) (A1F) (M1) (A1(F))	2	M1: Trigonometric equation to find α. A1F: Correct α. Follow through incorrect answer to (b). Ignore diagrams
	Total		**6**	

Question 4

4 A car, of mass 1400 kg, is towing a trailer, of mass 600 kg. The two vehicles accelerate together at $1.3\,\mathrm{m\,s^{-2}}$ along a straight horizontal road.



(a) Find the distance that the car and trailer would travel while accelerating from rest to $13\,\mathrm{m\,s^{-1}}$. *(3 marks)*

(b) A forward driving force, of magnitude 3900 N, acts on the car. A resistance force, of magnitude 800 N, also acts on the car.

(i) A resistance force, of magnitude P newtons, acts on the trailer. Find P. *(3 marks)*

(ii) Find the magnitude of the force that the car exerts on the trailer. *(3 marks)*

Student Response

Leave blank

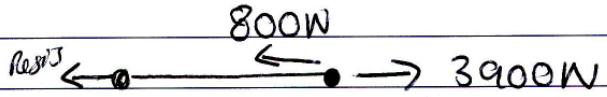
4a) $a = 1.3$

$u = 0 \quad v = 13$

$v^2 = u^2 + 2as$

$13^2 = 2 \times 1.3 \times s$

$s = 65$ ✓ ✓ 3



i) $3900 - 2000g - f = ma$ ✗

$3900 - 19600 + f = ma$

2600

$2000g - 3900 - f = ma$

$f = 18300 - 800$ ✗

$f = 17500$ 0

2

ii) $T - mg - f = ma$

$T - 5880 - 17500 = 26$ ✗

$T = 23406N$ 0

3

Commentary

This candidate gives a correct solution to part (a), which is carefully laid out. In part (b), there were quite a few incorrect or confused solutions. Tis candidates illustrate one error, which was to include the weight, in this casethat of the car and trailer in part (i) and just the trailer in part (ii). Including the weight when only the horizontal forces should be considered leads to no credit being given.

THis example illustrates how difficult some candidats find it to apply Newton's Second Law in this type of connected particle problem.

Mark Scheme

4(a)	$13^2 = 0^2 + 2\times 1.3s$	M1		M1: Use of a constant acceleration equation to find distance.
		A1		A1: Correct equation
	$s = \frac{13^2}{2.6} = 65$ m	A1	3	A1: Correct distance
(b)(i)	$3900 - 800 - P = 2000\times 1.3$	M1		M1: Four term equation of motion for car and trailer.
		A1		A1: Correct equation
	$P = 3900 - 800 - 2600 = 500$ N	A1	3	A1: Correct value for P
(b)(ii)	$T - 500 = 600\times 1.3$	M1		M1: Three term equation of motion for trailer.
		A1F		A1: Correct equation
	$T = 500 + 780 = 1280$ N	A1F	3	A1: Correct tension
	Total		**9**	

Question 5

5 A particle moves on a smooth horizontal plane. It is initially at the point A, with position vector $(9\mathbf{i} + 7\mathbf{j})$ m, and has velocity $(-2\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$. The particle moves with a constant acceleration of $(0.25\mathbf{i} + 0.3\mathbf{j})\,\mathrm{m\,s^{-2}}$ for 20 seconds until it reaches the point B. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find the velocity of the particle at the point B. *(3 marks)*

(b) Find the velocity of the particle when it is travelling due north. *(4 marks)*

(c) Find the position vector of the point B. *(3 marks)*

(d) Find the average velocity of the particle as it moves from A to B. *(2 marks)*

Student Response

5a. $u=(-2i+2j)$ $\quad v=u+at$

$a=(0.25i+0.3j)$

$t=20$

$v=(-2i+2j)+(0.25i+0.3j)20$

$v=(-2i+2j)+(5i+6j)$

$v=(3i+8j)\,ms^{-1}$ ✓ ✓ 3

b. When travelling north i vector = 0

$u=2j$

$a=0.3j$

$t=20$

$v=u+at$

$v=2j+0.3j \times 20$

$v=8j$

$v=(0i+8j)\,ms^{-1}$ ✗ 0

or

$v=2j+0.3jt\ ms^{-1}$

c. distance = speed x time

$(3i+8j)\times 20$

distance $=(60i+160j)$

$(9i+7j)+(60i+160j)$ ✗ 0

$=(69i+167j)\,m$

d. $\frac{v-u}{t} = \frac{(3i+8j)-(-2i+2j)}{20}$ ✗ 0

$=\frac{(5i+6j)}{20} = (0.25i+0.3j)\,ms^{-1}$ ③

Commentary

This candidate provides a correct solution for part (a), as did many other candidates. A number of errors are made in the later parts of the question.

In part (b) the candidate assumes that the particle is travelling due north when $t = 20$, calculate the **j** component of the velocity at this time and writes the **i** component as zero without checking this in any way.

In part (c), the candidate assumes that the particle is not accelerating, but in part (d) uses a formula that gives the acceleration of the particle.

This example illustrates some of the problems that candidates exreience when with working with positions and velocities in vector form.

Mark Scheme

5(a)	$\mathbf{v}=(-2\mathbf{i}+2\mathbf{j})+(0.25\mathbf{i}+0.3\mathbf{j})\times 20$	M1 A1		M1: Finding velocity using $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ A1: Correct expression
	$\mathbf{v}=3\mathbf{i}+8\mathbf{j}$	A1	3	A1: Correct velocity in simplest form
(b)	$-2+0.25t=0$ $t=8$ s	M1A1 A1		M1: One component equal to zero (either **i** or **j** component). A1: Correct equation A1: Correct time
	$\mathbf{v}=(2+0.3\times 8)\mathbf{j}=4.4\mathbf{j}$	A1	4	A1: Correct velocity
(c)	$\mathbf{r}=(-2\mathbf{i}+2\mathbf{j})\times 20+\frac{1}{2}(0.25\mathbf{i}+0.3\mathbf{j})\times 20^2+(9\mathbf{i}+7\mathbf{j})$ OR $\mathbf{r}=\frac{1}{2}\big((-2\mathbf{i}+2\mathbf{j})+(3\mathbf{i}+8\mathbf{j})\big)\times 20+(9\mathbf{i}+7\mathbf{j})$	M1 A1		M1: Finding position vector using a constant acceleration equation with or without the initial position with $t=20$. A1: Correct expression for position vector including initial position.
	$\mathbf{r}=19\mathbf{i}+107\mathbf{j}$	A1	3	A1: Correct position vector in simplest form.
(d)	$\mathbf{v}_{AVERAGE}=\frac{(19\mathbf{i}+107\mathbf{j})-(9\mathbf{i}+7\mathbf{j})}{20}$ $=\frac{10\mathbf{i}+100\mathbf{j}}{20}$ $=0.5\mathbf{i}+5\mathbf{j}$	M1 A1F	 2	M1: Finding average velocity based on change of position. Subtraction of initial position must be seen or implied. Division by 8 scores M0 A1F: Correct average velocity. Follow through incorrect answers from part (c). Allow $\frac{\mathbf{u}+\mathbf{v}}{2}$
	Total		**12**	

Question 6

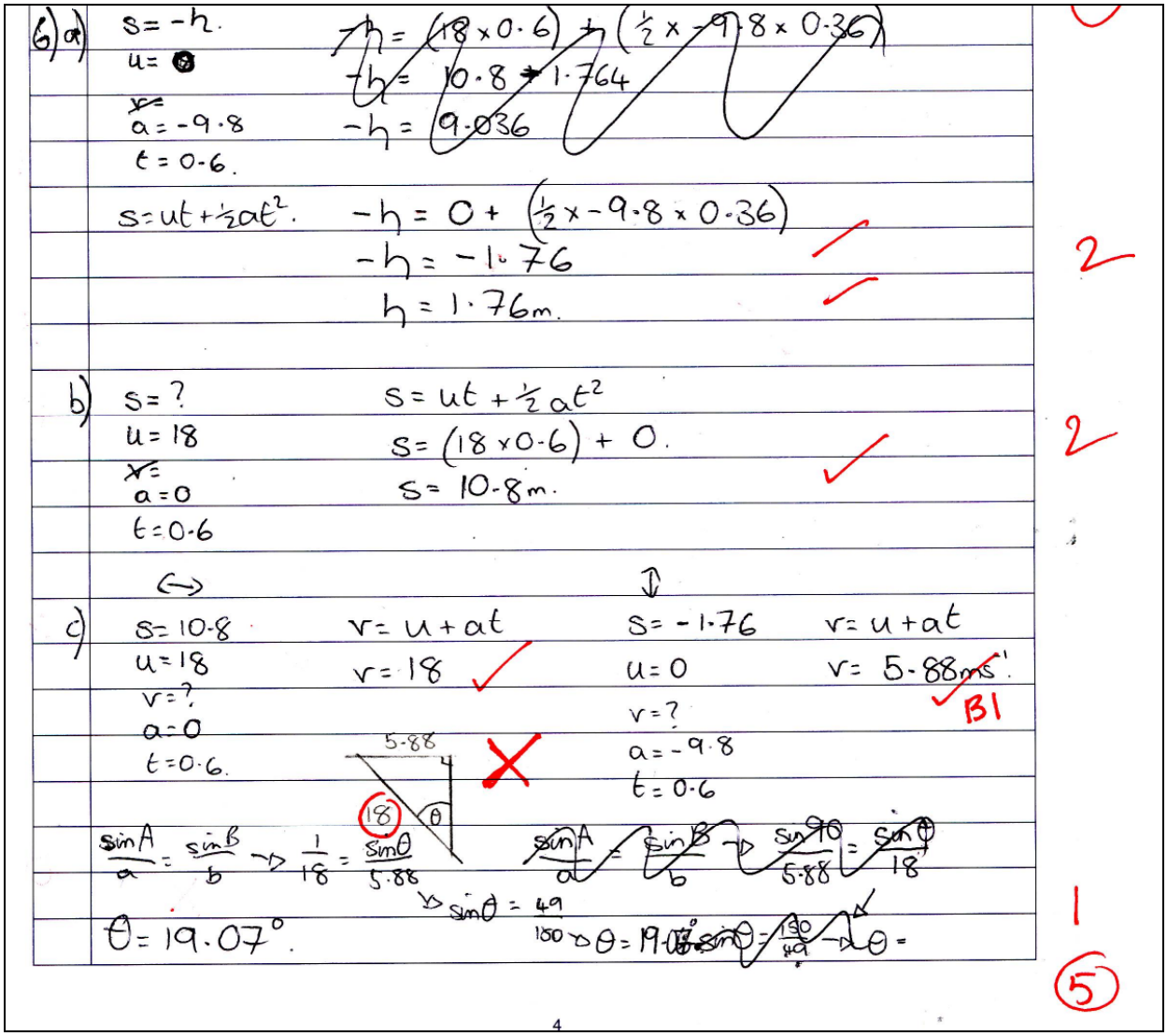
6 An arrow is fired horizontally at a speed of $18\,\text{m s}^{-1}$ from a point at a height of h metres above horizontal ground. The arrow hits the ground after it has been moving for 0.6 seconds. Model the arrow as a particle that moves only under the influence of gravity.

(a) Show that $h = 1.76$, correct to three significant figures. *(2 marks)*

(b) Find the horizontal distance travelled by the arrow during its flight. *(2 marks)*

(c) Find the speed of the arrow and the direction in which it is moving when it hits the ground. *(6 marks)*

Student Response



Commentary

In parts (a) and (b) of the question, this candidates provides correct solutions which are clearly laid out. In part (a) the incorrect first attempt has been crossed out and replaced so that no marks are lost, which would have been the case if both solutions had been presented.

In the more challenging part (c) the candidate makes little progress. The vertical component of the velocity is calculated correctly as 5.88 and the B1 mark is awarded for this. The triangle that the candidate has drawn shows the confusion that is experienced at this stage. The horizontal component of 18 is shown as the resultant and 5.88 as a horizontal component instead of a vertical one. Due to this fundamental error no further marks can be awarded to this candidate.

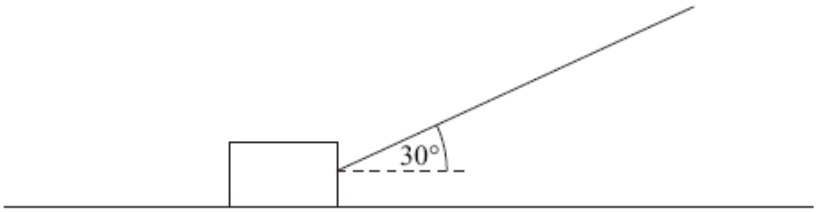
This example illustrates how many candidates find harder parts of projectile questions difficult to visualise and complete.

Mark Scheme

6(a)	$h=\frac{1}{2}\times 9.8\times 0.6^2=1.76$ m AG	M1 A1	2	M1: Equation to find h A1: Correct value from correct working
(b)	$x=18\times 0.6=10.8$ m	M1 A1	2	M1: Calculating range A1: Correct value
(c)	$v_y=9.8\times 0.6=5.88$ ms^{-1}	B1		B1: Correct vertical
	$v=\sqrt{5.88^2+18^2}=18.9$ ms^{-1}	M1 A1		M1: Calculating speed A1: Correct speed
	$\theta=\tan^{-1}\left(\frac{5.88}{18}\right)=18.1°$	M1 A1		M1: Calculating angle using tan A1: Correct angle
	18.9 ms^{-1} at 18.1° below the horizontal.	B1	6	B1: States below horizontal
	Total		**10**	

Question 7

7 The diagram shows a block, of mass 20 kg, being pulled along a rough horizontal surface by a rope inclined at an angle of 30° to the horizontal.



The coefficient of friction between the block and the surface is μ. Model the block as a particle which slides on the surface.

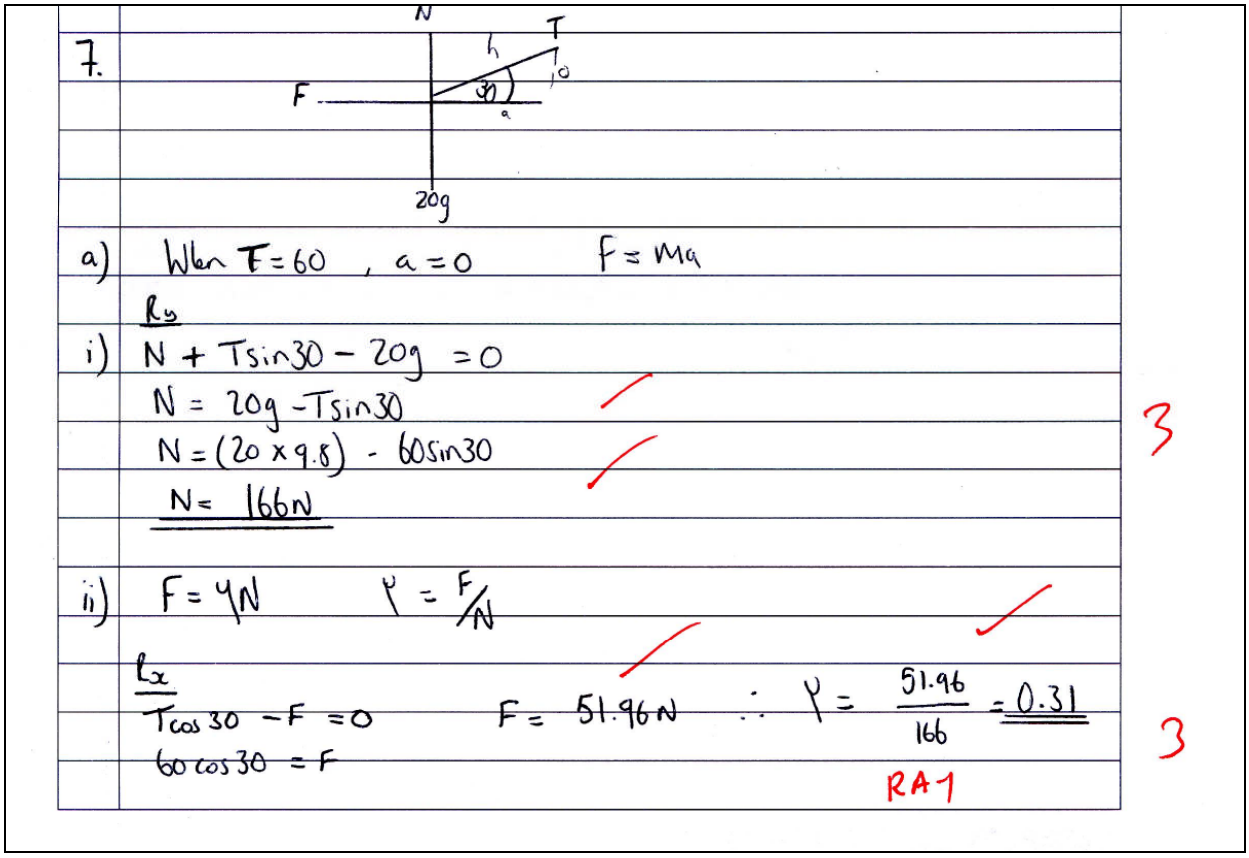
(a) If the tension in the rope is 60 newtons, the block moves at a constant speed.

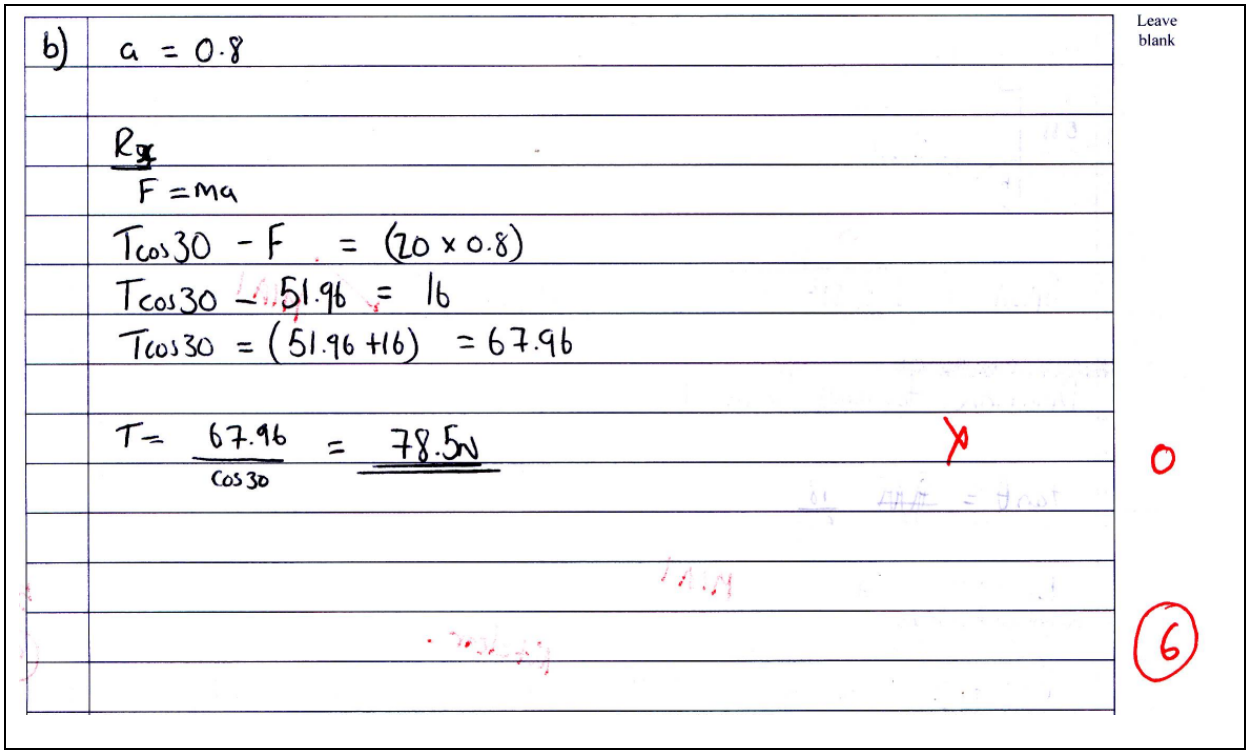
(i) Show that the magnitude of the normal reaction force acting on the block is 166 N. *(3 marks)*

(ii) Find μ. *(4 marks)*

(b) If the rope remains at the same angle and the block accelerates at $0.8\ \text{m s}^{-2}$, find the tension in the rope. *(5 marks)*

Student Response





Commentary

This example shows a very common response to this question. Part (a) contains the correct working and one mark has been lost due to the accuracy of the answer for the coefficient of friction. The question paper asks candidates to give their answers correct to three significant figures, so one mark is lost here as the answer is only given to two significant figures. Please note that only one mark per script can be lost for this reason.

In part (b) the candidate uses the value for the friction from part (a). As the candidates does not realise that the magnitude of the normal reaction force depends on the tension, no marks are awarded. This type of response was very common for this question.

This example illustrates how candidates can lose marks by not giving answers to the specified degree of accuracy and that marks are also lost for not considering the normal reaction correctly.

Mark Scheme

7(a)(i)	$20\times9.8=R+60\sin30°$ $(R=)20\times9.8-60\sin30°=166$ N AG	M1 A1 A1	3	M1: Equation or expression for normal reaction with mg or $20g$ or 196 and $60\sin30°$ or $60\cos30°$. A1: Correct equation or expression with correct signs. A1: Correct value from correct working. Must be positive. Don't penalise use of $g = 9.81$ if already done earlier on script. Should still get 166, but from 166.2.
(ii)	$166\mu=60\cos30°$ $\mu=\dfrac{60\cos30°}{166}$ $=0.313$	M1 M1A1 A1	4	M1: Use of $F=\mu R$, with $R = 166$ or 166.2. Do not allow inequalities here. M1: Resolving horizontally with $\cos30°$ or $\sin30°$ oe A1: Correct equation Examples: $166\mu=60$ M1M0A0 $166\mu=-60\cos30°$ M1M1A0 A1: Correct coefficient of friction.
(b)	$20\times0.8=T\cos30°-0.313(20\times9.8-T\sin30°)$ $T=\dfrac{20\times0.8+0.313\times20\times9.8}{\cos30°+0.313\sin30°}=75.6$ N	B1 M1 A1F dM1 A1F	5	B1: $20g-T\sin30°$ oe seen. M1: Three term equation of motion, where normal reaction is dependent on T. A1F: Correct equation dM1: Solving for T including factorisation. A1F: Correct tension AWRT 75.6 Follow through incorrect values of μ from part (a). Don't penalise use of $g = 9.81$ if already done earlier on script. Should get 75.7. Allow 75.8 if intermediate values rounded.
	Total		**12**	