Teacher Support Materials 2008

Maths GCE

Paper Reference MPC3

Question 1c

1 Find $\frac{dy}{dx}$ when:

(a) $y = (3x+1)^5$; *(2 marks)*

(b) $y = \ln(3x+1)$; *(2 marks)*

(c) $y = (3x+1)^5 \ln(3x+1)$. *(3 marks)*

Student Response

c) $y = (3x+1)^5 . \ln(3x+1)$

$\frac{dy}{dx} = (3x+1)^5 . \frac{3}{3x+1} + \ln(3x+1) . 15(3x+1)^4$ 2

$= 3(3x+1)^4 + 15\ln(3x+1)^5$ 6

$\frac{dy}{dx} = (9x+3)^4 + 75\ln(3x+1)$

Commentary

Candidates will be penalised for incorrect algebra.
This candidate correctly answered parts (a) and (b), then applied the product rule correctly to obtain the correct answer. Unfortunately the candidate proceeded to simplify their answer and produced some very poor work. They have taken an algebraic factor inside the ln expression.
The candidate was penalised the final accuracy mark.

Mark scheme

(c)	$\frac{dy}{dx} =$ $(3x+1)^5 \times \frac{3}{3x+1} + \ln(3x+1) \times 15(3x+1)^4$ $\left(= (3x+1)^4\left[3 + 15\ln(3x+1)\right]\right)$ $\left(= 3(3x+1)^4\left[1 + 5\ln(3x+1)\right]\right)$	M1 A1 A1	3	product rule $uv' + u'v$ (from (a) and (b)) either term correct CSO with no further errors

Question 2

2 (a) Solve the equation $\sec x = 3$, giving the values of x in radians to two decimal places in the interval $0 \leqslant x < 2\pi$. *(3 marks)*

(b) Show that the equation $\tan^2 x = 2\sec x + 2$ can be written as $\sec^2 x - 2\sec x - 3 = 0$. *(2 marks)*

(c) Solve the equation $\tan^2 x = 2\sec x + 2$, giving the values of x in radians to two decimal places in the interval $0 \leqslant x < 2\pi$. *(4 marks)*

Student response

Leave blank

2a) $\sec x = 3.$ $\quad 0 - 2\pi$

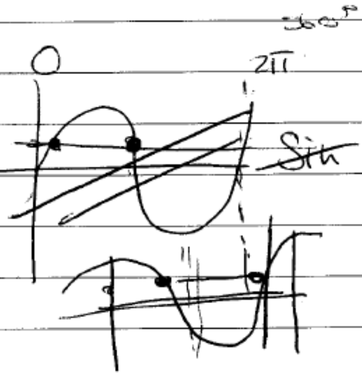
$\sec x = 3$

$\frac{1}{\cos} x = 3.$

$\frac{1}{3} = \cos x.$

$70°$

$0.999.$



1.230^c 2

4.3725552071

$= 1.23^c, 4.37^c.$ ✗

b) $\tan^2 x = 2\sec x + 2.$

~~tan~~

$\sec^2 x - 2\sec x = 3.$ ✗ 0

c) $(\sec x + 1)(\sec x - 3) = 0$ 31

$\sec x + 1 = 0$

$\sec x = -1.$

$\sec x - 3 = 0$

$\sec x = +3.$

cos (50).

Question number

Leave blank

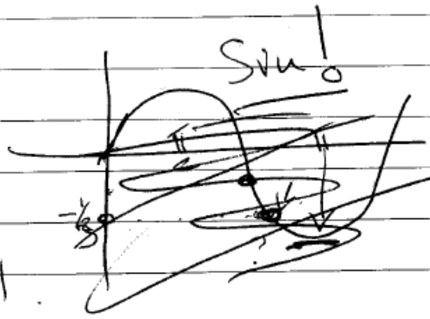
cont. $\sec x = -1$

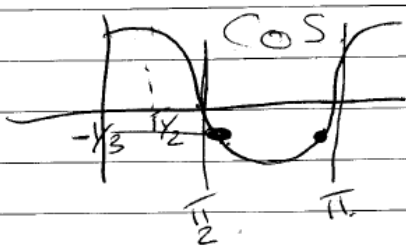
$\frac{1}{\cos x} = -1$

$\frac{1}{-1} = \cos x$

$\cos x = -1$

$x = \pi$.





When $\sec x = +3$

$\frac{1}{\cos x} = +3$ $\Rightarrow$ $\cancel{\frac{1}{+3} = \cos x}$

$x =$

$\cancel{\cos^{-1} x = 1.910}$

$\cancel{+ \pi = 5.0522}$

$\cancel{x = 1.91^c}$

$\cancel{x = 5.05^c}$

4

$\frac{1}{\cos x} = 3$ $\quad \frac{1}{3} = \cos$

$= 1.230959$

$= 4.37255$

$x = 1.23^c, 4.37^c$

6

Commentary

Candidates sometimes make mistakes under exam pressure but then maintain that 'mistake'.

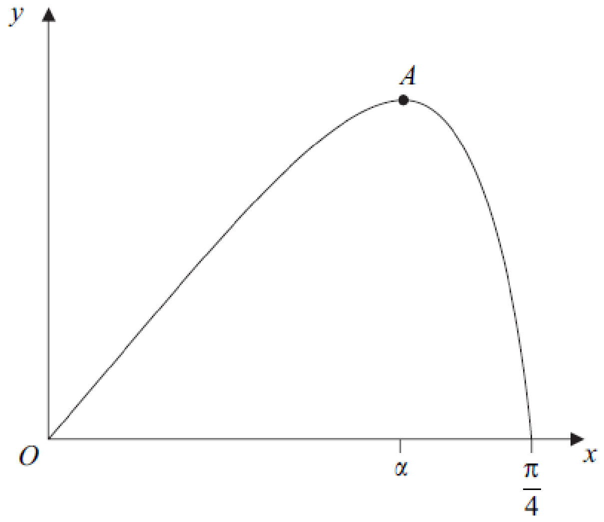
If that is the case they will, normally, only be penalised once in the question.
This candidate made amistake in part (a) of the question but they 'carried forward' their answers to part (c) and this was then recorded. They did lose extra marks in this part through not solving the secong part of the quadratic equation.

Mark Scheme

2(a)	$x=\cos^{-1}\frac{1}{3}$	M1		PI
	$=1.23,\ 5.05 \quad (0.39\pi,\ 1.61\pi)$	A1,A1	3	AWRT (– 1 for each error in range) SC 70.53, 289.47 B1
(b)	$\sec^2 x-1=2\sec x+2$	M1		use of $\sec^2 x=1+\tan^2 x$
	$\sec^2 x-2\sec x-3=0$	A1	2	AG; CSO
(c)	$\sec^2 x-2\sec x-3=0$			
	$(\sec x-3)(\sec x+1)=0$	M1		attempt to solve
	$\cos x=\frac{1}{3}$ or -1 o.e	A1		
	$x=1.23,\ 5.05,$	B1f		(2 answers in range from (a)) AWRT
	$3.14 \quad (\pi)$	B1	4	all correct and no extras in range SC 70.53, 289.47, 180 B1
	Total		**9**	

Question 3(a)(iii)

3 A curve is defined for $0 \leqslant x \leqslant \frac{\pi}{4}$ by the equation $y = x\cos 2x$, and is sketched below.



(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

Student Response

Question number

Leave blank

3) a)

$y = x\cos 2x$

$\frac{dy}{dx} = x.-2\sin 2x + \cos 2x$

$= -2x\sin 2x + \cos 2x$

2

i) $1-2\alpha \tan 2\alpha = 0$ ✗ $\tan 2\alpha = \frac{1}{2}$

$+2\alpha\tan 2\alpha = +1$

0

ii) ~~$f(x) = x\cos 2x$~~ ~~$f(0.4) = 0.2786$~~

$f(x) = 1-2\alpha\tan 2\alpha$

$f(0.4) = 0.17628$

$f(0.5) = -0.5574$

1

as there is a change of sign it lies between the points

iii) $1-2x\tan 2x = 0$

$-2x\tan 2x = -1$

$x\tan 2x = \frac{+1}{+2}$

$x\ \cancel{\tan 2x} = \frac{1}{2} \times \frac{1}{\tan 2x}$

$= \frac{1}{2} \times \left(\frac{1}{\tan} \times \frac{1}{2x}\right)$ ✗

$= \frac{1}{2}\tan^{-1}\left(\frac{1}{2x}\right)$

0

Commentary

On this paper candidates will have parts of questions where the answer will be given and candidates have to show the result.
Examiners check working carefully.
Regularly candidates will make major blunders in a solution and then 'miraculously' arrive at the correct answer.
These candidates as in the shown example will not score the marks.

Mark Scheme

Q	Solution	Marks	Total	Comments
3(a)	$\frac{dy}{dx} = -x\,2\sin 2x + \cos 2x$	M1 A1	2	product rule $kx\sin 2x \pm \cos 2x$ no further incorrect working

Question 3(c)

(c) Use integration by parts to find $\int_0^{0.5} x\cos 2x\,dx$, giving your answer to three significant figures. *(5 marks)*

Student Response

Question number

Leave blank

iv. when $n=1$, $x=2$: $\therefore x_2 = \frac{1}{2}\tan^{-1}\left(\frac{1}{2\times 0.4}\right) = 0.45$ M1 A1

when $n=2$, $x=3$ $\therefore x_3 = \frac{1}{2}\tan^{-1}\left(\frac{1}{2\times 0.45}\right) = 0.42$ 2

c) $\int_0^{0.5} x\cos 2x\,dx \Rightarrow u = x, \quad v = \frac{\sin 2x}{2}$

$\frac{du}{dx} = 1$

$\therefore \int_0^{0.5} x\cos 2x\,dx = uv - \int v\frac{du}{dx}\,.dx$ M1

m1

$= x\cdot\frac{\sin 2x}{2} - \int \frac{\sin 2x}{2}\cdot 1\,dx$

$= \left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]_0^{0.5}$ A1

$= \left[\frac{0.5\sin 1}{2} + \frac{\cos 1}{4}\right] - \left[0 + \frac{1}{4}\right]$

m1

$= \frac{0.8415}{4} + \frac{0.5403}{4} - \frac{1}{4}$ 4

$= 0.2104 + 0.1351 - 0.25$

$= 0.096$ AD

(13)

Commentary

In questions that require numerical evaluation candidates must ensure that they answer to the required degree of accuracy. This candidate has correctly used integration by parts, substituted values into their expression correctly but then has written the final answer to 2 significant figures (or to 3 decimal places!) and hence have lost the final accuracy mark.

Mark Scheme

(c)	$y = x\cos 2x$			
	$\left.\begin{array}{ll} u = x & du = 1 \\ dv = \cos 2x & v = \dfrac{\sin 2x}{2} \end{array}\right\}$	M1		$\left.\begin{array}{l}\text{differentiate one term} \\ \text{integrate one term}\end{array}\right\}$ must be $k\sin 2x$
		m1		correct substitution of their values into parts formula using $u = x$
	$\int = \dfrac{x\sin 2x}{2} - \int \dfrac{\sin 2x}{2}\,(dx)$			
	$= \left[\dfrac{x\sin 2x}{2} + \dfrac{\cos 2x}{4}\right]_{(0)}^{(0.5)}$	A1		
	$= \left(\dfrac{\sin 1}{4} + \dfrac{\cos 1}{4}\right) - \left(\dfrac{\cos 0}{4}\right)$	m1		correctly substituting values from previous 2 method marks
	$= 0.0954$	A1	5	AWRT

Question 4(c)

4 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^2, \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{2x-3}, \qquad \text{for real values of } x,\ x \neq \frac{3}{2}$$

(a) State the range of f. *(1 mark)*

(b) (i) The inverse of g is g^{-1}. Find $\mathrm{g}^{-1}(x)$. *(3 marks)*

(ii) State the range of g^{-1}. *(1 mark)*

(c) Solve the equation $\mathrm{fg}(x) = 9$. *(3 marks)*

Student Response

Question number

Leave blank

$$2x = \frac{1+3y}{y}$$

$$x = \frac{1}{2} \times \frac{1+3y}{y}$$

$$x = \frac{1+3y}{2y}$$

$$g^{-1}(x) = \frac{1+3x}{2x}$$

3

ii) $g^{-1}(x) = 0$

$g(x) \geq 1$

0

c) $fg(x) = 9$

$$fg(x) = \left(\frac{1}{2x-3}\right)^2$$

$$9 = \left(\frac{1}{2x-3}\right)^2$$

$$9 = \frac{1}{4x^2 - 12x + 9}$$

$$9(4x^2 - 12x + 9) = 1$$

$$36x^2 - 108x + 81 = 1$$

$$36x^2 - 108x + 80 =$$

$a = 36$
$b = -108$
$c = 80$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{108 \pm \sqrt{11664 - 4 \times 36 \times 80}}{72} = \frac{108 \pm \sqrt{144}}{72} =$$

Question number

Leave blank

$$x = \frac{108 \pm \sqrt{144}}{72}$$

$$x = \frac{108 \pm 12}{72}$$

$$x = \frac{120}{72} \qquad x = \frac{96}{72}$$

$$x = 1\tfrac{2}{3} \qquad x = 1\tfrac{1}{3}$$

3

Commentary

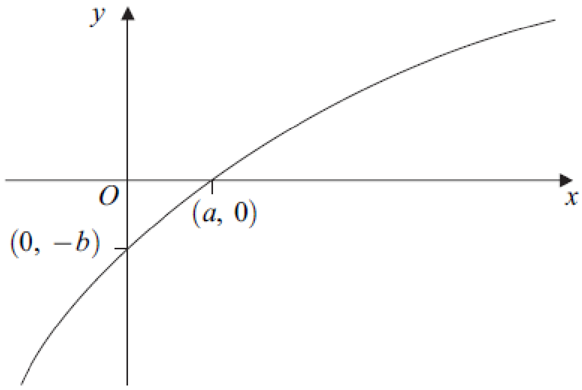
Although the mark scheme solved this part of the question using simple modulus functions it was possible to solve the question by using a quadratic equation.
This candidate has produced a perfect solution.
The working has taken more than the other printed solution but this candidate was able to solve the quadratic carefully.
A candidate could have scored full marks if they arrived at the correct quadratic equation and then written down the answers using a graphical calculator.

Mark Scheme

	(c)	$\left(\frac{1}{2x-3}\right)^2 = 9$	B1		
		$2x-3 = \pm\frac{1}{3}$	M1		square root and invert (condone missing $\pm$) **alternative:** attempt to solve a quadratic that comes from $4x^2 - 12 + 9 = \frac{1}{9}$ o.e.
		$x = \frac{5}{3}, \frac{4}{3}$ o.e.	A1	3	

Question 5(a)

5 (a) The diagram shows part of the curve with equation $y = f(x)$. The curve crosses the x-axis at the point $(a, 0)$ and the y-axis at the point $(0, -b)$.



On separate diagrams, sketch the curves with the following equations. On each diagram, indicate, in terms of a or b, the coordinates of the points where the curve crosses the coordinate axes.

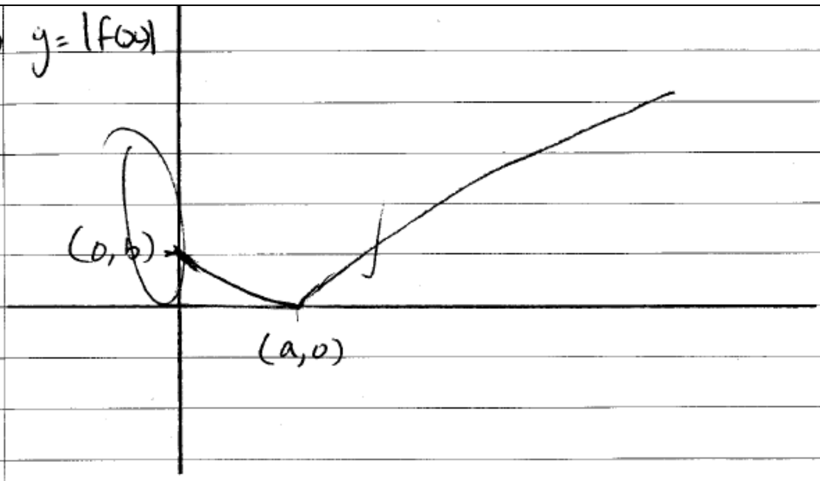
(i) $y = |f(x)|$. *(2 marks)*

(ii) $y = 2f(x)$. *(2 marks)*

Student Response

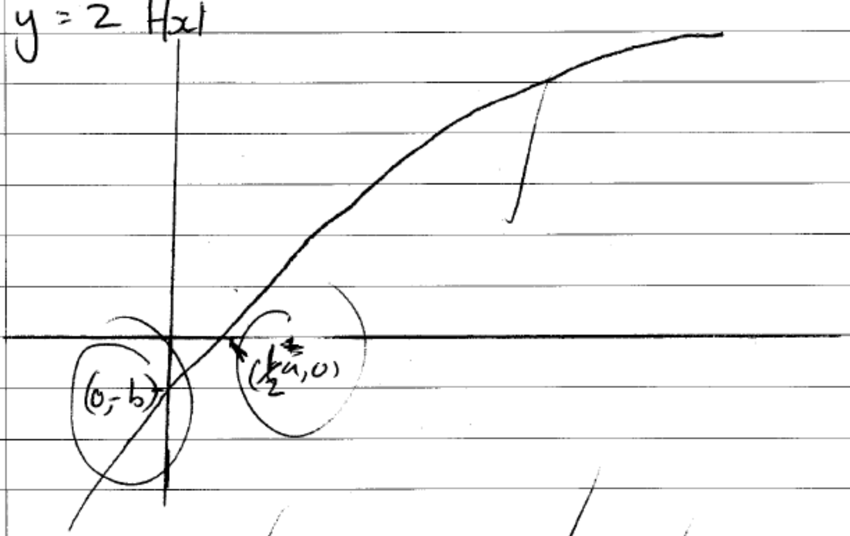
Leave blank

5a) $y = |f(x)|$



1

$y = 2f(x)$



1

bi) transformation of vector $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$

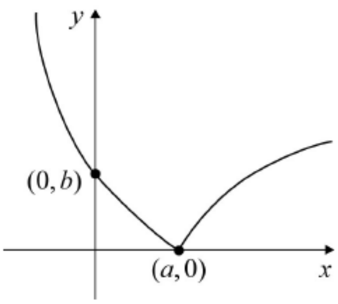
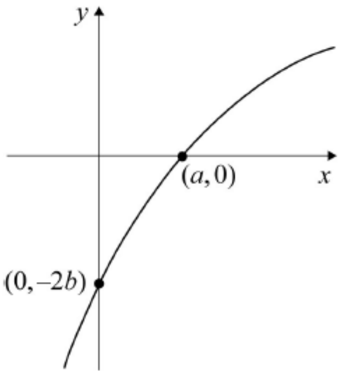
6

Stretch on the y axis of scale factor 4

Commentary

This question was poorly answered with the majority of candidates unaware of the modulus function.
The first sketch initially followed the correct shape but the candidate obviously believed that a modulus function meant that ‘y’ could not be negative. The candidate still scored 1 mark for the correct coordinates on the axex.
The second sketch is the correct shape but only scores 1 mark as the intersections on the axes are incorrect.
The transformation has been incorrectly interepreted.

Mark Scheme

5(a)(i)		B1 B1	 2	shape coordinates
(ii)		B1 B1	 2	shape coordinates

Question 5(b)(i)

(b)	(i)	Describe a sequence of geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4\ln(x+1) - 2$. *(6 marks)*

Student Response

bi) $y = \ln x \to y = 4\ln(x+1) - 2.$

translation $\begin{bmatrix}-1\\ 2\end{bmatrix}$ with a stretch in

y direction s/f 4.

3

Commentary

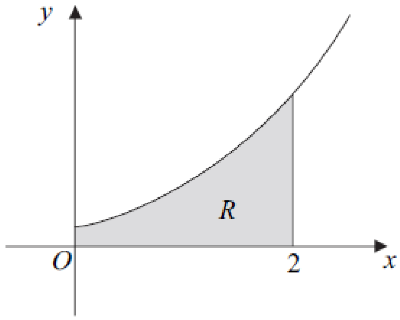
Candidates must 'think through' graphical transformations.
The function needed the translation to be split into 2 distinct parts either side of the stretch
Or to adjust the original function first.
The order of transformations is vital.

Mark Scheme

(b)(i)	Translation	M1			**OR** I stretch M1 I + (II or III)
	$\begin{bmatrix}-1\\0\end{bmatrix}$	A1			II SF 4 III // y-axis A1 (I + II + III)
	Stretch I	M1		I+(II or III)	Translation M1
	SF 4 II // y-axis III	A1		I + II + III	$\begin{bmatrix}-1\\-2\end{bmatrix}$ A1 B1
	Translation $\begin{bmatrix}0\\-2\end{bmatrix}$	B1		both	
	All correct and no mistakes on order etc	A1	6		All correct A1
	Alternative: $y = 4\ln(x+1) - 2 = 4\left[\ln(x+1) - \frac{1}{2}\right]$	(B1)			
	Translation	(M1)			
	$\begin{bmatrix}-1\\-\frac{1}{2}\end{bmatrix}$	(A1)			
	Stretch I	(M1)		I+(II or III)	
	SF 4 II // y-axis III	(A1)		I + II + III	
	All correct and no mistakes on order etc	(A1)	(6)		

Question 6(b)

6 The diagram shows the curve with equation $y = (e^{3x} + 1)^{\frac{1}{2}}$ for $x \geqslant 0$.



(a) Find the gradient of the curve $y = (e^{3x} + 1)^{\frac{1}{2}}$ at the point where $x = \ln 2$. *(5 marks)*

(b) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^2 (e^{3x} + 1)^{\frac{1}{2}}\, dx$, giving your answer to three significant figures. *(4 marks)*

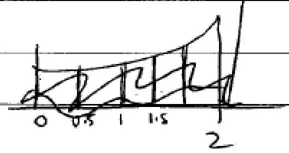
(c) The shaded region R is bounded by the curve, the lines $x = 0$, $x = 2$ and the x-axis.

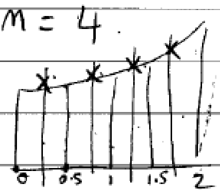
Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the x-axis. *(4 marks)*

Student Response

6 b) $\int_0^2 \left(e^{3x}+1\right)^{1/2}\,dx$

$n = 4.$





$h = \dfrac{b-a}{n} = \dfrac{2-0}{4} = 0.5$

mid-ordinat

$x_0 = 0.$

$\rightarrow x_{0\frac{1}{2}} = 0.25 \longrightarrow y_{\frac{1}{2}} = 1.77$

$x_1 = 0.5$

$\rightarrow x_{1\frac{1}{2}} = 0.75 \longrightarrow y_{\frac{3}{2}} = 3.24$

$x_2 = 1$

$\rightarrow x_{2\frac{1}{2}} = 1.25 \longrightarrow y_{5/2} = \cancel{6.59}\ 6.59$

$x_3 = 1.5$

$\rightarrow x_{3\frac{1}{2}} = 1.75 \longrightarrow y_{7/2} = 13.80$

$x_4 = 2$

$\cancel{x_{4\frac{1}{2}}} =$

$\int_0^2 \left(e^{3x}+1\right)^{1/2}\,dx = 0.5\left(1.77 + 3.24 + 6.59 + 13.8\right) =$

$= 12.7\cancel{0} \quad \cancel{\approx}\ \underline{\underline{12.7}}\ (3sf).$

3

Commentary

Numerical method questions need students to carefully study the required degree of accuracy and then work to a greater degree of accuracy.
This script has worked to 3 significant figures, but then the candidate cannot be sure that the final answer is correct ot 3 significant figures.
If the student had listed the functions not evaluated, they wouldn't have been penalised.

(b)	x / y: 0.25, 1.765(5); 0.75, 3.238(5); 1.25, 6.597(1); 1.75, 13.84(1)		B1		correct x values
			B1		3 or 4 correct y values 4 s.f. or better
	$\int = 0.5 \times \sum y$	P.I	M1		
	$= 12.7$		A1	4	sc 12.7 with no working $\frac{2}{4}$

x	y
0.25	1.765(5)
0.75	3.238(5)
1.25	6.597(1)
1.75	13.84(1)

Question 6(c)

(c) The shaded region R is bounded by the curve, the lines $x = 0$, $x = 2$ and the x-axis.

Find the exact value of the volume of the solid generated when the region R is rotated through 360° about the x-axis. *(4 marks)*

Student Response

c) $x = 0 \quad x = 2$

$V = \int_0^2 \pi y^2 \qquad y = (e^{3x}+1)^{\frac{1}{2}} \qquad y^2 = e^{3x}+1$

$V = \int_0^2 \pi (e^{3x}+1)$

$V = \pi \int_0^2 e^{3x}+1$

$V = \pi \int_0^2 \frac{1}{3} e^{3x} + x$

$V = \pi \left[\frac{1}{3}e^{3x} + x\right]_0^2$

$V = \pi \left[\frac{1}{3}e^{6} + 2\right] - \left[\frac{1}{3}e^{0} + 0\right]$ 3

$V = \pi \left[\cancel{136.47}\right] - \frac{1}{3}$ 8

$V = 428.4$

Commentary

Numerical method questions need students to carefully study the required degree of accuracy.
This particular question required an exact answer.
The candidate arrived at the correct expression after substituting in the limits but then failed to tidy up the expression exactly and instead resorted to the calculator for a final answer.

Mark Scheme

	(c)	$v = \pi\int y^2 \mathrm{d}x$ $= (\pi)\int\left(\mathrm{e}^{3x}+1\right)(\mathrm{d}x)$ $= (\pi)\left[\frac{1}{3}\mathrm{e}^{3x}+x\right]_{(0)}^{(2)}$ $= (\pi)\left[\left(\frac{1}{3}\mathrm{e}^{6}+2\right)-\left(\frac{1}{3}\mathrm{e}^{0}+0\right)\right]$ $= \pi\left[\frac{1}{3}\mathrm{e}^{6}+\frac{5}{3}\right]$ $\left(=\frac{\pi}{3}(e^{6}+5)\right)$	 M1 A1 m1 A1	 4	 $k\mathrm{e}^{3x}+x$ correct substitution into f ($\int e^{3x}$) CSO

Question 7(c)

7 (a) Given that $y = \dfrac{\sin\theta}{\cos\theta}$, use the quotient rule to show that $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \sec^2\theta$. *(3 marks)*

(b) Given that $x = \sin\theta$, show that $\dfrac{x}{\sqrt{1-x^2}} = \tan\theta$. *(2 marks)*

(c) Use the substitution $x = \sin\theta$ to find $\displaystyle\int \frac{1}{(1-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$, giving your answer in terms of x.

(5 marks)

Student Response

7c $\displaystyle\int \frac{1}{(1-x^2)^{\frac{3}{2}}} \, dx$ $\qquad x = \sin\theta$

$\qquad \dfrac{dx}{d\theta} = \cos\theta$

$$\int \frac{1}{(1-\cancel{\cos}\,\sin^2\theta)^{\frac{3}{2}}} \times \cos\theta \quad d\theta$$

$$\cancel{= \int (1-\cos^{-3}\theta)(\cos\theta) \, d\theta}$$

$$\cancel{= \int \cos\theta - \cos^{-2}\theta \, d\theta}$$

$$\cancel{= \sin\theta -}$$

$$= \int \frac{1}{(\cos^2\theta)^{\frac{3}{2}}} \times \cos\theta \quad d\theta$$

$$\cancel{= \int \sin^{-3}\theta \times \cos\theta \, d\theta}$$

$$= \int \frac{\cos\theta}{\cos^3\theta} \, d\theta = \int \frac{1}{\cos^2\theta} \, d\theta$$

$$= \int \sec^2\theta \, d\theta = \underline{\underline{\tan\theta}}$$

Leave blank

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

if $x = \sin\theta$ $\quad x^2 = \sin^2\theta$

$x^2 = 1 - \cos^2\theta$

$x^2 = 1 - \cos^2\theta$

$\cancel{x^2 - 1 = -\cos^2\theta}$

$\cos^2\theta = 1 - x^2$

$\cos\theta = \sqrt{1 - x^2}$

$\therefore \quad = \dfrac{x}{\sqrt{1 - x^2}}$ dam didn't see answer to part (b)!

4

9

Commentary

Questions that are split into a number of parts often are linked.
This final part of the question brought together work in parts (b) and (c). This candidate didn't study the question and started the final part from scratch. After solving the question they realised their 'error' as shown by their comment.
Questions are normally structured and candidates should be aware of this.

Mark Scheme

	(c)	$\int \frac{1}{\left(1-x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x$ $x=\sin\theta$ $\mathrm{d}x=\cos\theta\ \mathrm{d}\theta$ o.e.	M1		$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\pm\cos\theta$
		$\int=\int \frac{\cos\theta\ (\mathrm{d}\theta)}{\left(1-\sin^2\theta\right)^{\frac{3}{2}}}$	m1		all in terms of θ
		$=\int \frac{\cos\theta}{\left(\cos^2\theta\right)^{\frac{3}{2}}}\,(\mathrm{d}\theta)$	A1		
		$=\int \sec^2\theta\ (\mathrm{d}\theta)$ $=\tan\theta$	A1		
		$=\frac{x}{\sqrt{1-x^2}}\ (+c)$	A1	5	CSO including $\mathrm{d}\theta$'s