Teacher Support Materials 2009

Maths GCE

Paper Reference MM04

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Dr Michael Cresswell, Director General.

Question 1

1 The cylindrical drum in a spin dryer rotates about its vertical axis. Initially, the drum is at rest. It then rotates with a constant acceleration and reaches its maximum angular speed of 1200 revolutions per minute in 10 seconds.

(a) Show that the magnitude of the angular acceleration is $4\pi \text{ rad s}^{-2}$. *(4 marks)*

(b) A couple of constant magnitude 100π N m causes the drum to rotate with this angular acceleration. Find the moment of inertia of the drum about the axis of rotation. *(2 marks)*

Student Response

1a) Max. angular speed is 1200 rev/min $= \frac{1200 \text{ rev}}{1 \text{ min}} = \frac{1200(2\pi) \text{ rad}}{60 \text{ sec}}$

$= \left(\frac{2400\pi}{60}\right) \text{rad}\cdot\text{s}^{-1} = 40\pi \text{ rad}\cdot\text{s}^{-1}$

Constant angular accn, so we can use angular 'uvats' equations.

$\Omega = 0 \text{ rad}\cdot\text{s}^{-1}$

$\omega = 40\pi \text{ rad}\cdot\text{s}^{-1}$

$t = 10$ sec

$\omega = \Omega + \alpha t$] $40\pi = 0 + \alpha(10) = 10\alpha \Rightarrow \alpha = \frac{40\pi}{10} = 4\pi \text{ rad}\cdot\text{s}^{-2}$ ✓

number

1b) $C = I\ddot{\theta}$] $100\pi = I(4\pi) \Rightarrow I = \frac{100\pi}{4\pi} = 25 \text{ kg m}^2$

2

Commentary

This question proved less of a straight forward opener for some candidates. A small number did not seem to be aware of the two aspects need to solve this problem. On average candidates scored just under 80% of the total mark available.

The solution shown above illustrates an excellent solution. Units are used throughout the conversion. Variables are defined clearly and the appropriate formulas are used. A 'perfect' answer.

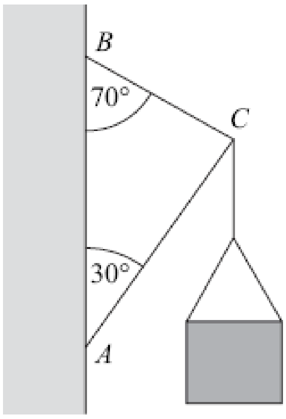
Mark scheme

1(a)	1200 rev per min = $\frac{1200\times 2\pi}{60}$ rad s^{-1}	M1		Attempt to convert to rad s^{-1}
	$= 40\pi$	A1		
	Using $\omega = \omega_0 + \ddot{\theta}t$	M1		Use of constant acceleration formula
	$\ddot{\theta} = \frac{40\pi - 0}{10}$			
	$= 4\pi$	A1	4	AG
(b)	Using $C = I\ddot{\theta}$	M1		Attempt to use $C = I\ddot{\theta}$
	$100\pi = 4\pi I$			
	$I = 25$ $(kg\,m^2)$	A1F	2	ft $\ddot{\theta}$ from (a)
	Total		**6**	

Question 2

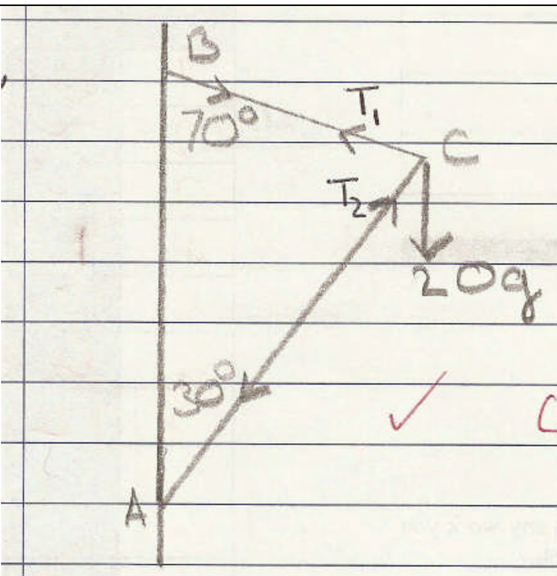
2 Two light smoothly-jointed rods, AC and BC, support a shop sign of mass 20 kg.

The two rods are smoothly hinged to a vertical wall at A and B, with B directly above A. Angle BAC is 30° and angle ABC is 70°. The shop sign hangs in equilibrium from C, as shown in the diagram.

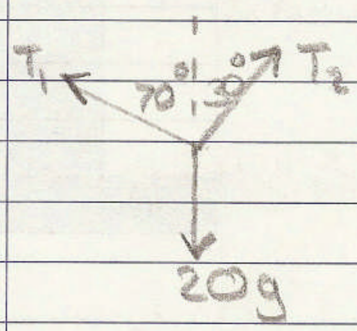


Find the magnitudes of the forces in rods AC and BC, stating whether the rods are in tension or compression. *(7 marks)*

Student response



✓ Compression / tension on diagram.



TCS
OAO
AHH

$$T_2\cos 30 + T_1\cos 70 = 20g$$ ✓

$$T_2\sin 30 = T_1\sin 70$$ ✓

$$T_1 = \frac{T_2\sin 30}{\sin 70}$$

$$T_2\cos 30 + \left(\frac{T_2\sin 30}{\sin 70}\right)\cos 70 = 20g$$

$$\frac{\sqrt{3}T_2}{2} + T_2(0.182) = 20g$$

$$T_2 = \frac{20g}{\left(\frac{\sqrt{3}}{2} + 0.182\right)} = \underline{187\text{N}}$$ ✓

$T_1 \sin 70 = 187 \sin 30$

$T_1 = \frac{187 \sin 30}{\sin 70}$

$T_1 = 99.5N$ ✓

~~Tension~~ Force in BC is 99.5N, and is in tension ✓

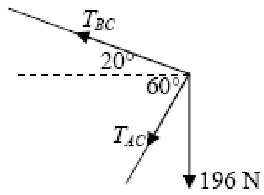
~~Tension~~ Force in AC is 187N and is in compression. ✓

Commentary

On average candidates scored just under 80% of the mark available for this question. A good response though candidates who refuse to use the aid of a sketch always come unstuck. Candidates should be encouraged to label diagrams with clear directions for tensions or compressions.

In the example above the candidate has used clearly labelled diagrams. They identified the tension and compression at the start of the question. Almost all candidates correctly identified AC in compression and BC in tension. Some candidates formulated the correct equations but then slipped up on the Pure Mathematics skills required to solve them. In this example clear working out is shown with equations stated. The solution scored full marks but it could still be improved by stating both where and in which direction the forces are to be resolved, before forming the equations.

Mark Scheme

2				
	Resolve horizontally at C	M1		Resolve in one direction – one correct component
	$T_{BC}\cos 20° + T_{AC}\cos 60° = 0$	A1		Fully correct equation
	Resolving vertically at C	M1		Resolve in second direction – one correct component
	$T_{BC}\sin 20° = T_{AC}\sin 60° + 196$	A1		Fully correct equation
	Solving gives: $\lvert T_{AC}\rvert = 187\text{N}$	M1		Attempt to solve their pair of equations – eliminate a variable
	$\lvert T_{BC}\rvert = 99.5\text{N}$	A1		Both correct; accept ±
	AC in compression **and** BC in tension	B1	7	Both correct
	Total		7	

Question 3

3 The forces $\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, $2\mathbf{i} - 7\mathbf{j} - \mathbf{k}$ and $4\mathbf{j} - 2\mathbf{k}$ act at the points with coordinates $(2, 1, 0)$, $(1, 13, -2)$ and $(6, 4, -7)$ respectively. The resultant of the three forces is a single force $\mathbf{F}$.

(a) Show that the magnitude of $\mathbf{F}$ is 7. *(3 marks)*

(b) The point P has coordinates $(3, 6, -3)$.

(i) Find the moment of the force $4\mathbf{j} - 2\mathbf{k}$ about P. *(4 marks)*

(ii) Given that the resultant of the two forces $\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ and $2\mathbf{i} - 7\mathbf{j} - \mathbf{k}$ acts through P, state the moment of $\mathbf{F}$ about P, giving a reason for your answer. *(2 marks)*

Student Response

number — Leave blank

3a) $F_1 \begin{pmatrix}1\\5\\-3\end{pmatrix}$ at $\begin{pmatrix}2\\1\\0\end{pmatrix}$ $F_2 \begin{pmatrix}2\\-7\\-1\end{pmatrix}$ at $\begin{pmatrix}1\\13\\-2\end{pmatrix}$ $F_3 \begin{pmatrix}0\\4\\-2\end{pmatrix}$ at $\begin{pmatrix}6\\4\\-7\end{pmatrix}$

$F_1 + F_2 + F_3 = F$

$F = \begin{pmatrix}1\\5\\-3\end{pmatrix} + \begin{pmatrix}2\\-7\\-1\end{pmatrix} + \begin{pmatrix}0\\4\\-2\end{pmatrix}$

$F = \begin{pmatrix}1+2\\5-7+4\\-3-1-2\end{pmatrix}$

$F = \begin{pmatrix}3\\2\\-6\end{pmatrix}$ ✓

~~Resultant of F =~~ Magnitude of $F = \sqrt{3^2+2^2+(-6)^2}$

$= \sqrt{9+4+36}$

$= \sqrt{49}$ ✓

$= 7$ — 3

b)(i) $P \begin{pmatrix}3\\6\\-3\end{pmatrix}$ Moment F_3 about $P = \begin{vmatrix}i & j & k\\3 & 6 & -3\\0 & 4 & -2\end{vmatrix}$ (SC1) — B0

$= (6\times-2-4\times-3)i - (3\times-2+0)j$ — M1

$+ (3\times4+0)k$ — A F

$= 0i + 6j + 12k$ — A0

(ii) Forces F_1 and F_2 act at P and so don't produce a moment

(?) Moment of F about P = Moment of F_3 about P — B1F

Moment of F about $P = 0i + 6j + 12k$ ✓ — E1

Commentary

On average candidates scored 75% of the mark available. Part a) proved very successful with all candidates. Several types of errors occurred in part b):

- Fxr rather than **r** x **F**
- Use of the wrong **r** – either the negative of the correct **r** or the coordinates of P
- Expanding the correct determinant but omitting a crucial negative sign

Many candidates gave good explanations for b)ii) – candidates who dropped marks in b)i) did recover here on a follow through principle.

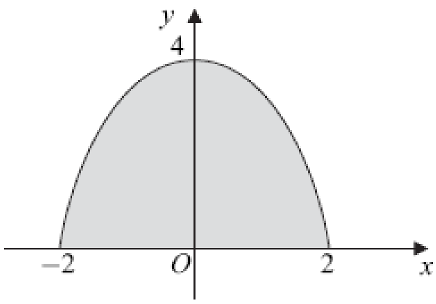
In the example shown the candidate has used he coordinates of P rather than the correct **r** vector. They still score two marks out of the four available as they have shown their ability to expan a determinant. In b)ii) the candidate scores both marks as they have simply used their incorrect answer from b)i) – hence no further error. The explanation is good too.

Mark Scheme

3(a)	$\mathbf{F} = \begin{pmatrix}1\\5\\-3\end{pmatrix}+\begin{pmatrix}2\\-7\\-1\end{pmatrix}+\begin{pmatrix}0\\4\\-2\end{pmatrix}=\begin{pmatrix}3\\2\\-6\end{pmatrix}$	B1		Correct total
	$\lvert\mathbf{F}\rvert=\sqrt{3^2+2^2+6^2}$	M1		Attempt to find $\lvert\mathbf{F}\rvert$
	$=7$	A1	3	AG
(b)(i)	$\mathbf{r} = \begin{pmatrix}6\\4\\-7\end{pmatrix}-\begin{pmatrix}3\\6\\-3\end{pmatrix}=\begin{pmatrix}3\\-2\\-4\end{pmatrix}$	B1		Correct **r**
	Moment $=\mathbf{r}\times\mathbf{F}$ $=\begin{vmatrix}\mathbf{i} & 3 & 0\\ \mathbf{j} & -2 & 4\\ \mathbf{k} & -4 & -2\end{vmatrix}$	M1		Attempt at $\mathbf{r}\times\mathbf{F}$ or $\mathbf{F}\times\mathbf{r}$
	$=\begin{pmatrix}20\\6\\12\end{pmatrix}$	A2,1,0	4	One component correct $\Rightarrow$ A1 All components correct $\Rightarrow$ A2 SC1: $\mathbf{F}\times\mathbf{r}\Rightarrow$ M1A1A0 SC2: Use of $\begin{pmatrix}-3\\2\\4\end{pmatrix}$ to get $\begin{pmatrix}-20\\-6\\-12\end{pmatrix}$ scores B0 M1 A1 A1F SC3: Use of $\mathbf{r}=\begin{pmatrix}3\\6\\-3\end{pmatrix}$ to get $\begin{pmatrix}0\\6\\12\end{pmatrix}$ scores B0 M1 A1F A0
(ii)	Moment $=\begin{pmatrix}20\\6\\12\end{pmatrix}$	B1F		ft (b)(i)
	since resultant of other two forces acts through given point (therefore 0 moment)	E1	2	
	Total		**9**	

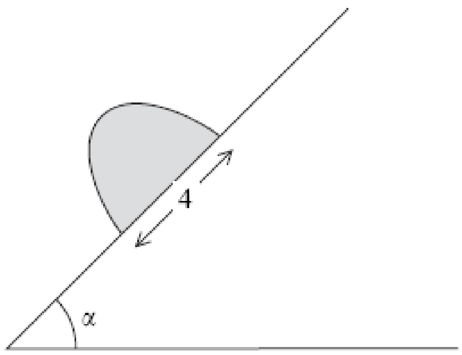
Question 4

4 (a) A uniform lamina is bounded by the curve $y = 4 - x^2$ and the x-axis, as shown in the diagram.



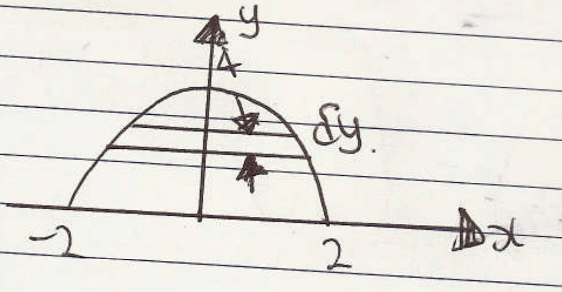
Given that the area of the lamina is $\frac{32}{3}$ square units, find the y-coordinate of the centre of mass of the lamina. *(5 marks)*

(b) The cross-section of a uniform prism is the same shape as the lamina in part (a). The prism is placed on a plane inclined at an angle α to the horizontal with the rectangular base of the prism in contact with the inclined plane, as shown in the diagram.



Given that the prism is just about to topple and that no slipping occurs, find the value of α, giving your answer to the nearest degree. *(4 marks)*

Student Response



considering one strip, width δy, the area is

$2x \, \delta y \, x \sqrt{4-y}$

(as the curve is symmetrical about the y axis, and the length is given by $2 \times x$ (co-ord)

so mass of strip $= 2(4-y)^{1/2} \delta y \, \rho$

$= 2(4-y)^{1/2} \delta y \cdot \frac{3m}{32}$

$= \frac{3m}{16}(4-y)^{1/2} \delta y$

take moments about the x axis $\Rightarrow$

$m\bar{y} = \lim_{\delta y \to 0} \sum_0^4 \frac{3m}{16}(4-y)^{1/2} \delta y \cdot y$

$\therefore \bar{y} = \int_0^4 \frac{3}{16} y (4-y)^{1/2} \, dy$

parts $\Rightarrow$ $= \left[\frac{3}{16} y \left[\frac{\frac{2}{3}(4-y)^{3/2}}{-1}\right]\right]_0^4 - \int_0^4 \frac{3}{16} \cdot \frac{\frac{2}{3}(4-y)^{3/2}}{-1} \, dy$

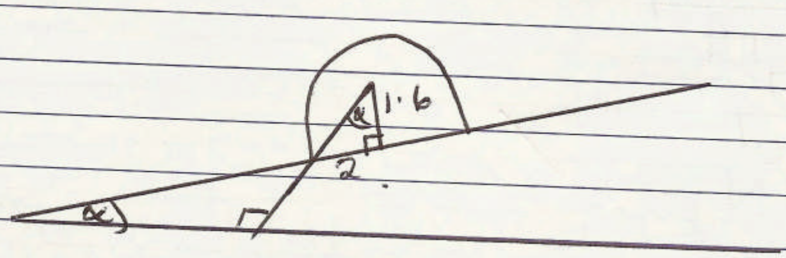
$$= \frac{3}{16}(4)\left(-\frac{2}{3}(0)^{3/2}\right) - 0 + \frac{1}{8}\int_0^4 (4-y)^{3/2}\,dy$$

$$= \frac{1}{8}\left[\frac{\frac{2}{5}(4-y)^{5/2}}{-1}\right]_0^4$$

Alternative

$$= \frac{1}{8}\left[0 + \frac{2}{5}(4)^{5/2}\right]$$

$$= 1.6 = \frac{8}{5}$$



point of toppling $\Rightarrow$

$$\tan\alpha = \frac{2}{1.6} = 1.25$$

$$\therefore \alpha = \tan^{-1}(1.25)$$
$$= 51.3^\circ$$
$$= 51^\circ \text{ (to } 1^\circ)$$

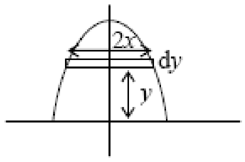
Commentary

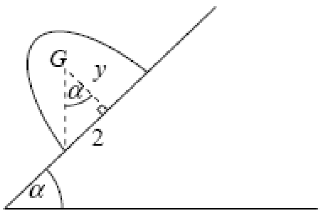
On average candidates scored 67% of the mark available. It was disappointing to see a significant number of candidates made heavy weather of this question. It is crucial that formulas which are stated in the formula book should be learnt. A number of candidates started from first principles and needed to do a lot of work to score the first mark. Many who chose this approach could not always complete the integration particularly if they had left the integrand as a function of y.

In the example shown the candidate does not state any formula to start with for part a). However they do build up the required integral correctly. Good step by step explanation. They have also correctly recognised the need for integration by parts with the correct limits. It is very rare to see this method fully used correctly.

Part b) was successful for all candidates.

Mark Scheme

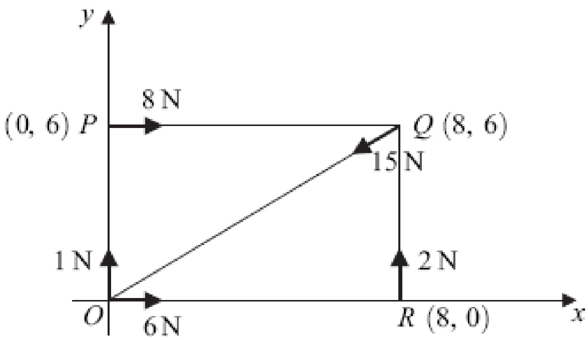
Q	Solution	Marks	Total	Comments
4(a)	$\frac{1}{2}\int_{-2}^{2} y^2\,dx$			
	$=\frac{1}{2}\int_{-2}^{2}\left(16-8x^2+x^4\right)dx$	M1		Attempt to integrate y^2 as a function of x
	$=\frac{1}{2}\left[16x-\frac{8x^3}{3}+\frac{x^5}{5}\right]_{-2}^{2}$	A1		Correct integration
	$=\frac{256}{15}$	A1F		Correct limits applied to their integral
	$\overline{y}=\frac{256/15}{32/3}$	m1		$\overline{y}=\frac{\frac{1}{2}\int_a^b y^2\,dx}{\text{Area}}$
	$=\frac{8}{5}$	A1	5	
	Alternative 1: 			
	$I=\int_{x=2}^{x=0} 2xy\,dy=\int_{2}^{0} 2x\left(4-x^2\right)(-2x)\,dx$			
	$=\int_{2}^{0} 4x^4-16x^2\,dx$	(M1)		Attempt to integrate $2xy$ as a function of x
	$I=\left[\frac{4x^5}{5}-\frac{16x^3}{3}\right]_{2}^{0}$	(A1)		Correct integration
	$I=\frac{256}{15}$	(A1F)		Correct limit applied to their integral
	$\overline{y}=\frac{I}{32/3}=\frac{8}{5}$	(m1) (A1)		Their evaluated $I\div$ area
	Alternative 2:			
	$I=\int_{y=0}^{y=4} 2xy\,dy=\int_{0}^{4} 2\sqrt{(4-y)}\,y\,dy$	(M1)		Attempt to integrate $2xy$ as a function of y by parts
	$I=\left[-\frac{4y}{3}(4-y)^{\frac{3}{2}}\right]_0^4+\int_0^4 \frac{4}{3}(4-y)^{\frac{3}{2}}\,dy$			
	$I=\left[-\frac{8}{15}(4-y)^{\frac{5}{2}}\right]_0^4$	(A1)		Fully integrated
	$I=\frac{256}{15}$	(A1F)		Correct limit applied to their integral
	$\overline{y}=\frac{I}{32/3}=\frac{8}{5}$	(m1) (A1)		Their evaluated $I\div$ area

4(a) cont	**General for all three versions:** M1 Attempt to integrate an appropriate function of x or y (must apply a full method) A1 Correct integration A1F Correct limits applied to their integral m1 Their evaluated integral $\div$ area A1 Correct answer $\frac{8}{5}$			
(b)				
	$\tan\alpha = \frac{2}{\bar{y}}$	M1 A1F		$\tan\alpha$ seen Correct structure – ft error in (a)
	$\tan\alpha = \frac{2}{8/5} = 1.25$	m1		Substitute and use of $\tan^{-1}$ – dependent on first M1
	$\alpha = 51°$	A1F	4	ft error in (a)
	Total		**9**	

Question 5

5 The points O, P, Q and R have coordinates $(0, 0)$, $(0, 6)$, $(8, 6)$ and $(8, 0)$ respectively. The units of length are metres.

A force of 1 N acts at O along OP.
A force of 2 N acts at R along RQ.
A force of 6 N acts at O along OR.
A force of 8 N acts at P along PQ.
A force of 15 N acts at Q along QO.



(a) Show that the resultant of the five forces has magnitude $2\sqrt{10}$ N. *(5 marks)*

(b) An anticlockwise couple of magnitude 20 N m together with these five forces is equivalent to a single force of magnitude $2\sqrt{10}$ which has a line of action passing through the point $(0, d)$.

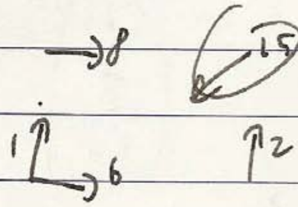
(i) Find d. *(5 marks)*

(ii) Determine the equation of the line of action of the resultant, giving your answer in vector form. *(3 marks)*

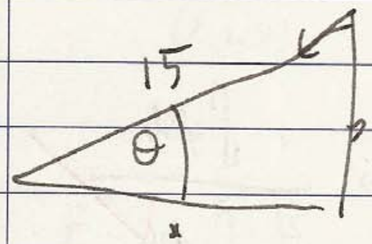
Student Response

5.

a.



$\Rightarrow \uparrow 36$ ... $\rightarrow 14.$



$\tan\theta = \frac{6}{8}$

$\therefore \theta = 36.9°$

$\therefore x = 12.0 \qquad y = 9.$

$\Rightarrow$ total $= 14 - 12 = 2 \rightarrow$

$9 - 9 ... = -6 \uparrow$

~~to magnitude $= \sqrt{12^2 + 26^2} = \sqrt{820}$~~

$\Rightarrow$ magnitude $= \sqrt{(-6)^2 + 2^2} = \sqrt{40}$

$= \sqrt{4 \times 10} = 2\sqrt{10}$

5

bi. The moment of these forces is $0 \times 1 + 0 \times 6 - 8 \times 6 + 2 \times 8$

$= -32.$

M1 $\therefore$ total magnitude of turning effect (about O) $= -32 + 20 = -12.$

A1

A1 $\therefore$ d acts at y, where $-(2\sqrt{10} \times y) = -12 \therefore y = \frac{6}{\sqrt{10}}$

d $(0, \frac{6}{\sqrt{10}})$

M0

A0 ii. line of action of resultant $= \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -6 \end{pmatrix}$

Commentary

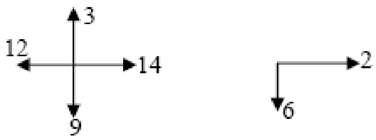
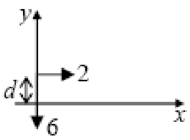
This proved to be of equal difficulty as the rotational dynamics questions. Part a) was the most successful though candidates must be aware to show full working when a printed answer is given. In the second part the principles of moments were understood though not always applied fully successfully. Common errors were:

- Inconsistent directions
- Failure to understand what to do with the couple of magnitude 20NM
- Using the magnitude of the resultant rather than the required x component

It was disappointing to see very few candidates attempt a vector equation. When it was attempted quite often only the right hand side was given. On average candidates scored 60% of the mark available.

In the example shown the candidate completes part a) correctly. They have used a sketch of a right angled triangle to deal with the exact trigonometrical values (rather than a calculator and rounded values). In part b)i) the total moment of the system is obtained correctly with consistent use of signs. The error occurs by using the resultant magnitude of the forces rather than the x component. In b)ii) the candidate correctly states the vector equation, although misses out the '**r** = '. No penalty as given for this as few candidates managed to even identify the structure.

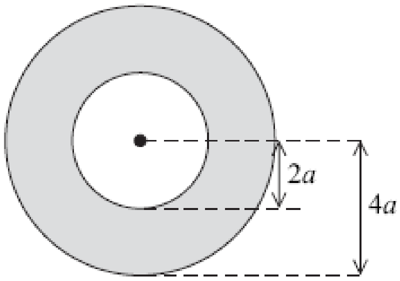
Mark Scheme

Q	Solution	Marks	Total	Comments
5(a)	Let resultant be $\begin{pmatrix} X \\ Y \end{pmatrix} = R$			
	$X = 8 + 6 - 15\cos\theta$ $Y = 1 + 2 - 15\sin\theta$	M1		Attempt at X and Y; must involve use of $15\sin\theta$ or $15\cos\theta$
	with $\cos\theta = \frac{8}{10}$ and $\sin\theta = \frac{6}{10}$ or $\theta = 36.9°$	A1		Either 12 or 9 seen as components of the 15N force
	$\Rightarrow X = 2,\ Y = -6$	A1		Both X and Y correctly evaluated including direction
	$\lvert R \rvert = \sqrt{2^2 + 6^2} = \sqrt{40}$	m1		Attempt at $\lvert R \rvert$
	$= 2\sqrt{10}$	A1	5	AG; must see $\sqrt{40}$ or $\sqrt{4 \times 10}$
	Alternative – using diagrams:			
		(M1) (A1) (A1) (m1) (A1)		4 components shown 12 or 9 seen Resultant components - correct direction shown As above As above
(b)(i)	Moments about O for system $20 + 2(8) - 8(6) = -12$	M1 A2,1,0		Attempt at moments for system –1 each error or omission
	(ie 12 Nm clockwise)			
	 Moment of resultant $2d = 12$ $d = 6$	M1 A1	5	Form equation – must be of form x-component $\times d$ = moment for system
(ii)	$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + t\begin{pmatrix} 1 \\ -3 \end{pmatrix}$	M1		Correct structure on RHS (**a** + t**b**)
		A1F		$\begin{pmatrix} 0 \\ 6 \end{pmatrix}$; ft d value from (b)(i)
		A1F	3	$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ OE; ft components from (a)
				Condone omission of $\begin{pmatrix} x \\ y \end{pmatrix}$ or **r** on LHS
	Total		**13**	

Question 6

6 (a) Show, by integration, that the moment of inertia of a uniform disc, of mass m and radius r, about an axis through its centre and perpendicular to the plane of the disc is $\frac{mr^2}{2}$. *(5 marks)*

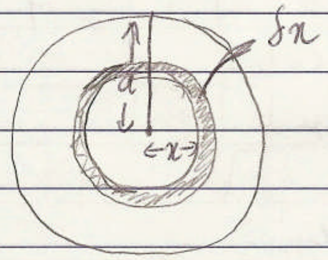
(b) A disc, of radius $2a$, is removed from the centre of a uniform disc, of radius $4a$. The resulting ring has mass M and is shown in the diagram.



Using the result from part (a), or otherwise, show that the moment of inertia of the ring about an axis through its centre and perpendicular to the plane of the ring is $10Ma^2$. *(5 marks)*

(c) Determine the moment of inertia of the ring about an axis along a diameter, stating any theorem that you use. *(3 marks)*

Student Response



Total mass of disc:

$m = \pi a^2 \rho$

$\rightarrow \rho = \frac{m}{\pi a^2}$

Area of the shaded region above is

$\pi (x + \delta x)^2 - \pi x^2$

$= \pi (x^2 + 2x\delta x + \delta x^2 - x^2)$

$= \pi (2x\delta x + \delta x^2)$

Since $2x\delta x >> \delta x^2$ we can ignore δx^2 ~~the~~ ~~2xδx~~ in equation.

Thus area is $\pi 2x \delta x$ ✓

$\rightarrow$ mass ~~of~~ is $\rho \pi 2x\delta x = \frac{m \pi 2x \delta x}{\pi a^2} = \frac{2mx\delta x}{a^2}$ ✓

Moment of inertia of small disc is δI

~~$\delta I =$~~

Leave blank

$\delta I = m x^2$

$= \frac{2mx\,\delta x}{a^2} x^2$

$\frac{\delta I}{\delta x} = \frac{2mx^3}{a^2}$

As $\delta x \to 0 \Rightarrow \frac{dI}{dx} = \frac{2mx^3}{a^2}$

$\Rightarrow I = \frac{2m}{a^2}\int_0^a x^3\,dx$

$= \frac{2m}{a^2}\left[\frac{1}{4}x^4\right]_0^a = \frac{2m}{a^2}\left[\frac{1}{4}a^4\right] = \frac{ma^2}{2}$ ✓ 5

b) ~~Moment of inert~~

~~Mass~~ ~~Since~~ Mass removed $= \frac{1}{4}$ mass of whole disc ✓

(since $m \propto a^2$)

	Mass	Moment of inertia
Whole	4m	$4m \times \frac{(4r)^2}{2} = 32mr^2$
~~Lost~~		
Lost	m	$\frac{m(2r)^2}{2} = 2mr^2$
Remain	$3m (= M)$ ✓	Unknown

B1 M1 A1 A1 A1

Alternative method

Moment of inertia of remaining section

$32mr^2 - 2mr^2 = 30mr^2$

$= 10Mr^2$ ✓ (Condone r not a)

c) $I = \frac{10Ma^2}{2} = 5Ma^2$ ✓

Perpendicular axes theorem used ✓ 3

Commentary

Very much a hit or miss question. Many candidates were awarded full marks others struggled to gain just a few. On average candidates scoed just under 60% of the mark available. Part a) was the best attempted part with good clear explanations from most candidates. The example shown was one of the best seen – step by step explanation.

Part b) resulted in varied methods with the most successful using the ratio of areas in relation to the various masses. Those who used integration again with different limits were often successful. This candidate has chosen the ratio approach and explains the method fully – very efficient.

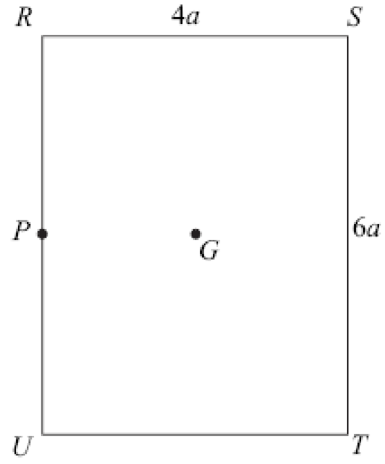
Part c) wrong footed many candidates who thought that the parallel axis theorem should be used. This candidate correctly isdentifies the perpendicular axis theorem.

Mark Scheme

6(a)	$m = \pi r^2 \rho \Rightarrow \rho = \dfrac{m}{\pi r^2}$	B1		ρ and m linked – used anywhere
	Mass of elemental ring $= 2\pi x\,\delta x\,\rho$	M1		Attempt at mass
	MI of elemental ring $= (2\pi x\,\delta x\,\rho)x^2$	A1		Correct use of mr^2
	MI of disc $= \int_0^r 2\pi x^3 \rho\,\mathrm{d}x = \int_0^r \dfrac{2mx^3}{r^2}\mathrm{d}x$	m1		Attempt to integrate – dependent on first M1and **must** be of form $\int kx^3\,\mathrm{d}x$
	$= \left[\dfrac{mx^4}{2r^2}\right]_0^r = \dfrac{mr^2}{2}$	A1	5	AG
(b)	MI required = $\text{MI}_{\text{large disc}} - \text{MI}_{\text{small disc}}$ $= \dfrac{(4a)^2\pi\rho(4a)^2}{2} - \dfrac{(2a)^2\pi\rho(2a)^2}{2}$	M1		Attempt at difference of MIs – $4a$, $2a$ substituted for r_1, r_2 $\dfrac{M(4a)^2}{2} - \dfrac{m(2a)^2}{2}$ ok for M1
		A1		Correct MI for either disc - must involve correct masses or ratios
	$= 120\pi a^4 \rho$	A1		Correct difference
	$M = 12a^2\pi\rho$	B1		Mass of ring
	$\Rightarrow \text{MI} = 10Ma^2$	A1	5	AG
	Alternative 1:			
	$M = 12a^2\pi\rho \Rightarrow \rho = \dfrac{M}{12a^2\pi}$	(B1)		ρ and M linked – used anywhere
	MI of hoop = $\int_{2a}^{4a} 2\pi x^3 \rho\,\mathrm{d}x = \int_{2a}^{4a} \dfrac{2\pi x^3 M}{12a^2\pi}\mathrm{d}x = \int_{2a}^{4a} \dfrac{Mx^3}{6a^2}\mathrm{d}x$	(M1)		Integral with correct limits - any form given here
	$= \left[\dfrac{Mx^4}{24a^2}\right]_{2a}^{4a} = \dfrac{M(4a)^4}{24a^2} - \dfrac{M(2a)^4}{24a^2}$	(A1)		Correct integration
	$= 10Ma^2$	(M1) (A1)		Use of correct limits AG
	Alternative 2: Mass removed = ¼ of mass of whole disc (as mass is proportional to radius ^{2})			
	Let masses be $4m$ and m; remaining mass = $3m$	(B1)		Ratio of masses
	$\text{MI}_{\text{large disc}} = \dfrac{4m(4a)^2}{2} = 32ma^2$	(M1)		MI of either
	$\text{MI}_{\text{small disc}} = \dfrac{m(2a)^2}{2} = 2ma^2$	(A1)		Both correct
	Difference = $30ma^2$	(A1)		Difference
	$= 10(3m)a^2 = 10Ma^2$	(A1)		Converting answer
6 cont (c)	Using the perpendicular axis theorem	E1		
	$10Ma^2 = \text{I}_\text{D} + \text{I}_\text{D}$	M1		
	$\therefore\ \text{I}_\text{D} = 5\text{M}a^2$	A1	3	
	Total		**13**	

Question 7

7 A uniform rectangular lamina, $RSTU$, has mass M, with $RS = 4a$ and $ST = 6a$. The centre of mass of the lamina is G, and the mid-point of RU is P.



(a) (i) Show that the moment of inertia of the rectangular lamina about an axis perpendicular to its plane and passing through G is $\frac{13Ma^2}{3}$. *(2 marks)*

(ii) Hence find the moment of inertia of the lamina about an axis perpendicular to its plane and passing through P. *(2 marks)*

(b) The lamina is smoothly hinged at P. It is free to rotate in a vertical plane about a fixed horizontal axis which is perpendicular to its plane and passes through P. Initially, the lamina is held with PG horizontal and then released. At time t after release, PG makes an angle θ with the horizontal.

(i) Show that $\dot{\theta}^2 = \frac{12g\sin\theta}{25a}$. *(5 marks)*

(ii) Hence, or otherwise, determine an expression for $\ddot{\theta}$, in terms of a, g and θ. *(3 marks)*

(iii) Show that the magnitude of the component in the direction GP of the force at P which the hinge exerts on the lamina is $\frac{49Mg\sin\theta}{25}$. *(3 marks)*

(iv) Find, in terms of M, g and θ, the magnitude of the component in the direction PR of the force at P which the hinge exerts on the lamina. *(3 marks)*

Student Response

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7ai $I = \text{'}\frac{1}{3}m(a^2+b^2)\text{'}$

$= \frac{1}{3}M\left((2a)^2+(3a)^2\right)$

$= \frac{1}{3}M\left(4a^2+9a^2\right)$

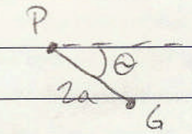
$= \frac{13}{3}Ma^2$ 2

ii Parallel axis theorem

$I = \frac{13}{3}Ma^2 + M(2a)^2$ 2

$= \frac{25}{3}Ma^2$

bi



$GPE_{lost} = KE_{gained}$

$Mg(2a\sin\theta) = \frac{1}{2}I\dot{\theta}^2$

$4aMg\sin\theta = \frac{25}{3}Ma^2\dot{\theta}^2$

$\frac{12g\sin\theta}{25a} = \dot{\theta}^2$ 5

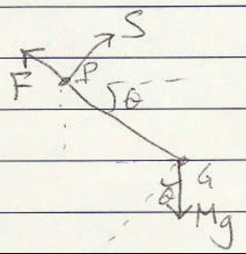
ii $\frac{d(\dot{\theta}^2)}{d\theta} = \frac{d}{d\theta}\left(\frac{12g\sin\theta}{25a}\right)$

$2\dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{12g\cos\theta}{25a}$

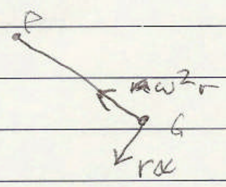
$\frac{d\theta}{dt}\cdot\frac{d\dot{\theta}}{d\theta} = \frac{6g\cos\theta}{25a}$

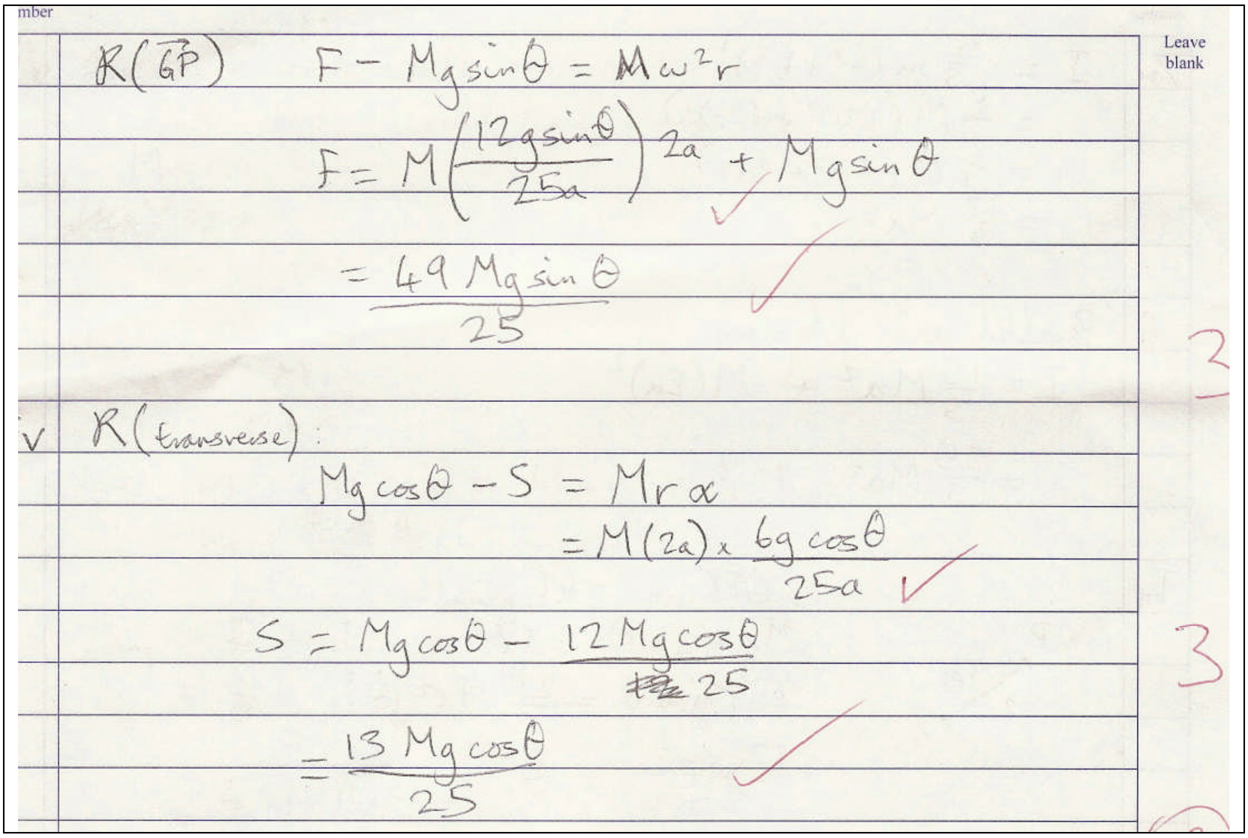
$\ddot{\theta} = \frac{6g\cos\theta}{25a}$ ✓ 3

iii Force:



Acceleration:





mber

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$R(\overrightarrow{GP})$ $\quad F - Mg\sin\theta = M\omega^2 r$

$$F = M\left(\frac{12g\sin\theta}{25a}\right)2a + Mg\sin\theta$$

$$= \frac{49\,Mg\sin\theta}{25}$$

2

v $R(\text{transverse})$:

$$Mg\cos\theta - S = Mr\alpha$$

$$= M(2a) \times \frac{6g\cos\theta}{25a}$$

$$S = Mg\cos\theta - \frac{12\,Mg\cos\theta}{25}$$

$$= \frac{13\,Mg\cos\theta}{25}$$

3

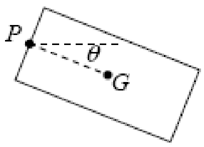
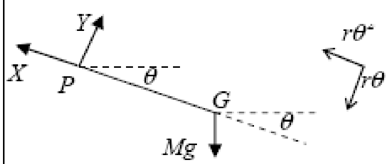
Commentary

On average candidates scored just under 60% of the mark available. An improvement on last year. Parts a) and b)i) were well attempted with good clear reasoning and explanation. This candidate clearly shows the formulas used and then applies them correctly.

In b)ii) candidates sometimes struggled in their attempts to differentiate – surprising given that it is a standard procedure. This candidate has chosen to do this in an unusual way with and application of the chain rule. Correct notation is used and the correct answer is obtained.

In iii) and iv) a mark was sometimes dropped through incorrect signs, particularly in iv). Only the stronger candidates were able to attempt these parts. This candidate uses clearly labelled diagrams – essential if the correct answer is to be obtained.

Mark Scheme

7(a)(i)	Use $I = \frac{1}{3}m\left(a^2+b^2\right)$			
	With $'a' = 2a \quad 'b' = 3a$	M1		Use of formulae booklet
	$I = \frac{1}{3}M\left(4a^2+9a^2\right) = \frac{13Ma^2}{3}$	A1	2	AG
(ii)	$I_M = I_G + Md^2$			
	$= \frac{13Ma^2}{3} + M(2a)^2$	M1		Use of Parallel Axis Theorem
	$= \frac{25Ma^2}{3}$	A1	2	
(b)(i)	 KE gained $= \frac{1}{2}I\dot{\theta}^2$			
	$= \frac{25Ma^2}{6}\dot{\theta}^2$	B1F		ft from (a)(ii)
	PE lost $= mgh = Mg\,2a\sin\theta$	B1		
	C of E $\Rightarrow \frac{25Ma^2}{6}\dot{\theta}^2 = 2Mga\sin\theta$	M1 A1F		Forms equation: KE gained = PE lost ft their expressions - 2 terms
	$\dot{\theta}^2 = \frac{12g\sin\theta}{25a}$	A1	5	AG
(ii)	Differentiating $2\dot{\theta}\ddot{\theta} = \frac{12g}{25a}\cos\theta\dot{\theta}$	M1A1		M1 RHS, A1 LHS
	Cancelling $\ddot{\theta} = \frac{6g}{25a}\cos\theta$	A1	3	
	Alternative:			
	$C = I\ddot{\theta}$ gives $Mg\cos\theta.2a = \frac{25Ma^2}{3}\ddot{\theta}$	(M1) (A1)		M1 one side correct A1 fully correct
	$\ddot{\theta} = \frac{6g}{25a}\cos\theta$	(A1)		
7 cont (b)(iii)				
	Along *GP*:			
	$X - Mg\sin\theta = M(2a)\frac{12g}{25a}\sin\theta$	M1A1		$X \pm$ component $= \pm Mr\dot{\theta}^2$ M1 one side, A1 both sides correct (structure)
	$X = Mg\sin\theta + \frac{24Mg}{25}\sin\theta = \frac{49Mg}{25}\sin\theta$	A1	3	AG
(iv)	Along *PR*:			
	$Y - Mg\cos\theta = -M(2a)\frac{6g}{25a}\cos\theta$	M1 A1		$Y \pm$ component $= \pm Mr\ddot{\theta}$ M1 one side, A1 both sides correct (structure)
	$Y = -\frac{12Mg}{25}\cos\theta + Mg\cos\theta$			
	$= \frac{13Mg}{25}\cos\theta$	A1	3	Must be simplified
	Total		**18**	