Teacher Support Materials 2009

Maths GCE

Paper Reference MD02

Question 1

1 [**Figure 1**, printed on the insert, is provided for use in this question.]

A decorating project is to be undertaken. The table shows the activities involved.

Activity	Immediate Predecessors	Duration (days)
A	–	5
B	–	3
C	–	2
D	A, B	4
E	B, C	1
F	D	2
G	E	9
H	F, G	1
I	H	6
J	H	5
K	I, J	2

(a) Complete an activity network for the project on **Figure 1**. *(3 marks)*

(b) On **Figure 1**, indicate:

(i) the earliest start time for each activity; *(2 marks)*

(ii) the latest finish time for each activity. *(2 marks)*

(c) State the minimum completion time for the decorating project and identify the critical path. *(2 marks)*

(d) Activity F takes 4 days longer than first expected.

(i) Determine the new earliest start time for activities H and I. *(2 marks)*

(ii) State the minimum delay in completing the project. *(1 mark)*

Student Response

1a) Fig 1
1b) (i) Fig 1
(ii) Fig 1
1c) minimum completion time 22 days
critical path: B, E, G, H, I, k
1d) (i) 13+4=17 so H starts 17 days
17+1=18 and I starts 18 days
(ii) 18+6=24+2=26
26−22=4 days ∴ there is a delay of 4 days in completing the project.

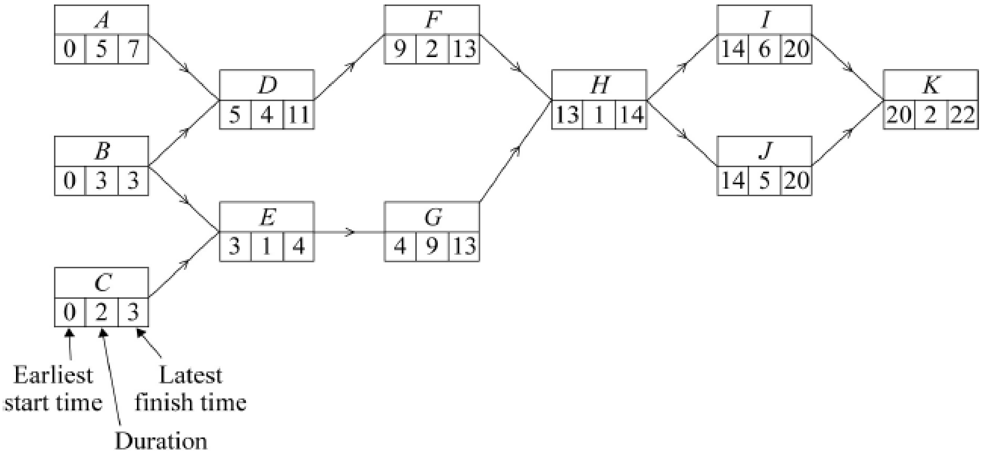
M0

Commentary

Almost every candidate completed the activity diagram correctly and this candidate scored full marks for the network and for indicating the values of the earliest start times and latest finish times on Figure 1. The correct minimum completion time and critical path were then written down. It was not sufficient to merely write 22 in the final box of the activity diagram.

In part (d), the candidate added 4 days to the latest finish time of *F*(13 days) and obtained 17 days, instead of considering the earliest start time for *F* (9 days) plus the duration of *F* (2 days) together with the 4 day delay, thus giving 9+2+4 =15 days as the new earliest start time for *H*. This had an impact on the earliest start time for *I* (now 16 days) and the overall delay to complete the project was stated as 4 days when it should have been 2 days.

Mark scheme

Q	Solution	Marks	Total	Comments
1				
(a)	Network attempted (3 more activities)	M1		SCA
	Up to 2 slips (boxes or connections)	A1		
	Correct network	A1	3	Condone missing arrows if sequence is clear
(b)(i)	Forward pass	M1		up to 1 slip ft
	Correct	A1	2	
(ii)	Backward pass	M1		up to 1 slip ft
	Correct	A1	2	
(c)	Minimum completion time 22 days	B1		Must be stated – not simply in K box
	Critical path B E G H I K	B1	2	and no others
(d)(i)	New start time for H is 15 days	M1		For H, their (F earliest time 9) + (2 + 4)
	New start time for I is 16 days	A1	2	both correct
(ii)	Minimum delay is 2 days	B1	1	Condone new completion time 24 days
	Total		**12**	

Question 2

2 Two people, Rowena and Colin, play a zero-sum game.

The game is represented by the following pay-off matrix for Rowena.

Colin

	Strategy	C_1	C_2	C_3
	R_1	-4	5	4
Rowena	R_2	2	-3	-1
	R_3	-5	4	3

(a) Explain what is meant by the term 'zero-sum game'. *(1 mark)*

(b) Determine the play-safe strategy for Colin, giving a reason for your answer. *(2 marks)*

(c) Explain why Rowena should never play strategy R_3. *(1 mark)*

(d) Find the optimal mixed strategy for Rowena. *(7 marks)*

Student response

Question number

Leave blank

2)a. A zero-sum game is when one player's gain = the other player's loss.

1

b.

R\C.	C_1	C_2	C_3	~~col max~~
R_1	-4	5	4	~~-4~~
R_2	2	-3	-1	~~-3~~
R_3	-5	4	3	~~-5~~
col max	2	5	4	

2

Colin should play C_1, because 2 is the min. col max. $2 < 5$ and $2 < 4$.

c) R_3 is dominated by R_1

$(-5, 4, 3) < (-4, 5, 4)$.

$\Rightarrow$ Rowena should not play R_3.

1

d) let R play R_1 with probability P and R_2 with probability $(1-P)$

Expected gain when C plays:

C_1: $-4P + 2(1-P)$

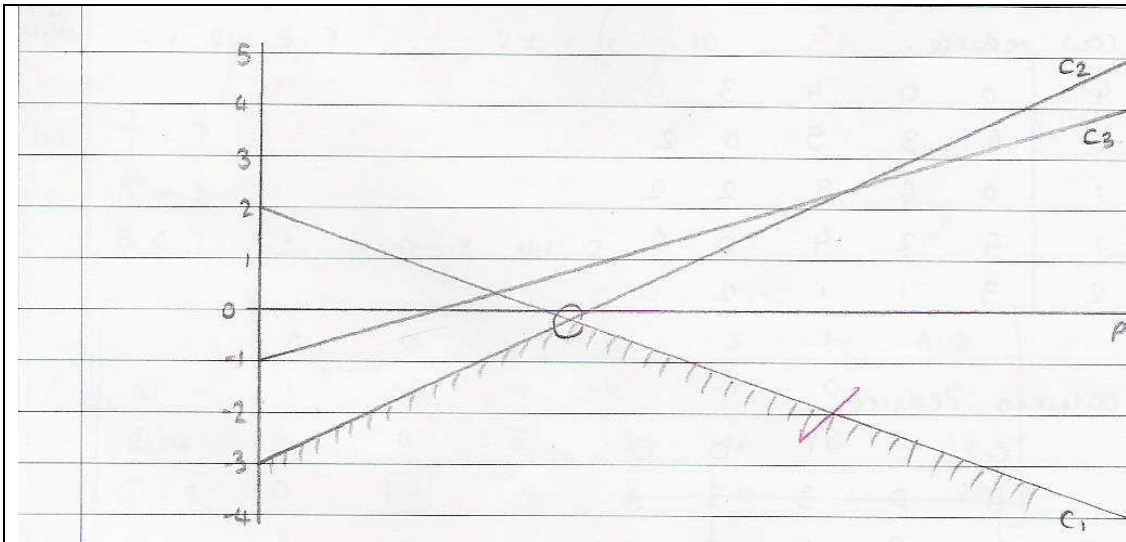
$= -6P + 2$

C_2: $5P - 3(1-P)$

$= 8P - 3$

C_3: $4P - (1-P)$

$= 5P - 1$



$-6p + 2 = 8p - 3$

$-14p = -5.$

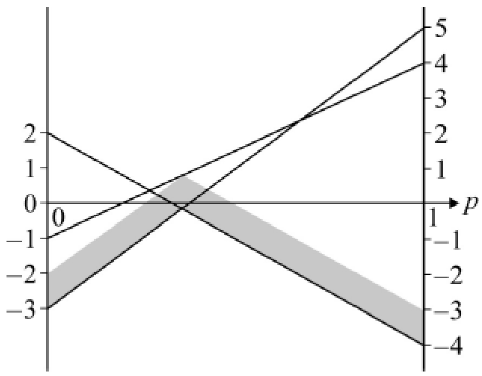
$p = \frac{5}{14}.$

$\Rightarrow$ Rowena should play R_1 with probability $\frac{5}{14}$, R_2 with probability $\frac{9}{14}$ and never R_3.

Commentary

This is an example of a good solution for this question.
(a) The explanation of a zero-sum game was sufficient to score the mark but it would have been even better if the statement had included the words "for each outcome".
(b) The row minima had also been calculated and then crossed out by the candidate, since these were not required. Many left these in their solution and this was not penalised. The minimum of the column maxima was indicated with an arrow and further explanation showed why C_1 was Colin's play-safe strategy and so this answer also earned full marks. Most candidates only scored one mark out of the two for this part of the question.
(c) The reason for not playing strategy R_3 was explained in detail by using both the phrase "dominated by" and then showing the various inequalities. Either of these two lines would have earned the mark but it was good to see the detailed solution when many candidates seemed to choose a minimalist approach.
(d) It was particularly good to see the initial statement defining the variable p. Many candidates neglected to do this but in future marks may be given for this opening statement. Expressions for the expected gains were carefully calculated and simplified. These expected gains were plotted against p and the omission of a scale on the right hand side (when $p = 1$) was condoned since there was a clear scale when $p = 0$. The highest point of the region was indicated clearly and the two appropriate expressions equated in order to find the value of p. It was also important to make a statement about the mixed strategy for Rowena and this candidate once again completed an excellent solution to secure full marks.

Mark Scheme

Q	Solution	Marks	Total	Comments
2(a)	(For each outcome)			
	Rowena's gain + Colin's gain = 0	E1	1	One player's loss is other's gain
(b)	(Column maxima 2, 5, 4)			Withhold E mark if any value
	$\Rightarrow \min(\text{col max}) = 2$ (OE but strict)	E1		incorrect; accept column minimax = 2
	$\Rightarrow$ Colin's play-safe strategy is C_1	B1	2	
(c)	R_3 is dominated by R_1	E1	1	$-5 < -4;\ \ 4 < 5$ and $3 < 4$
				E0 if R_2 mentioned as well
(d)	Let Rowena play R_1 with prob p			
	and R_2 with prob $1-p$			
	Expected gain when Colin plays			
	$C_1: -4p + 2(1-p) = 2-6p$			
	$C_2: 5p - 3(1-p) = -3+8p$	M1		attempt at least 2 with one correct
	$C_3: 4p - (1-p) = -1+5p$	A1		all 3 correct unsimplified
	Plot expected gains against p for $0 \leqslant p \leqslant 1$	M1		All 3 drawn ft their exp gains
		A1		correct
	$\Rightarrow 2-6p = -3+8p$	M1		Using "correct" equation Choosing highest point of region
	$\Rightarrow p = \frac{5}{14}$	A1		
	Therefore Rowena plays R_1 with prob $\frac{5}{14}$			
	and R_2 with prob $\frac{9}{14}$	E1√	7	ft their p
	Total		**11**	

Question 3

3 Five lecturers were given the following scores when matched against criteria for teaching five courses in a college.

	Course 1	Course 2	Course 3	Course 4	Course 5
Ron	13	13	9	10	13
Sam	13	14	12	17	15
Tom	16	10	8	14	14
Una	11	14	12	16	10
Viv	12	14	14	13	15

Each lecturer is to be allocated to exactly one of the courses so as to maximise the total score of the five lecturers.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $17 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 17. Hence show that, by reducing **rows first** and then columns, the resulting table of values is as below.

0	0	3	3	0
4	3	4	0	2
0	6	7	2	2
5	2	3	0	6
3	1	0	2	0

(3 marks)

(c) Show that the zeros in the table in part (b) can be covered with two horizontal and two vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) Hence find the possible allocations of courses to the five lecturers so that the total score is maximised. *(4 marks)*

(e) State the value of the maximum total score. *(1 mark)*

Student Response

③ a) ~~Max number~~ = 17
Hungarian algorithm minimises so ~~by~~
17 − x gives measurement of criteria not
met therefore minimizing this then gives maximum.

b)

	1	2	3	4	5	
R	2	4	8	7	4	(4)
S	4	3	5	0	2	(0)
T	1	7	9	3	3	(1)
U	6	3	5	1	7	(1)
V	5	3	3	4	2	(2)

	1	2	3	4	5
R	0	0	4	3	0
S	4	3	5	0	2
T	0	6	8	2	2
U	5	2	4	0	6
V	3	1	1	2	0
	(0)	(0)	(1)	(0)	(0)

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
R & 0 & 0 & 3 & 3 & 0 \\
S & 4 & 3 & 4 & 0 & 2 \\
T & 0 & 6 & 7 & 2 & 2 \\
U & 5 & 2 & 3 & 0 & 6 \\
V & 3 & 1 & 0 & 2 & 0
\end{array}
$$

Min no. uncovered = 2 such so subtract to uncovered and add to crossing lines

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
R & 2 & 0 & 3 & 5 & 0 \\
S & 4 & 4 & 2 & 0 & 0 \\
T & 0 & 4 & 5 & 2 & 0 \\
U & 5 & 0 & 1 & 0 & 4 \\
V & 5 & 1 & 0 & 4 & 0
\end{array}
$$

T1 = 16   T1 = 16
V3 = 14   V3 = 14
R2 = 13   U2 = 14
S5 = 15   S4 = 17
U4 = 16   R5 = 13

e) max score = 74

Commentary

This was a good solution to the question using the Hungarian Algorithm.
(a) The candidate mentioned both important points: the Hungarian Algorithm is used to minimise total scores; the expression 17–*x* measures the criteria not met by each lecturer. It was rare to see a candidate score both marks in this opening part of the question.
(b) & (c) The candidate made a slip initially but recovered to complete the row and column reductions correctly. The augmentation was not only performed accurately but the candidate stated clearly that the minimum number not covered by the four lines and then explained what augmentation was needed.
(d) & (e) Both allocations were listed by the candidate and the correct total score was stated. It was very common to see candidates presenting only one of these two allocations and so it was good to see a solution that showed both a good understanding of the algorithm and the correct interpretation of the final matrix.

Mark Scheme

Q	Solution	Marks	Total	Comments
3(a)	Hungarian algorithm minimises.	E1		Or changes maximising to minimising problem
	$17-x$ gives measure of criteria not met (which need minimising in order to maximise scores)	E1	2	Explanation of what each new entry or 17- x represents (as something which can be minimised)
(b)	$\begin{array}{ccccc} 4 & 4 & 8 & 7 & 4 \\ 4 & 3 & 5 & 0 & 2 \\ 1 & 7 & 9 & 3 & 3 \\ 6 & 3 & 5 & 1 & 7 \\ 5 & 3 & 3 & 4 & 2 \end{array}$	B1		array with $17-x$ values
	$\begin{array}{ccccc} 0 & 0 & 4 & 3 & 0 \\ 4 & 3 & 5 & 0 & 2 \\ 0 & 6 & 8 & 2 & 2 \\ 5 & 2 & 4 & 0 & 6 \\ 3 & 1 & 1 & 2 & 0 \end{array} \rightarrow \begin{array}{ccccc} 0 & 0 & 3 & 3 & 0 \\ 4 & 3 & 4 & 0 & 2 \\ 0 & 6 & 7 & 2 & 2 \\ 5 & 2 & 3 & 0 & 6 \\ 3 & 1 & 0 & 2 & 0 \end{array}$	M1 A1	3	reduce rows first – condone one slip then columns; AG
(c)	Top and bottom rows and 1st & 4th columns covered	B1		Zeros covered with 2 horizontal and 2 vertical lines
	$\begin{array}{ccccc} 2 & 0 & 3 & 5 & 0 \\ 4 & 1 & 2 & 0 & 0 \\ 0 & 4 & 5 & 2 & 0 \\ 5 & 0 & 1 & 0 & 4 \\ 5 & 1 & 0 & 4 & 0 \end{array}$	M1 A1CSO	3	augment by subtracting 2 from each uncovered and adding 2 to each double covered – condone one slip (may earn if 4 different lines are drawn)
(d)	T1, R2, V3, U4, S5 T1, U2, V3, S4, R5	M1 A1 M1 A1	4	3 items correctly matched First matching correct 3 items correct in second matching Second matching correct and no other matches attempted
(e)	Maximum total score = 74	B1	1	
	Total		**13**	

Question 4

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 4x + y + kz$, where k is a constant. The initial Simplex tableau is given below.

P	x	y	z	s	t	*value*
1	-4	-1	$-k$	0	0	0
0	1	2	3	1	0	7
0	2	1	4	0	1	10

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, write down **two** inequalities involving x, y and z for this problem. *(1 mark)*

(b) (i) The first pivot is chosen from the **x-column**. Identify the pivot and perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Given that the optimal value of P has not been reached after this first iteration, find the possible values of k. *(2 marks)*

(c) Given that $k = 10$:

(i) perform one further iteration of the Simplex method; *(4 marks)*

(ii) interpret the final tableau. *(3 marks)*

Student Response

4a $x + 2y + 3z \leq 7$

$2x + y + 4z \leq 10$

4bi $\frac{10}{2} < \frac{7}{1}$ so the value of 2 (from row 3) is used

	P	x	y	z	s	t	Value	Equation
	1	0	1	8−k	0	2	20	4 = 1 + 4 × 6
	0	0	$\frac{3}{2}$	1	1	$-\frac{1}{2}$	2	5 = 2 − 6
	0	1	$\frac{1}{2}$	2	0	$\frac{1}{2}$	5	6 = 3 × $\frac{1}{2}$

4bii $k > 8$

4ci	P	x	y	z	s	t	Value	Equation
	1	0	4	0	2	1	24	7 = 4 + 2 × 8
	0	0	$\frac{3}{2}$	1	1	$-\frac{1}{2}$	2	8 = 5
	0	1	$-\frac{5}{2}$	0	−2	$\frac{3}{2}$	1	9 = 6 − 2 × 8

1 is the pivot (from row 5) as $\frac{2}{1} < \frac{5}{2}$

B2 E0

4cii $P = 24$ when $x = 1$, $z = 2$, $y = 0$, $s = 0$, $t = 0$

Commentary

(a) Both inequalities were correct. This was intended as an easy opening part but many weaker candidates were unable to answer this correctly.
(b)(i) Several candidates showed the calculations 10/2 and 7/1 but then drew a wrong conclusion about the pivot. Although this candidate does not use the word pivot, it is clear that the entry 2 has been identified from the third row.
The row operations were clearly explained on the right hand side and these were performed accurately. This is an example of good practice.
(ii) Although there was no explanation, full marks were scored for the correct inequality $k > 8$.
(c)(i) This is another good example of the correct use of the Simplex Method where fractions were used and the row operations were performed accurately. Extra information was given regarding the pivot being used for the second iteration, which was not actually credited but was good to see in the overall solution.
(ii) The correct values of *P, x, y* and *z* were stated but the candidate lost a mark for failing to state that the optimum value of *P* had now been achieved. Many candidates lost this explanation mark which is a key aspect of interpreting the final tableau.

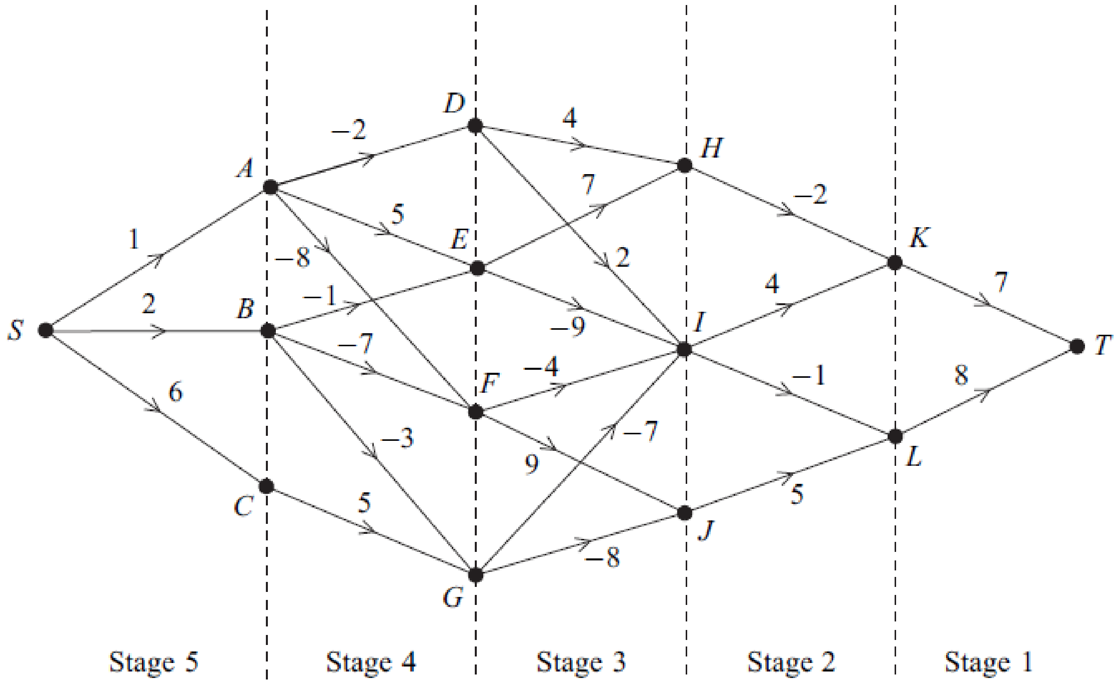
Mark Scheme

Q	Solution	Marks	Total	Comments
4(a)	$x+2y+3z \leqslant 7$ $2x+y+4z \leqslant 10$	B1	1	Exactly this
(b)(i)	Pivot is 2 in x-column	B1		Must be ringed or clearly indicated or stated – **not** simply implied
	$\begin{array}{ccccccc} P & x & y & z & s & t & value \\ 1 & 0 & 1 & 8-k & 0 & 2 & 20 \\ 0 & 0 & 1\frac{1}{2} & 1 & 1 & -\frac{1}{2} & 2 \\ 0 & 1 & \frac{1}{2} & 2 & 0 & \frac{1}{2} & 5 \end{array}$	M1 A1 A1	4	row operations (even with incorrect pivot) condone one slip Top or 2nd row correct using correct pivot All correct (condone multiples of rows)
(ii)	$8-k<0$	M1		Their $f(k)<0$
	$\Rightarrow k>8$	A1	2	SC B1 for $k \geq 9$
(c)(i)	New pivot from z-column in second row	B1√		Stated or possibly implied from tableau
	$\begin{array}{ccccccc} P & x & y & z & s & t & value \\ 1 & 0 & 4 & 0 & 2 & 1 & 24 \\ 0 & 0 & 1\frac{1}{2} & 1 & 1 & -\frac{1}{2} & 2 \\ 0 & 1 & -2\frac{1}{2} & 0 & -2 & 1\frac{1}{2} & 1 \end{array}$	M1 A1 A1	4	row operations using "their" **correct pivot** condone 1 slip one row (other than pivotal row) correct all correct (condone multiples of rows)
(ii)	$P = 24$	B1√		Provided no negatives in top row
	Optimum now reached	E1		Or $P_{\max} = \ldots\ldots$
	$x=1,\ y=0,\ z=2$	B1√	3	Only ft if no more than 2 slips in final tableau
	Total		**14**	

Question 5

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

A company has a number of stores. The following network shows the possible actions and profits over the next five years. The number on each edge is the expected profit, in millions of pounds. A negative number indicates a loss due to investment in new stores.

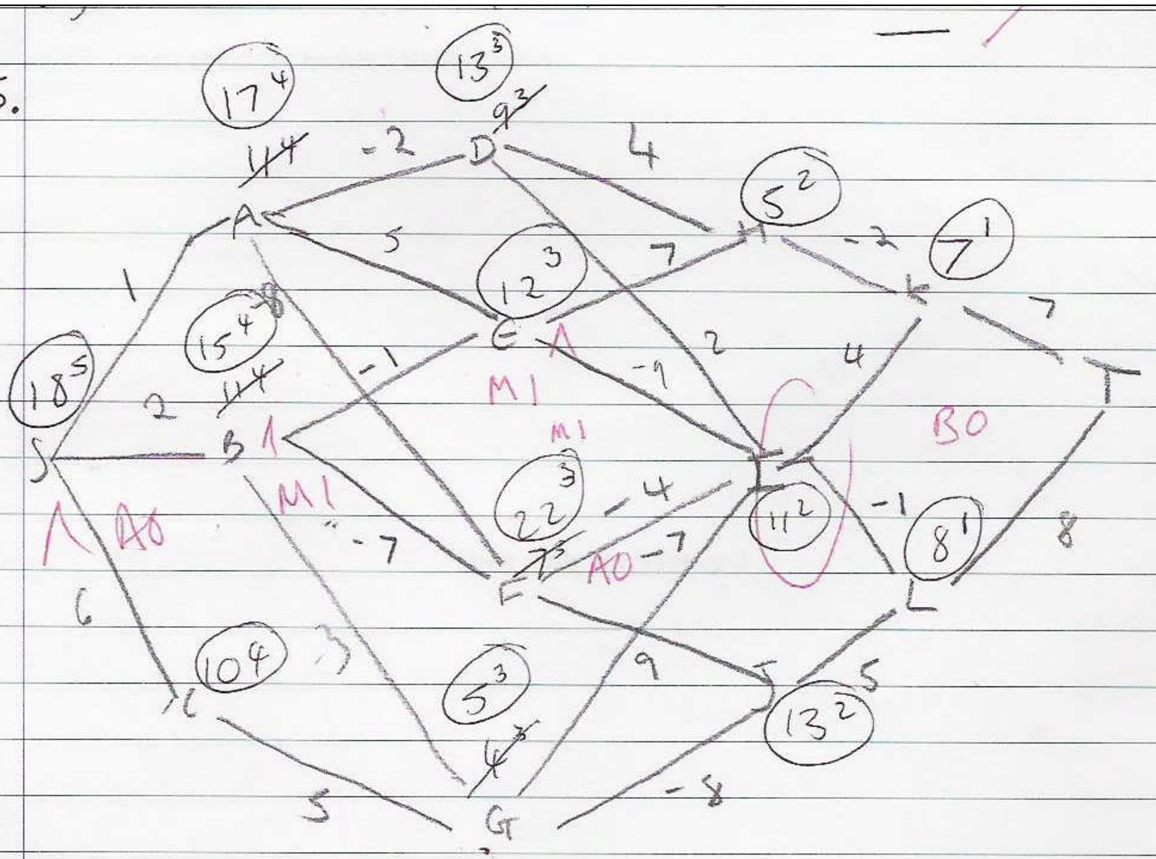


(a) **Working backwards from** T, use dynamic programming to maximise the expected profits over the five years. You may wish to complete the table on **Figure 2** as your solution. *(7 marks)*

(b) State the maximum expected profit and the sequence of vertices from S to T in order to achieve this. *(2 marks)*

Student Response

5.



b) £18 million. ~~pa~~

SAEHKT

Commentary

(a) Those candidates who used the table on the insert provided often scored full marks and even those who made a slip in their working usually scored much better than those who insisted on using a network diagram to present their solution.
The example above is typical of many who used a network approach. There is no key to notation such as 12^3 which presumably means a value of 12 after 3 stages, but this notation gives no indication of vertices visited and so would be impossible to use in order to trace back through the network to find the optimum solution.
One of the important things about dynamic programming is the ability to show how the value at any stage depends only on the maximum value (in this problem) from the previous stage. It must be evident that a candidate has performed the correct number of calculations and recorded these at each stage and that the answer has not been obtained by a complete enumeration. It is actually possible to record all this information on a network but failure to do this can result in a heavy penalty. For instance the first mark in the mark scheme is lost because this candidate failed to identify where the 11 at vertex *I* came from and there was no indication that a value of $-1 + 8 = 7$ has been considered when reaching *I* via vertex *L*. Three generous method marks were awarded for this attempt, but no accuracy marks were earned.
In future candidates may be required to produce a table similar to that on the insert showing the values for different stages and states.
(b) The candidate correctly recorded the maximum profit and the sequence of actions *SAEHKT*.

Mark Scheme

<table>
<tr><th>Q</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>5(a)</td><td>Completing stage 2 values
(condone unsimplified)

At least 6 values at stage 3
using only “their” max I value from stage 2
All stage 3 values correct

Using only max at D, E, F, G from stage 3 in stage 4 (at least 3 of these values used)
All stage 4 values correct

All stage 5 values correct and all other values correct unsimplified</td><td>B1

M1
m1

A1

M1

A1

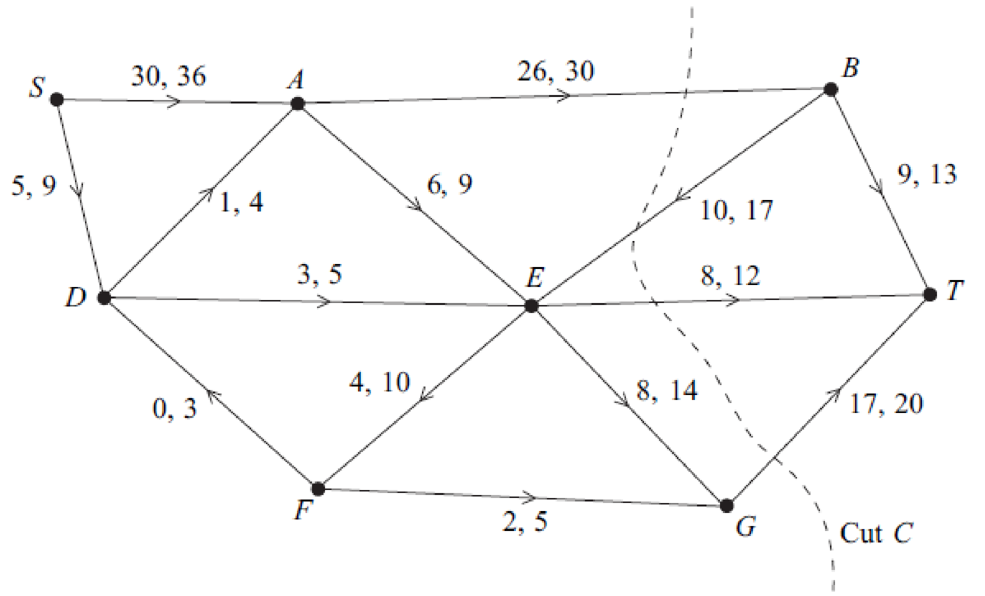
A1CSO</td><td>

7</td><td>
<table>
<tr><th>Stage</th><th>State</th><th>From</th><th>Value</th><th></th></tr>
<tr><td>1</td><td>K</td><td>T</td><td>7</td><td></td></tr>
<tr><td></td><td>L</td><td>T</td><td>8</td><td></td></tr>
<tr><td>2</td><td>H</td><td>K</td><td>$-2 + 7 = 5$</td><td></td></tr>
<tr><td></td><td>I</td><td>K
L</td><td>$4 + 7 = 11$
$-1 + 8 = 7$</td><td>*</td></tr>
<tr><td></td><td>J</td><td>L</td><td>$5 + 8 = 13$</td><td></td></tr>
<tr><td>3</td><td>D</td><td>H
I</td><td>$4 + 5 = 9$
$2+11 = 13$</td><td>*</td></tr>
<tr><td></td><td>E</td><td>H
I</td><td>$7 + 5 = 12$
$-9+11 = 2$</td><td>*</td></tr>
<tr><td></td><td>F</td><td>I
J</td><td>$-4+11 = 7$
$9+13 = 22$</td><td>*</td></tr>
<tr><td></td><td>G</td><td>I
J</td><td>$-7+11 = 4$
$-8+13 = 5$</td><td>*</td></tr>
<tr><td>4</td><td>A</td><td>D
E
F</td><td>$-2+13=11$
$5+12 = 17$
$-8+22=14$</td><td>*</td></tr>
<tr><td></td><td>B</td><td>E
F
G</td><td>$-1+12=11$
$-7+22=15$
$-3 + 5 = 2$</td><td>*</td></tr>
<tr><td></td><td>C</td><td>G</td><td>$5 + 5 = 10$</td><td></td></tr>
<tr><td>5</td><td>S</td><td>A
B
C</td><td>$1+17 = 18$
$2+15 = 17$
$6+10 = 16$</td><td>*</td></tr>
</table>
</td></tr>
<tr><td>(b)</td><td>Maximum profit £18m
Sequence of actions $S\ A\ E\ H\ K\ T$</td><td>B1
B1</td><td>
2</td><td>condone 18</td></tr>
<tr><td></td><td>**Total**</td><td></td><td>**9**</td><td></td></tr>
</table>

Question 6

6 [**Figures 3, 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) Find the value of the cut C. *(2 marks)*

(b) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 40 litres per second from S to T. Indicate, on **Figure 3**, the flows along the edges AE, EF and FG. *(3 marks)*

(c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 4**. *(3 marks)*

(ii) Use flow augmentation on **Figure 4** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

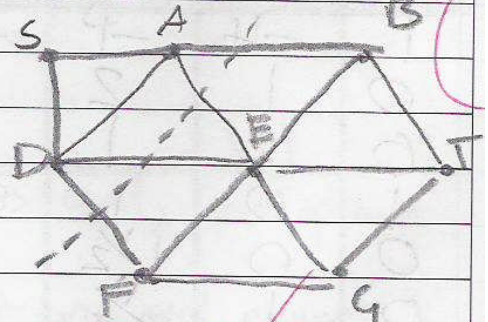
(d) Illustrate the maximum flow on **Figure 5**. *(2 marks)*

(e) Find a cut with value equal to that of the maximum flow. *(2 marks)*

Student Response

6a) Cut = (Max capacity to T) − (Min capacity to S)
$= (30+12+20) - (10)$
$= 52$ Litres per ~~secon~~ second

~~6~~ cii) Max flow = 44



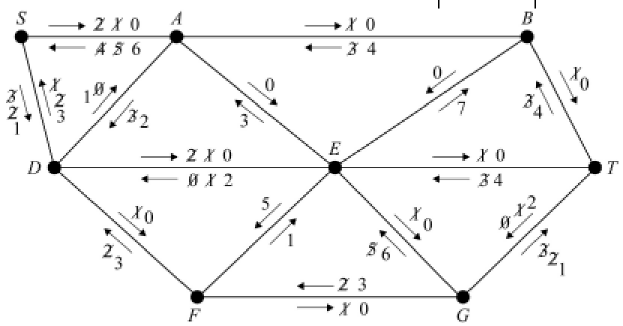
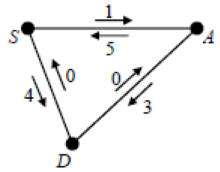
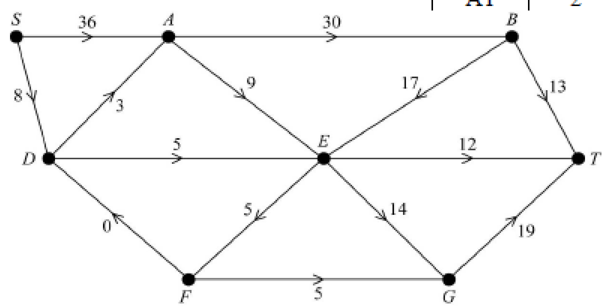
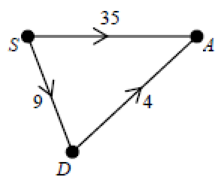
e) Cut : AB, AE, DE, DF

Value of cut = $(30+9+5) - (0)$
$= 44$

Commentary

(a) This is a good example of how to calculate the value of a cut when the edges have upper and lower capacities. Most candidates were unable to find the correct value of the cut and justified the two marks allocated to this part.
(b) Almost all candidates managed to find the correct values of the missing flows along the edges *AE*,*EF* and *FG*. This was answered correctly on the insert by this candidate.
(c) Future candidates would benefit from studying carefully the model solution in the mark scheme where the potential forward and backward flows are marked on the edges to form an initial flow. This is best done by candidates using ink for the initial values and then any adjustments can be made using pencil. A misconception evident in many solutions was that it was not possible to augment the flow by more than 3. In order to do this, it was necessary to reduce the flow on certain edges and it was clear that many candidates did not feel comfortable doing this.
(d)This candidate successfully augmented the flows to obtain a correct maximum flow of 44 and produced a solution identical to that in the mark scheme. Another misconception was that the final flow diagram could be used to identify a cut having a value of 44; this is not the case. Candidates needed to consider their saturated edges after flow augmentation or to calculate the values of the various cuts on the original diagram printed in the question paper. This candidate redrew the network in order to indicate a correct cut but then in addition listed the edges through which the cut passed.

Mark Scheme

Q	Solution	Marks	Total	Comments
6(a)	Value of cut = 30 – 10 + 12 + 20 = 52	M1 A1	2	Full marks for correct answers without working
(b)	$AE = 9$; $EF = 5$; $FG = 4$	B1 B1 B1	3	
(c)(i)	Attempt at forward and backward flows SA 2 & 4 ; AB 1 & 3 ; BT 1 & 3 SD 3 & 1 ; DA 0 & 3 ; AE 0 & 3 BE 0 & 7 ; DE 2 & 0 ; ET 1 & 3 FD 2 & 1 ; EF 5 & 1 ; EG 1 & 5 FG 1 & 2 ; GT 3 & 0	M1 A1 A1	3	At least 5 pairs correct 10 pairs correct all correct
(ii)	First flow augmenting path and correct flow on table Table correct Adjusting flows – forward and back Correct 	M1 A1 M1 A1	4	May end up with  Path — Extra flow *SABT* — 1 *SADET* — 1 — Or *SDET* *SDFGT* — 1 *SDEGT* — 1
(d)	Max flow of 44 shown on figure 5 	M1 A1	2	up to 2 slips all correct May have 
(e)	Cut through their saturated arcs Cut passes through AB, AE, DE and DF	M1 A1	2	Or BT, ET, EG, and FG
	Total		**16**	
	TOTAL		**75**	