Teacher Support Materials 2008

Maths GCE

Paper Reference MM2B

Question 1

1 A particle moves in a straight line and at time t seconds has velocity $v\,\mathrm{m\,s^{-1}}$, where

$$v = 6t^2 + 4t - 7, \quad t \geqslant 0$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. *(2 marks)*

(c) When $t = 0$, the displacement of the particle from the origin is 5 metres.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

Student Response

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1a) $v = 6t^2 + 4t - 7$ $t \geqslant 0$

x
v
$\Rightarrow a = 12t + 4$ ✓ a 2

b) $m = 3\text{kg}$

$F = 3(12t + 4)$ {when $t = 4$} 2
$F = 36t + 12$ ✓
$F = (36 \times 4) + 12 = 156\text{N}$

Question number

Leave blank

1c) when $t=0$, $x=5$ ← Initial conditions

$V = 6t^2 + 4t - 7$

$x = \int 6t^2 + 4t - 7 \, dt$

$x = \frac{6t^3}{3} + \frac{4t^2}{2} - 7t + c$

$x = 3t^3 + 2t^2 - 7t + c$ M1

when $t=0$, $x=5$

so $5 = 3.0^3 + 2.0^2 - 7.0 + c$

$5 = c$ ✓ M1

so $x = 3t^3 + 2t^2 - 7t + 5$ (6)

#

x = displacement

Commentary

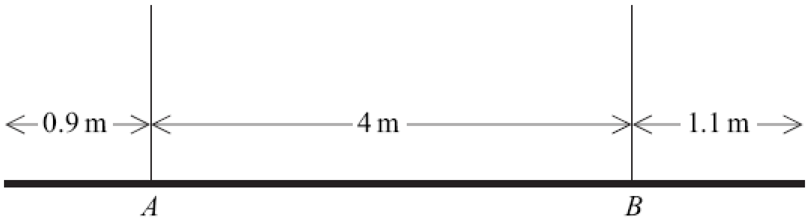
Most candidates answered this question well; this script shows one of a small proportion of candidates who made an algebraic or arithmetical error in part (c).

Mark scheme

Q	Solution	Marks	Total	Comments
1(a)	$a = \frac{dv}{dt} = 12t + 4$	M1 A1	2	
(b)	Using $F = ma$,			
	Force $= 3 \times (12t + 4)$	M1		
	When $t = 4$, force $= 3\,(12 \times 4 + 4)$			
	Force $=$ 156 N	A1	2	
(c)	$r = 2t^3 + 2t^2 - 7t + c$	M1 A1		
	When $t = 0$, $r = 5$, $\therefore c = 5$	M1		
	$\therefore r = 2t^3 + 2t^2 - 7t + 5$	A1	4	SC3 if no '+c' seen
	Total		**8**	

Question 2

2 A uniform plank, of length 6 metres, has mass 40 kg. The plank is held in equilibrium in a horizontal position by two vertical ropes attached to the plank at A and B, as shown in the diagram.



(a) Draw a diagram to show the forces acting on the plank. *(1 mark)*

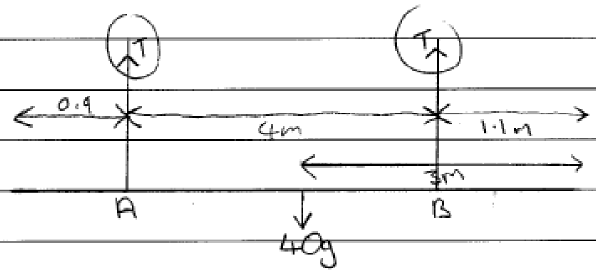
(b) Show that the tension in the rope attached to the plank at B is $21g$ N. *(3 marks)*

(c) Find the tension in the rope that is attached to the plank at A. *(2 marks)*

(d) State where in your solution you have used the fact that the plank is uniform. *(1 mark)*

Student response

2a.



B0

b. $1.9T = 40g$

$T = \frac{40g}{1.9} = 21g$ N

B0 M0

c. $2.1T = 40g$

$T = \frac{40g}{2.1} = 19g$ N ✗

M0

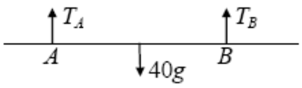
d. mass relates to length.

B0

Commentary

This question was answered well by many candidates. As in this example, a few lost a mark in part (a) by not showing that the two tensions were different. This candidate, in common with a number of others, used incorrect moments in part (b), but "obtained" the given answer by approximating $\frac{40g}{1.9}$ to 21g. Most of these then correctly resolved vertically and thus gained marks in part (c); however this candidate used a similar, incorrect moment equation in part (c) and hence scored no marks for this part, and his answer to part (d) was not relevant.

Mark Scheme

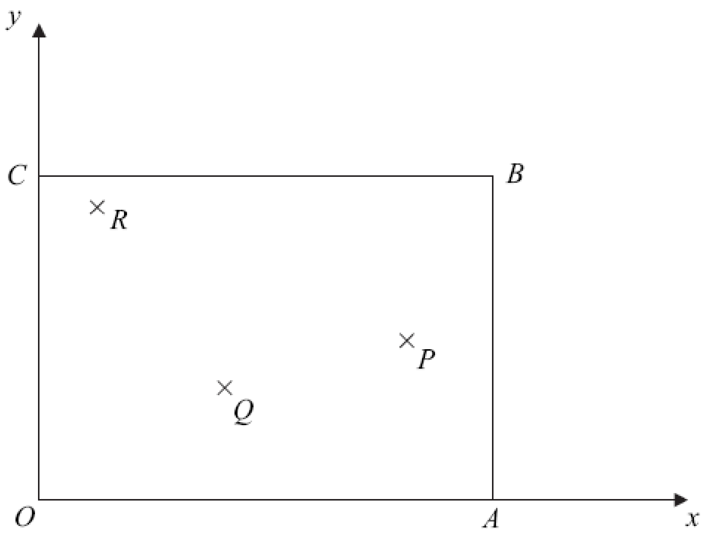
2(a)		B1	1	
(b)	Taking moments about A $2.1 \times 40g = T_B \times 4$ $T_B = 21g$	M1 B1 A1	3	B1 for 2.1
(c)	Resolve vertically $T_A + T_B = 40g$ $T_A = 19g$ or 186 N	M1 A1	2	
(d)	Gravitational force acts through mid point of the rod	E1	1	
	Total		7	

Question 3

3 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

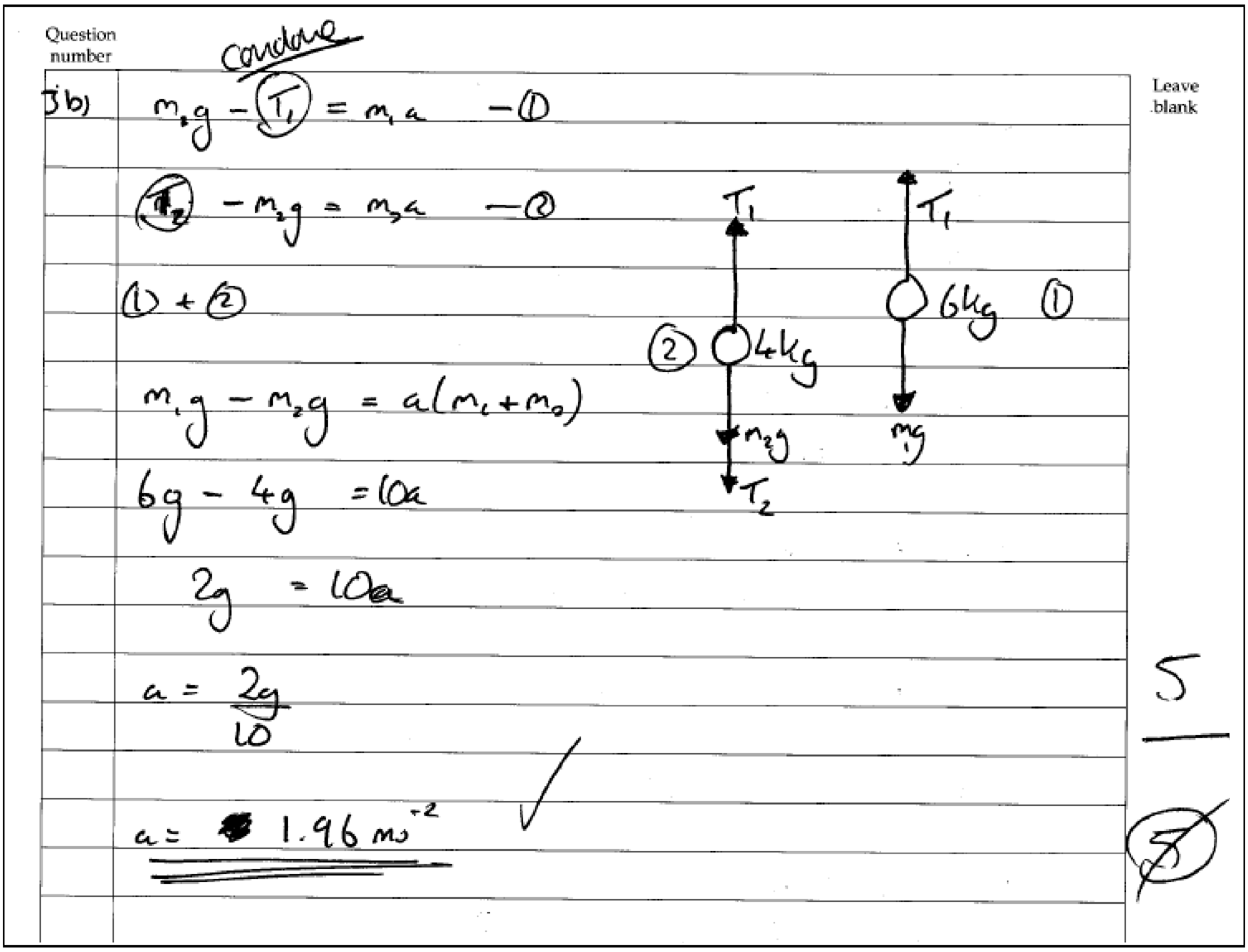
Take OA and OC as the x- and y-axes, as shown.

Particle P has mass 1 kg and is attached at the point (25, 10).
Particle Q has mass 4 kg and is attached at the point (12, 7).
Particle R has mass 5 kg and is attached at the point (4, 18).



Find the coordinates of the centre of mass of the three particles. *(4 marks)*

Student Response



Question number

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Continue

3b) $m_1 g - T_1 = m_1 a$ — ①

$T_2 - m_2 g = m_2 a$ — ②

① + ②

$m_1 g - m_2 g = a(m_1 + m_2)$

$6g - 4g = 10a$

$2g = 10a$

$a = \frac{2g}{10}$

$a = 1.96 \, ms^{-2}$ ✓

Commentary

As in this example, most candidates answered this question well.

Mark Scheme

3	$\bar{X} = \frac{25\times1+12\times4+4\times5}{1+4+5}$	M1		Two terms on top correct (+third) and denominator correct
	$= \frac{93}{10}$ or 9.3	A1		
	$\bar{Y} = \frac{10\times1+7\times4+18\times5}{10}$	M1		
	$= \frac{128}{10}$ or 12.8	A1	4	SC3 for interchanged $\bar{X}$ and $\bar{Y}$
	$\therefore$ Centre of mass is at (9.3, 12.8)			
	Total		**4**	

Question 4

4 A van, of mass 1500 kg, has a maximum speed of 50 m s^{-1} on a straight horizontal road. When the van travels at a speed of $v \text{ m s}^{-1}$, it experiences a resistance force of magnitude $40v$ newtons.

(a) Show that the maximum power of the van is 100 000 watts. *(2 marks)*

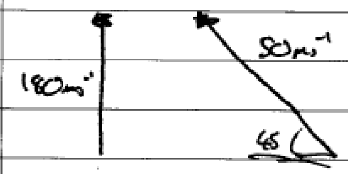
(b) The van is travelling along a straight horizontal road.

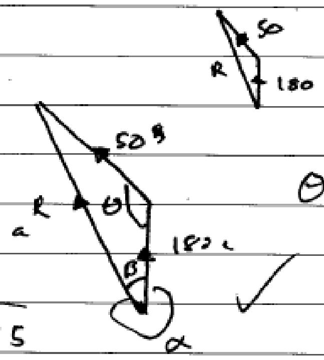
Find the maximum possible acceleration of the van when its speed is 25 m s^{-1}. *(3 marks)*

(c) The van starts to climb a hill which is inclined at $6°$ to the horizontal. Find the maximum possible constant speed of the van as it travels in a straight line up the hill. *(6 marks)*

Student Response

4)





$\theta = 135°$ ✓

a)

$R^2 = b^2 + c^2 - 2bc\cos R$

$R = \sqrt{50^2 + 180^2 - 2 \times 50 \times 180 \times \cos 135}$

$R = 218.2382232$

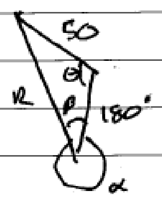
$R = 218\ ms^{-1}$ ✓

4

Question number

Leave blank

4 b)



$\frac{\sin\beta}{50} = \frac{\sin\theta}{R}$

$\sin\beta = 50 \times \left(\frac{\sin 135}{218.2382232}\right)$

$\sin\beta = 0.162180454$

$\beta = 9.333480312$

$360 - \beta = \alpha$

$\alpha = 350.6665197 \Rightarrow 351$ ✓

plane flies on bearing 351° at 218 ms⁻¹

4

8

Commentary

In general, part (a) was answered well, but again a number of candidates created the answer; if they had obtained 80 000, a factor of $\frac{5}{4}$ clearly was needed to give the printed result.
This script shows part (a) being answered correctly. In part (b), using the formula:
Power = Force × Velocity, the candidate calculates the correct force of 4000N exerted by the van's engine at 25ms^{-1}. Unfortunately, the common error shown here was to forget the resistance force and thus use 4000 = 1500*a*.

Mark Scheme

4(a)	Using power = force × velocity Power = (40 × 50) × 50 $\therefore$ = 100,000 watts	 M1 A1	 2	
(b)	When speed is 25, max force exerted is $\frac{100000}{25}$ = 4000N $\therefore$Accelerating force is 3000N Using $F = ma$ 3000 = 1500 a a = 2 ms^{-2}	 B1 M1 A1	 3	 Need 3 terms eg '4000' ± 1000 = ma or 2000 ± 1000 = ma M0 for 1000 = ma
(c)	When van is at maximum speed force against gravity is $mg\sin 6$ (parallel to slope) Force against gravity and resistance is $mg\sin 6 + 40v$ = 1536.6+ 40 v Speed is maximum when $1536.6 + 40v = \frac{100000}{v}$ $40v^2 + 1536.6v - 100\,000 = 0$ Speed is 34.4 ms^{-1}	 B1 M1 A1 M1 A1 A1	 6	 For 3 terms; $\frac{100000}{v}$ and 1 other term correct CAO
	Total		**11**	

Question 5

5 A particle moves on a horizontal plane in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the particle's position vector, **r** metres, is given by

$$\mathbf{r} = 8\left(\cos\tfrac{1}{4}t\right)\mathbf{i} - 8\left(\sin\tfrac{1}{4}t\right)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Show that the speed of the particle is a constant. *(3 marks)*

(c) Prove that the particle is moving in a circle. *(2 marks)*

(d) Find the angular speed of the particle. *(2 marks)*

(e) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(f) State the magnitude of the acceleration of the particle. *(1 mark)*

Student Response

Question number		Leave blank
5d	$\omega = \dfrac{v}{r}$	
	$\omega = \dfrac{(-2\sin\frac{1}{4}t)\mathbf{i} - (2\cos\frac{1}{4}t)\mathbf{j}}{(8\cos\frac{1}{4}t)\mathbf{i} - (8\sin\frac{1}{4}t)\mathbf{j}}$	
	$= (-\frac{1}{4}\tan\frac{1}{4}t)\mathbf{i} + (\frac{1}{4}\sec\frac{1}{4}t)\mathbf{j}$ ✗	0
5e	$a = \dfrac{dv}{dt} = (-\frac{1}{2}\cos\frac{1}{4}t)\mathbf{i} + (\frac{1}{2}\sin\frac{1}{4}t)\mathbf{j}$ ✓	2

Commentary

In part (d), $v = \omega r$ and $v = \dfrac{\omega^2}{r}$, were used in equal numbers. As shown in this example, the values of r and v which candidates substituted were often in vector form, with random attempts made at the division of the two vectors.

Mark Scheme

5(a)	$\mathbf{v} = \dfrac{d\mathbf{r}}{dt}$			
	$\mathbf{v} = -2\sin\frac{1}{4}t\,\mathbf{i} - 2\cos\frac{1}{4}t\,\mathbf{j}$	M1 A1	2	No **i**, **j**: no marks
(b)	Speed is $\{(-2\sin\frac{1}{4}t)^2 + (-2\cos\frac{1}{4}t)^2\}^{\frac{1}{2}}$	M1		
	$= 2\left(\sin^2\frac{1}{4}t + \cos^2\frac{1}{4}t\right)^{\frac{1}{2}}$	m1		clear use of $\sin^2\theta + \cos^2\theta = 1$
	$=$ 2 which is a constant	A1	3	Use of 2 values SC1
(c)	Magnitude of **r** is			
	$\{(8\cos\frac{1}{4}t)^2 + (8\sin\frac{1}{4}t)^2\}^{\frac{1}{2}}$	M1		$\mathbf{a} = -k\mathbf{r} \Rightarrow$ circle SC2
	= 8 which is a constant	A1	2	
	∴ Particle is moving in a circle			
(d)	Using $v = a\omega$	M1		M1 for their $\frac{b}{c}$ if both found
	Angular speed is 0.25	A1	2	
(e)	$a = -\frac{1}{2}\cos\frac{1}{4}t\,\mathbf{i} + \frac{1}{2}\sin\frac{1}{4}t\,\mathbf{j}$	M1 A1	2	
(f)	Magnitude of acceleration is $\frac{1}{2}$	B1	1	
	Total		**12**	

Question 6

6 A car, of mass m, is moving along a straight smooth horizontal road. At time t, the car has speed v. As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

(a) Show that

$$\frac{dv}{dt} = -0.05v$$ *(1 mark)*

(b) When $t = 0$, the speed of the car is $20\,\text{m}\,\text{s}^{-1}$.

Show that $v = 20e^{-0.05t}$. *(4 marks)*

(c) Find the time taken for the speed of the car to reduce to $10\,\text{m}\,\text{s}^{-1}$. *(3 marks)*

Student Response

Question number | Leave blank

(6)

(a) $\dfrac{-0.05mv = m\dfrac{dv}{dt}}{m}$

$-0.05v = \dfrac{dv}{dt}$ ✓ 1

(b) $\int -0.05\ dt = \int \dfrac{dv}{v}$

$-0.05t + c = \ln v$ M1

$e^{-0.05t+c} = v$ ✓ A1

$e^{-0.05t} + e^{c} = v$

$ke^{-0.05t} = v$

$ke^{-0.05\times 0} = v$

$k = v$

$v = 20$

$v = 20e^{-0.05t}$ ✗

Commentary

Virtually all candidates obtained $\frac{dv}{dt}$ = – 0.05*v*, only a few ignored the required step $m\frac{dv}{dt}$ = –0.05*mv*.

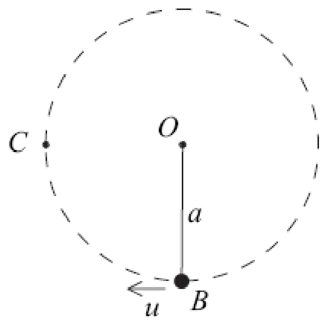
The equation: $\int\frac{dv}{v} = -\int 0.05\,dt$ was a necessary step which needed to be seen in part (b), as shown in this example. Candidates knew roughly how to obtain *v* = $20e^{-0.05t}$
Too often, algebraic skills were not sufficient and, as in this script, the equation ln *v* = – 0.05*t* + c regularly changed from *v* = $e^{-0.05t+c}$, to *v* = $e^{-0.05t}$ + *e* ^c^ before becoming *v* =$Ke^{-0.05t}$. This and similar errors were not condoned.

Mark Scheme

6(a)	Using $F = ma$ $-0.05mv = m\frac{dv}{dt}$ $\therefore \frac{dv}{dt} = -0.05v$	B1	1	Need to see *m* terms
(b)	$\int\frac{dv}{v} = -\int 0.05\,dt$	B1		
	$\ln v = -0.05t + c$ $v = Ce^{-0.05t}$	M1		Need first 2 terms
	When $t = 0$, $v = 20$, $\therefore C = 20$	M1		} fully correct solutions
	$v = 20e^{-0.05t}$	A1	4	
(c)	When $v = 10$, $10 = 20e^{-0.05t}$	M1		
	$e^{0.05t} = 2$	A1		
	$\therefore t = \frac{1}{0.05}\ln 2$ $= 13.9$	A1	3	Accept 20 ln 2
	Total		**8**	

Question 7

7 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string, of length a. The bead is then set into circular motion with the string taut at B, where B is vertically below O, with a horizontal speed u.



(a) Given that the string does not become slack, show that the least value of u required for the bead to make complete revolutions about O is $\sqrt{5ag}$. *(5 marks)*

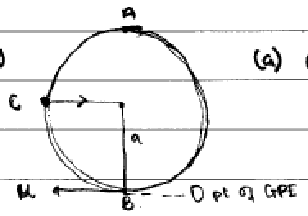
(b) In the case where $u = \sqrt{5ag}$, find, in terms of g and m, the tension in the string when the bead is at the point C, which is at the same horizontal level as O, as shown in the diagram. *(3 marks)*

(c) State one modelling assumption that you have made in your solution. *(1 mark)*

Student Response

Question number

7. (a)



(a) At B: GPE = 0, KE = $\frac{1}{2}mv^2$

At A: GPE = $mga \times 2$, KE = ~~$\frac{1}{2}mv^2$~~ = 0

Conservation of Energy:

At A: GPE = 0, KE = 0

At B: KE = $\frac{1}{2}mv^2$, GPE = $-2amg$

$2mga = \frac{1}{2}mv^2$

$(\times 2)$

$4ga = v^2$

$(\sqrt{\ })$ $v \geq 4ga$

(b) $v = \sqrt{5ag}$.

Resolve $(\rightarrow)$

$T = \frac{mv^2}{r}$

$T = \frac{m(5ag\ 5ag)}{a}$

$= 5mg$ N

(c) No air resistance.

Leave blank: 0, M1, 0, 1, (2)

Commentary

Many candidates made little progress in this question. As in this example, a number did not use conservation of energy correctly, with many using the potential energy at *B* to be zero, and assuming that the kinetic energy at the top was zero. These candidates ignored the fact that the bead could not complete full revolutions attached to a string with no speed at the top.

In part (b), the required components, *T* and $\frac{mv^2}{r}$, appeared frequently in the equation but often candidates, including this one, did not find the value of *v* when the bead was at *C*. Part (c) was usually answered well.

Mark Scheme

7(a)	At top, for complete revolutions:			
	$\frac{mv^2}{a} = mg$ where v is speed at top	M1		
	$\therefore\ v^2 = ag$	A1		
	Conservation of energy from B to top :			
	$\frac{1}{2}mv^2 + mg2a = \frac{1}{2}mu^2$	M1 A1		3 terms, 2 KE and PE
	$u^2 = 4ag + v^2$			
	$= 5ag$			
	$u = \sqrt{5ag}$	A1	5	AG
(b)	At C, speed of particle is $\sqrt{3ag}$	B1		
	Resolving horizontally at C:			
	$T = \frac{mv^2}{a}$	M1		Needs 2 correct terms
	$T = m\frac{3ag}{a}$			
	$T = 3mg$	A1	3	
(c)	No air resistance Bead is a particle	B1	1	
	Total		**9**	

Question 8

8 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\frac{\lambda x}{l}$ when its extension is x.

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\frac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point O.

(i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(ii) The particle is pulled down and held at the point P, which is 0.9 metres vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 11.25 J. *(2 marks)*

(iii) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\mathrm{m\,s^{-1}}$ when it is x metres **above *P***.

Show that, while the string is taut,

$$v^2 = 10.4x - 50x^2$$ *(7 marks)*

(iv) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(2 marks)*

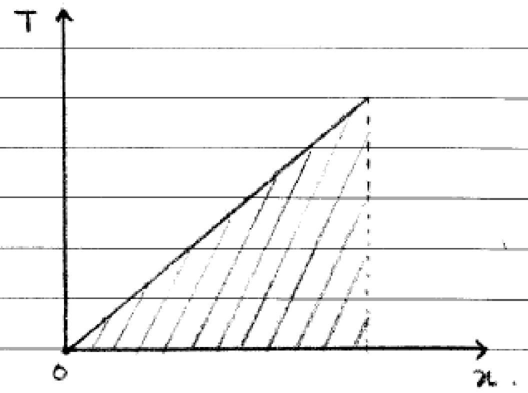
Student Response

8. a) Tension $= \dfrac{\lambda x}{l}$.

$x = e \quad \therefore \quad T = \dfrac{\lambda e}{l}$

Question number

Leave blank



Work done = Force × displacement

= area under the graph

$= \frac{1}{2} T x$

$= \frac{1}{2} \times \frac{\lambda e}{l} \times e$

$= \dfrac{\lambda e^2}{2l}$. ✓ 3

Commentary

Part (a) tested that part of the specification, work done = $\int F\ dx$. Few candidates found $\int_0^e \frac{\lambda x}{l} dx$ correctly; instead of integrating, a few candidates used the value of the integral to be the area under the line $y = \frac{\lambda x}{l}$ as shown in this example.

Unfortunately, many candidates used techniques which were not credited:

for example, elastic potential energy is $\frac{\lambda x^2}{2l}$ and $x = e$;

or work done = maximum force × half the distance moved, which is only valid if the force is linear and this was very rarely stated.

8(a)	Work done = $\int_0^e \frac{\lambda x}{l}\,dx$	M1		
	$= \left[\frac{\lambda x^2}{2l}\right]_0^e$	A1		Needs limit of 0
	$= \frac{\lambda e^2}{2l}$	A1	3	AG
	Or Area under a straight line = average force × distance = $\frac{\lambda e^2}{2l}$			
(b)(i)	Using $T = \frac{\lambda x}{l}$			
	$5g = \frac{150 \times x}{0.6}$	M1		
	Extension is 0.196 m	A1	2	
(ii)	EPE $= \frac{\lambda x^2}{2l}$			
	$= \frac{150 \times (0.3)^2}{2 \times 0.6}$	M1		
	= 11.25 J	A1	2	
(iii)	When x above P,			
	EPE $= \frac{150 \times (0.3 - x)^2}{2 \times 0.6}$	M1 A1		for $\frac{150 \times (\ldots - x)^2}{2 \times 0.6}$
	PE[relative to P] = $(-)5 \times g \times x$	M1		for $5 \times g \times$ distance
	KE + EPE [at new point] = EPE [at P] – gain in PE	M1		4 terms, all signs correct, 2 terms correct
	$\frac{1}{2}mv^2 + \frac{150 \times (0.3 - x)^2}{2 \times 0.6} =$			
	$\frac{150 \times (0.3)^2}{2 \times 0.6} - 5gx$	A1		
	$\frac{1}{2}mv^2 + \frac{150 \times (x^2 - 0.6x)}{2 \times 0.6} = -5gx$	m1		Equation involving terms in v^2, x^2 and x only
	$\frac{1}{2}.5.v^2 + 125x^2 - 75x = -49x$			
	$v^2 = 10.4x - 50x^2$	A1	7	
(iv)	Particle is at rest when $v = 0$			
	$10.4x - 50x^2 = 0$	M1		
	$x = 0$ [not required]			
	Or $x = \frac{10.4}{50} = 0.208$ m above P.	A1	2	
	Total		**16**	