Teacher Support Materials 2009

Maths GCE

Paper Reference MFP2

Question 1

1 Given that $z = 2e^{\frac{\pi i}{12}}$ satisfies the equation

$$z^4 = a(1 + \sqrt{3}\,i)$$

where a is real:

(a) find the value of a; *(3 marks)*

(b) find the other three roots of this equation, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(5 marks)*

Student Response

1) a) $z = 2e^{\frac{\pi}{12}i}$

$e^{\frac{\pi}{12}i} = 2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

$\left(2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right)^4$

$= 16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ ✓

$$16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 16\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$= 8(1 + \sqrt{3}i)$$

$$\Rightarrow a = 8$$

3

b) $z^4 = 8(1+3i)$

$z^4 = 16e^{\frac{i\pi}{3}}$

$z = \cancel{\quad}\ 2e^{\frac{i\pi}{12} + \frac{k2\pi}{4}i}$ $\quad k = 0, \pm 1, \pm 2$

when $k = 0$, $z = 2e^{\frac{i\pi}{12}i}$

$k = \cancel{+}1$ $\quad z = 2e^{\frac{7\pi}{12}i}$

$k = -1$ $\quad z = 2e^{-\frac{5\pi}{12}i}$

5

$k = \pm 2$ $\quad z = 2e^{-\frac{11}{12}\pi i}$

8

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Commentary

There were many methods used for solving the first part of this question, some of which led to incorrect answers. This candidate worked with z4 correctly to provide 16(cos π/3 + isinπ/3). The 16 was frequently left as 2, and another common approach was to rewrite a(1+ 3I) in exponential form, but this approach sometimes led to an incorrect value of a due to poor arithmetic. The values of z in part (b) were written down with clarity and care. There were many incorrect different expressions for π i/12 +2kπi/4 but not only did the candidate write the roots out in full, but he made sure that they were written in the correct range and that the magnitude of each root was still 2.

Mark scheme

1(a)	$z^4 = 16e^{\frac{4\pi i}{12}}$	M1		Allow M1 if $z^4 = 2e^{\frac{4\pi i}{12}}$
	$= 16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$	A1		OE could be $2ae^{\frac{\pi i}{3}}$ or $2a\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$
	$= 8 + 8\sqrt{3}i$; $a = 8$	A1F	3	ft errors in 2^4
(b)	For other roots, $r = 2$	B1		for realising roots are of form $2 \times e^{i\theta}$
	$\theta = \frac{\pi}{12} + \frac{2k\pi}{4}$	M1A1		M1 for strictly correct θ i.e must be $\left(\text{their } \frac{\pi}{3} + 2k\pi\right) \times \frac{1}{4}$
	Roots are $2e^{\frac{7\pi i}{12}}, 2e^{\frac{-5\pi i}{12}}, 2e^{\frac{-11\pi i}{12}}$	A2,1,0 F	5	ft error in $\frac{\pi}{12}$ or r $\left[\text{accept } 2e^{\left(\frac{\pi}{12}+\frac{2k\pi}{4}\right)i} \quad k = -1, -2, 1\right]$
	Total		**8**	

Question 2

2 (a) Given that

$$\frac{1}{4r^2-1} = \frac{A}{2r-1} + \frac{B}{2r+1}$$

find the values of A and B. *(2 marks)*

(b) Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{4r^2-1} = \frac{n}{2n+1}$$ *(3 marks)*

(c) Find the least value of n for which $\sum_{r=1}^{n} \frac{1}{4r^2-1}$ differs from 0.5 by less than 0.001 . *(3 marks)*

Student response

2 a) $\frac{1}{4r^2-1} = \frac{A}{2r-1} + \frac{B}{2r+1}$

$1 = A(2r+1) + B(2r-1)$ $\quad$ [$\times$ by $(2r+1)(2r-1)$

coefficients of r $\quad 0 = 2A + 2B$

constants $\quad 1 = A - B$

✓

Leave blank

$A = 1 + B$

Sub into coefficients of r $\quad 0 = 2(1+B) + 2B$

$0 = 2 + 4B$

$4B = -2$ ✓

$B = -\frac{1}{2}$

$A = 1 - \frac{1}{2}$

$A = \frac{1}{2}$ ✓

2

$A = \frac{1}{2} \quad B = -\frac{1}{2}$

b) $\sum_{r=1}^{n} \frac{1}{4r^2-1} = \sum_{r=1}^{n} \frac{1}{2(2r-1)} - \frac{1}{2(2r+1)}$

$= \frac{1}{2\times1} - \frac{1}{2\times3}$ [r=1 ✓

$+ \frac{1}{2\times3} - \frac{1}{2\times5}$ [r=2

$+ \frac{1}{2\times5} - \frac{1}{2\times7}$ [r=3

$\vdots \qquad \vdots$

$+ \frac{1}{2(2n-1)} - \frac{1}{2(2n+1)}$ [r=n ✓

Leave blank

$$= \frac{1}{2} - \frac{1}{2(2n+1)}$$

$$= \frac{2n+1}{2(2n+1)} - \frac{1}{2(2n+1)}$$

$$= \frac{2n}{4n+2}$$

$$= \frac{n}{2n+1}$$

3

c) $\sum_{r=1}^{n} \frac{1}{4r^2-1} = \cancel{0.5}\ \frac{1}{2} - \frac{1}{2(2n+1)}$

$$= 0.5 - \frac{1}{2(2n+1)}$$

$$\therefore \frac{1}{2(2n+1)} < \frac{1}{1000}$$

$$\therefore 2(2n+1) > 1000$$

$$4n+2 > 1000$$

$$4n > 998$$

$$n > 249.5$$

$\therefore$ least value of $n = 250$

3

8

Commentary

This candidate shows clear methods for all parts of the question.In particular, (as part (a) was completely correctly done by virtually all candidates) sufficient rows were written down by the candidate to show the cancellation. Sometimes rows were written as
1/2(2-1) – 1/2(2+1) followed by 1/2(4-1) – 1/2(4+1) with cancellations. Part (c) was particularly well done with the use of inequalities (not often used) and the number rounded up at the end to 250 (again not always seen).

Mark Scheme

2(a)	$A=\frac{1}{2}$, $B=-\frac{1}{2}$	B1, B1F	2	For either A or B For the other
(b)	Method of differences clearly shown	M1		
	$\text{Sum}=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$	A1		
	$=\frac{n}{2n+1}$	A1	3	AG
(c)	$\frac{1}{2(2n+1)}<0.001$ or $\frac{n}{2n+1}>0.499$	M1		Condone use of equals sign
	$1<0.004n+0.002$ or $n>0.998n+0.499$			
	$n>\frac{0.998}{0.004}$ or $0.004n>0.998$	A1		OE
	$n=250$	A1F	3	ft if say 0.4999 used If method of trial and improvement used, award full marks for a completely correct solution showing working
	Total		8	

Question 3

3 The cubic equation

$$z^3 + pz^2 + 25z + q = 0$$

where p and q are real, has a root $\alpha = 2 - 3\mathrm{i}$.

(a) Write down another non-real root, β, of this equation. *(1 mark)*

(b) Find:

(i) the value of $\alpha\beta$; *(1 mark)*

(ii) the third root, γ, of the equation; *(3 marks)*

(iii) the values of p and q. *(3 marks)*

Student Response

blank

3)a) $\alpha = 2 - 3i$ since $\sum \alpha\beta = 25$, which is real, one other root must be non-real, ie $\beta = 2 + 3i$

b)i) $\alpha\beta = (2+3i)(2-3i) = (4 - 9i^2) = 13$

ii) $13 + (2+3i)\gamma + (2-3i)\gamma = 25$

$13 + 2\gamma + 2\gamma = 25 \Rightarrow \gamma = 3$

iii) $p = -\sum\alpha = -[2+3i + 2-3i + 3] = -7$

$q = -\gamma\beta\gamma = -[13 \times 3] = -39$

8

Commentary

Although many candidates were awarded full marks for this question, this candidate produced one of the best most concise solutions completely correct. Part (b)(i) was not always correct. (2+3I)(2-3i) produced a number of answers, but here the intermediate step of 4-9i ² (not always evident) helped with the accuracy. In (b)(iii) the work was impressive, with appropriate signs to hand right from the start. Errors when they did occur in this part were errors of sign in the evaluation of p and q.

Mark Scheme

3(a)	$2+3\mathrm{i}$	B1	1	
(b)(i)	$\alpha\beta=13$	B1	1	
(ii)	$\alpha\beta+\beta\gamma+\gamma\alpha=25$	M1		M1A0 for -25 (no ft)
	$\gamma(\alpha+\beta)=12$	A1F		
	$\gamma=3$	A1F	3	ft error in $\alpha\beta$
(iii)	$p=-\sum\alpha=-7$	M1 A1F		M1 for a correct method for either p or q
	$q=-\alpha\beta\gamma=-39$	A1F	3	ft from previous errors p and q must be real for sign errors in p and q allow M1 but A0
	Alternative for (b)(ii) and (iii):			
(ii)	Attempt at $(z-2+3\mathrm{i})(z-2-3\mathrm{i})$	(M1)		
	$z^2-4z+13$	(A1)		
	cubic is $(z^2-4z+13)(z-3)\therefore\gamma=3$	(A1)	(3)	
(iii)	Multiply out or pick out coefficients	(M1)		
	$p=-7,\ q=-39$	(A1, A1)	(3)	
	Total		**8**	

Question 4

4 (a) Sketch the graph of $y = \tanh x$. *(2 marks)*

(b) Given that $u = \tanh x$, use the definitions of $\sinh x$ and $\cosh x$ in terms of e^x and e^{-x} to show that

$$x = \frac{1}{2}\ln\left(\frac{1+u}{1-u}\right)$$ *(6 marks)*

(c) (i) Show that the equation

$$3\,\text{sech}^2 x + 7\tanh x = 5$$

can be written as

$$3\tanh^2 x - 7\tanh x + 2 = 0$$ *(2 marks)*

(ii) Show that the equation

$$3\tanh^2 x - 7\tanh x + 2 = 0$$

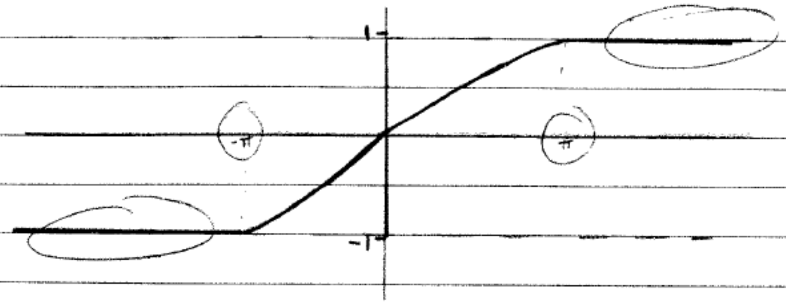
has only one solution for x.

Find this solution in the form $\frac{1}{2}\ln a$, where a is an integer. *(5 marks)*

Student Response

4

a



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B1

b

$u = \tanh x$

$u = \dfrac{\sinh x}{\cosh x}$

$u = \dfrac{\frac{1}{2}(e^{x} - e^{-x})}{\frac{1}{2}(e^{x} + e^{-x})}$ ✓

M1

$= \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$ ✓

A1

$= (e^{x} - e^{-x})(e^{-x} + e^{x})$??

$= e^{x}e^{-x} + e^{2x} - e^{-x}e^{-x} - e^{x}e^{-x}$

$= \cancel{1} + e^{2x} - e^{-2x} - \cancel{1}$

$u = e^{2x} - e^{-2x}$

$\frac{1}{2} \ln\left(\dfrac{1 + e^{2x} - e^{-2x}}{1 - e^{2x} + e^{-2x}}\right)$ ✗

Leave blank

c

i $3\operatorname{sech}^2 x + 7\tanh x = 5$

$\cosh^2 x - \sinh^2 x = 1$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$3\operatorname{sech}^2 x = 3 - 3\tanh^2 x$

$3 - 3\tanh^2 x + 7\tanh x = 5$

$3\tanh^2 x - 7\tanh x + 5 - 3 = 0$

2

$\therefore 3\tanh^2 x - 7\tanh x + 2 = 0$

ii $3\tanh^2 x - 7\tanh x + 2 = 0$

$(3\tanh x - 1)(\tanh x - 2)$

$\tanh x = \frac{1}{3}$ or $\tanh x = 2$

$\therefore \quad x = \frac{1}{2}\ln\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)$

$= \frac{1}{2}\ln\left(\frac{4/3}{2/3}\right)$

$= \frac{1}{2}\ln 2$

5

$x = \frac{1}{2}\ln\left(\frac{1+2}{1-2}\right)$

$= \frac{1}{2}\ln\left(-\frac{3}{1}\right)$

$= \frac{1}{2}\ln(-3) \leftarrow$ this is impossable as ln of a negative number doesnt exist. There is only one solution

10

$\therefore \quad x = \frac{1}{2}\ln 2$

Commentary

Sketches in part (a) were poor in general. Although this candidate had some idea of the general shape of the curve the diagram shows the curve running along its asymptotes rather than approaching them. Also the appearance of π on the diagram (a common occurrence) suggests some confusion between trigonometrical and hyperbolic functions. Again in part (b), as was commonly the case, after expressing tanhx in terms of e, poor algebraic techniques prevented the candidate from completing this part. Part (c)(i) was almost always completed correctly as was the solution of the ensuing quadratic equation, but in this case the candidate failed to see the relevance of the sketch to the rejection of tanhx = 2, but waited until ½ ln(-3) was arrived at.

Mark Scheme

4(a)	Sketch, approximately correct shape	B1		B0 if curve touches asymptotes
	Asymptotes at $y = \pm 1$	B1	2	lines of answer booklet could be used for asymptotes be strict with sketch
(b)	Use of $u = \dfrac{\sinh x}{\cosh x}$	M1		
	$= \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ or $\dfrac{e^{2x} - 1}{e^{2x} + 1}$	A1		
	$u\left(e^x + e^{-x}\right) = e^x - e^{-x}$	M1		M1 for multiplying up
	$e^{-x}(1+u) = e^x(1-u)$	A1		A1 for factorizing out e's or M1 for attempt at $1+u$ and $1-u$ in terms of e^x
	$e^{2x} = \dfrac{1+u}{1-u}$	m1		
	$x = \dfrac{1}{2}\ln\left(\dfrac{1+u}{1-u}\right)$	A1	6	AG
4(c)(i)	Use of $\tanh^2 x = 1 - \operatorname{sech}^2 x$	M1		
	Printed answer	A1	2	
(ii)	$(3\tanh x - 1)(\tanh x - 2) = 0$	M1		Attempt to factorise
	$\tanh x \neq 2$	E1		Accept tanh $x \neq 2$ written down but not ignored or just crossed out
	$\tanh x = \dfrac{1}{3}$	A1		
	$x = \dfrac{1}{2}\ln 2$	M1 A1F	5	ft
	Total		**15**	

Question 5

5 (a) Prove by induction that, if n is a positive integer,

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$ *(5 marks)*

(b) Hence, given that

$$z = \cos\theta + \mathrm{i}\sin\theta$$

show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$ *(3 marks)*

(c) Given further that $z + \frac{1}{z} = \sqrt{2}$, find the value of

$$z^{10} + \frac{1}{z^{10}}$$ *(4 marks)*

Student Response

Leave blank

5.

a) ~~$\cos\theta$~~

assume that this is true for $n=k$

$\therefore \quad (\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$

now if each side is multiplyied by $(\cos\theta + i\sin\theta)$:

$\Rightarrow \quad (\cos\theta + i\sin\theta)^{k+1} = (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$

$= \cos k\theta\cos\theta + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta) - \sin k\theta\sin\theta$

$= (\cos k\theta\cos\theta - \sin k\theta\sin\theta) + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$

using the compound angle trigonometry identities:

$\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$ where $A = k\theta$, $B = \theta$

$\sin(A+B) \equiv \sin A\cos B + \cos A\sin B$

$\Rightarrow (\cos\theta + i\sin\theta)^{k+1} = \cos(k\theta + \theta) + i\sin(k\theta + \theta)$

$= \cos((k+1)\theta) + i\sin((k+1)\theta)$

which is the same as $\cos k\theta + i\sin k\theta$ if k is replaced by $k+1$

so the expression is still valid for $k+1$. If it is true for a value of n then it must be true for every other ~~inde~~ positive integer.

$n=1$

$\Rightarrow \quad (\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta$ which is true

Therefore, by proof of induction, the expression is true for all positive integers.

5

Leave blank

b) $z = \cos\theta + i\sin\theta$

$\therefore z^n + \frac{1}{z^n} = \cancel{\cos n\theta + is}\ \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)$

$= \cos n\theta + \cos n\theta + i\sin n\theta - i\sin n\theta$

$= 2\cos n\theta$

3

c) $z + \frac{1}{z} = \sqrt{2}$

$\Rightarrow 2\cos\theta = \sqrt{2}$ (from part b))

$\therefore z^{10} + \frac{1}{z^{10}} = 2\cos 10\theta$

$\cos\theta = \frac{\sqrt{2}}{2} \Rightarrow \theta = \frac{\pi}{4}$

$\therefore 10\theta = \frac{10\pi}{4}$

$\Rightarrow 2\cos 10\theta = 2\cos\frac{10\pi}{4}$

$= 0$

$\therefore z^{10} + \frac{1}{z^{10}} = 0$

4

7

Commentary

An excellent proof by induction. Because candidates knew the result to be arrived at for n=k+1 was cos(k+1)Θ +isin(k+1)Θ, many candidates wrote down the answer without sufficient intermediate working. In this case, the candidate went into considerable detail when evaluating (coskΘ+isinkΘ)(cosΘ+isinΘ) even to the extent of quoting the trigonometrical formulæ.Also the explanation of the inductive process was clearly expressed. In part (b) the candidate demonstrated that (cosΘ+isinΘ)$^{-n}$ was cosnΘ – isinnΘ rather than merely quoting the result as happened in many cases.

Mark Scheme

5(a)	$(\cos\theta+\mathrm{i}\sin\theta)^{k+1}=$			
	$(\cos k\theta+\mathrm{i}\sin k\theta)(\cos\theta+\mathrm{i}\sin\theta)$	M1		
	Multiply out	A1		Any form
	$=\cos(k+1)\theta+\mathrm{i}\sin(k+1)\theta$	A1		Clearly shown
	True for $n=1$ shown	B1		
	$P(k)\Rightarrow P(k+1)$ and $P(1)$ true	E1	5	provided previous 4 marks earned
(b)	$\dfrac{1}{z^n}=\dfrac{1}{\cos n\theta+\mathrm{i}\sin n\theta}=\cos n\theta-\mathrm{i}\sin n\theta$	M1A1		or $z^{-n}=\cos(-n\theta)+\mathrm{i}\sin(-n\theta)$ SC $(\cos\theta+\mathrm{i}\sin\theta)^{-n}$ quoted as $\cos n\theta-\mathrm{i}\sin n\theta$ earns M1A1 only
	$z^n+\dfrac{1}{z^n}=2\cos n\theta$	A1	3	AG
(c)	$z+\dfrac{1}{z}=\sqrt{2}$			
	$2\cos\theta=\sqrt{2}$	M1		
	$\theta=\dfrac{\pi}{4}$	A1		
	$z^{10}+\dfrac{1}{z^{10}}=2\cos\left(\dfrac{10\pi}{4}\right)$	M1		M0 for merely writing $z^{10}+\dfrac{1}{z^{10}}=2\cos 10\theta$
	$=0$	A1F	4	
	Total		**12**	

Question 6

6 (a) Two points, A and B, on an Argand diagram are represented by the complex numbers $2+3\mathrm{i}$ and $-4-5\mathrm{i}$ respectively. Given that the points A and B are at the ends of a diameter of a circle C_1, express the equation of C_1 in the form $|z-z_0|=k$. *(4 marks)*

(b) A second circle, C_2, is represented on the Argand diagram by the equation $|z-5+4\mathrm{i}|=4$. Sketch on one Argand diagram both C_1 and C_2. *(3 marks)*

(c) The points representing the complex numbers z_1 and z_2 lie on C_1 and C_2 respectively and are such that $|z_1-z_2|$ has its maximum value. Find this maximum value, giving your answer in the form $a+b\sqrt{5}$. *(5 marks)*

Student Response

6a. diameter $=\sqrt{(2+4)^2+(3+5)^2}$

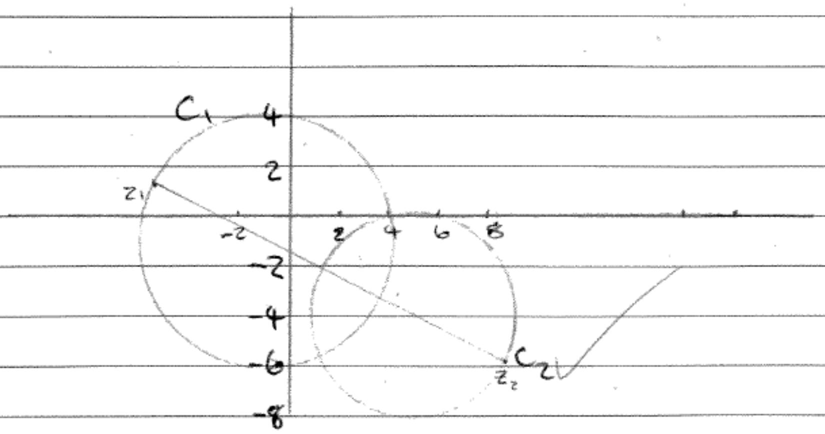
$=\sqrt{36+64} = \sqrt{100} = 10$ ✓

∴ radius $= 5$. ✓

Centre $=\left(\frac{(2-4)}{2}, \frac{(3-5)}{2}i\right) = (-1, -i)$ Condone ✓

∴ $|z-(-1-i)| = 5$. ✓ 4

b. $|z-(5-4i)| = 4$. $+4$ -8 $+4$ $+9$



✓ 3

6c. - Distance from 2 centres + radius C_1 + radius C_2

$5-4i \quad -1-i$

$\sqrt{(5+1)^2+(-4+1)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$3\sqrt{5}+4+5 = 9+3\sqrt{5}$

Leave blank

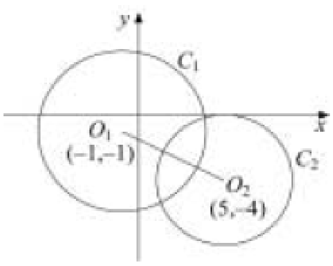
5

12

Commentary

This candidate is selected because overall the solution was good, clear and with a neat diagram. The candidate did not (as many did) confuse radius with diameter, but on the other hand, for the coordinates of the centre wrote (-1,-i) a common misunderstanding. The scale on the y-axis of the sketch did not contain i, as did many diagrams and the sketch was reasonably accurate with circles drawn using compasses. Many sketches had circles looking like anything but circles with candidates trying to plot points on their diagram and then joining up their points freehand. The final part of the question was well done with clear demonstration of the distance to be calculated, together with the method of showing how it was to be done.

Mark Scheme

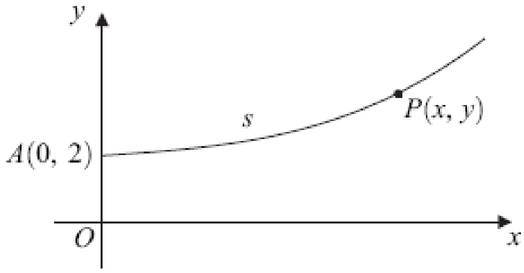
6(a)	Centre $-1-\mathrm{i}$ or $(-1,-1)$	B1		
	Radius 5	M1 A1F		ft incorrect centre if used
	$\lvert z+1+\mathrm{i}\rvert = 5$ or $\lvert z-(-1-i)\rvert = 5$	A1F	4	ft $\lvert z+1+\mathrm{i}\rvert = 10$ earns M0B1
(b)				
	C_1 correct centre, correct radius	B1F		ft errors in (a) but fit circles need to intersect and C_1 enclose $(0,0)$
	C_2 correct centre, correct radius	B1		
	Touching x-axis	B1F	3	error in plotting centre
(c)	$O_1O_2 = 3\sqrt{5}$	M1A1		allow if circles misplaced but O_1O_2 is still $3\sqrt{5}$
	Correct length identified	m1		
	Length is $9+3\sqrt{5}$	M1 A1F	5	ft if r is taken as 10
	Total		**12**	

Question 7

7 The diagram shows a curve which starts from the point A with coordinates $(0, 2)$. The curve is such that, at every point P on the curve,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}s$$

where s is the length of the arc AP.



(a) (i) Show that

$$\frac{\mathrm{d}s}{\mathrm{d}x} = \frac{1}{2}\sqrt{4+s^2}$$ *(3 marks)*

(ii) Hence show that

$$s = 2\sinh\frac{x}{2}$$ *(4 marks)*

(iii) Hence find the cartesian equation of the curve. *(3 marks)*

(b) Show that

$$y^2 = 4 + s^2$$ *(2 marks)*

Student Response

7a i) $\dfrac{ds}{dx} = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}$ ✓

$= \sqrt{1 + \tfrac{1}{4} s^2}$ ✓

$= \sqrt{\tfrac{1}{4}}\ \sqrt{4 + s^2}$

$= \tfrac{1}{2}\sqrt{4 + s^2}$ ✓

3

Question number

Leave blank

ii) $\dfrac{ds}{dx} = \dfrac{1}{2}\sqrt{4 + 4\sinh^2 \dfrac{x}{2}}$

$= \dfrac{1}{2}\sqrt{4(1 + \sinh^2 \dfrac{x}{2})}$

$[1 + \sinh^2 \dfrac{x}{2} = \cosh^2 \dfrac{x}{\cancel{4}2}]$

$= \dfrac{1}{2}\sqrt{4\cosh^2 \dfrac{x}{2}}$

Use of [illegible] is circled!

$\dfrac{ds}{dx} = \cosh \dfrac{x}{2}$

$s = \int \cosh \dfrac{x}{2}\, dx$

$s = 2\sinh \dfrac{x}{2}$ as required

0

Question number

Leave blank

iii) $\frac{dy}{dx} = \sinh\frac{x}{2}$

$y = \cancel{\tfrac{1}{2}\cosh\tfrac{x}{2}}\ 2\cosh\frac{x}{2}$ +c

M1
A1
A0

b) $y^2 = 4\cosh^2\frac{x}{2}$

$\cancel{[\cosh^2\tfrac{x}{2} = \tfrac{1}{2}(\cosh x + 1)]}$

$[\cosh^2\frac{x}{2} = 1 + \sinh^2\frac{x}{2}]$

$\cancel{y = 1/8(\cosh x + 1)}$

$y^2 = 4(\sinh^2\frac{x}{2} + 1)$

$[\sinh^2\frac{x}{2} = \frac{1}{4}s^2]$

$y^2 = 4(\frac{1}{4}s^2 + 1)$

$y^2 = 4 + s^2$

2

7

Commentary

The candidate starts off well with ds/dx = √(1+(dy/dx)²). Many candidates started with s=∫√(1+(dy/dx0²) dx but left the dx off or replaced it by ds.A common error in part a(ii) was to assume the answer in order to prove the result ie 2sinh(x/2) was substituted for s in ds/dx in order to prove that s=2sinh(x/2) at the end.In part (a)(ii), even when variables were separated as was intended for this part of the question, very few candidates indeed considered the constant of integration but just assumed that it was zero.The same applied to part (a)(iii) with no consideration being given to the constant of integration.Finally part (b) was well done.

7(a)(i)	$\frac{ds}{dx} = \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{s}{2}\right)^2}$	M1A1		Allow M1 for $s = \int\sqrt{1+\left(\frac{s}{2}\right)^2}\,dx$ then A1 for $\frac{dy}{dx}$
	$= \frac{1}{2}\sqrt{4+s^2}$	A1	3	AG
(ii)	$\int \frac{ds}{\sqrt{4+s^2}} = \int \frac{1}{2}\,dx$	M1		For separation of variables; allow without integral sign
	$\sinh^{-1}\frac{s}{2} = \frac{1}{2}x + C$	A1		Allow if C is missing
	$C = 0$	A1		
	$s = 2\sinh\frac{1}{2}x$	A1	4	AG if C not mentioned allow $\frac{3}{4}$ **SC** incomplete proof of (a)(ii), differentiating $s = 2\sinh\frac{x}{2}$ to arrive at $\frac{ds}{dx} = \frac{1}{2}\sqrt{4+s^2}$ allow M1A1 only $\left(\frac{2}{4}\right)$
(iii)	$\frac{dy}{dx} = \sinh\frac{1}{2}x$	M1		
	$y = 2\cosh\frac{1}{2}x + C$	A1		Allow if C is missing
	$C = 0$	A1	3	Must be shown to be zero and CAO
(b)	$y^2 = 4\left(1+\sinh^2\frac{x}{2}\right)$	M1		Use of $\cosh^2 = 1+\sinh^2$
	$= 4+s^2$	A1	2	AG
	Total		**12**	