Teacher Support Materials 2008

Maths GCE

MECHANICS MM03

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Dr Michael Cresswell, Director General.

Question 1

1 The speed, $v\,\mathrm{m\,s^{-1}}$, of a wave travelling along the surface of a sea is believed to depend on

the depth of the sea, d m,
the density of the water, $\rho\,\mathrm{kg\,m^{-3}}$,
the acceleration due to gravity, g, and
a dimensionless constant, k

so that

$$v = kd^{\alpha}\rho^{\beta}g^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, show that $\beta = 0$ and find the values of α and γ. *(6 marks)*

Student Response

1 $[v] = LT^{-1}$ $[d] = L$ $[\rho] = ML^{-3}$ $[g] = LT^{-2}$

Leave blank

$\therefore LT^{-1} = L^{\alpha}(ML^{-3})^{\beta}(LT^{-2})^{\gamma} = L^{\alpha}M^{\beta}L^{-3\beta}L^{\gamma}T^{-2\gamma}$ ✓

M's: $0 = \beta$ (since no M's on left hand side of equation) ✓

L's: $1 = \alpha - 3\beta + \gamma = \alpha + \gamma$

T's: $-1 = -2\gamma$ $\therefore \gamma = \frac{1}{2}$ ✓

$\alpha = 1 - \gamma = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore \alpha = \frac{1}{2}, \beta = 0, \gamma = \frac{1}{2}$ ✓

(6)

Commentary

The candidate writes the correct dimensions for *v, d,* ρ and *g* in terms of M, L and T, using square brackets for the physical quantities. The appropriate dimensions equation involving the indices α, β and γ is formed. The candidate is able to combine the indices outside the brackets with those inside brackets. The corresponding indices are then equated. The candidate provides a reason as to why $\beta = 0$. The equations involving α, β and γ are solved simultaneously to arrive at the correct results.

Mark scheme

Q	Solution	Marks	Total	Comments
1	$LT^{-1} = L^{\alpha} \times (ML^{-3})^{\beta}(LT^{-2})^{\gamma}$	M1		
	There is no M on the left hand side, so $\beta = 0$.	E1		
	$LT^{-1} = L^{\alpha+\gamma}T^{-2\gamma}$	m1		Dependent on M1
	$\alpha + \gamma = 1$ $-2\gamma = -1$	m1		Equating corresponding indices
	$\gamma = \frac{1}{2}$	A1		
	$\alpha = \frac{1}{2}$	A1	6	
	Total		**6**	

Question 2

2 The unit vectors **i** and **j** are directed due east and due north respectively.

Two runners, Albina and Brian, are running on level parkland with constant velocities of $(5\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$ and $(3\mathbf{i} + 4\mathbf{j})\,\text{m s}^{-1}$ respectively. Initially, the position vectors of Albina and Brian are $(-60\mathbf{i} + 160\mathbf{j})\,\text{m}$ and $(40\mathbf{i} - 90\mathbf{j})\,\text{m}$ respectively, relative to a fixed origin in the parkland.

(a) Write down the velocity of Brian relative to Albina. *(2 marks)*

(b) Find the position vector of Brian relative to Albina t seconds after they leave their initial positions. *(3 marks)*

(c) Hence determine whether Albina and Brian will collide if they continue running with the same velocities. *(3 marks)*

Student response

number

1 $[v] = LT^{-1}$ $[d] = L$ $[\rho] = ML^{-3}$ $[g] =$ [redacted] LT^{-2}

$\therefore LT^{-1} = L^{\alpha}(ML^{-3})^{\beta}(LT^{-2})^{\gamma} = L^{\alpha}M^{\beta}L^{-3\beta}L^{\gamma}T^{-2\gamma}$ ✓

M's: $0 = \beta$ (since no M's on left hand side of equation) ✓

L's: $1 = \alpha - 3\beta + \gamma = \alpha + \gamma$

T's: $-1 = -2\gamma$ $\therefore \gamma = \frac{1}{2}$ ✓

$\alpha = 1 - \gamma = 1 - \frac{1}{2} = \frac{1}{2}$ ✓

$\therefore \alpha = \frac{1}{2}$ $\beta = 0$ $\gamma = \frac{1}{2}$ ✓

Commentary

(a) The velocity of Brian relative to Albina is found correctly by subtracting the velocity of Albina from the velocity of Brian.

(b) The candidate knows that the position vector at any time is the sum of the initial position vector and the displacement vector and accordingly writes down the position vector of each runner at time *t*. The position vector of Brian relative to Albina at time *t* is then found by subtracting the position vector of the latter from the former.

(c) The candidate correctly states that for the two runners to collide, their relative position vector at time *t* would be the zero vector. As a result, **each** component of the relative position vector is set to zero and confirmed that **both** equations are satisfied when *t*=50. The candidate infers that the two runners would collide.

Mark Scheme

2(a)	${}_Av_B = v_B - v_A$ $= (3i + 4j) - (5i - j)$ $= -2i + 5j$	M1 A1	2	
(b)	${}_Ar_{0B} = (40i - 90j) - (-60i + 160j)$ $= 100i - 250j$ ${}_Ar_B = (100i - 250j) + (-2i + 5j)t$	M1 m1 A1F	3	Simplification not necessary **ALTERNATIVE:** $r_A = (60i + 160j) + (5i - j)t$ M1 $r_B = (40i - 90j) + (3i + 4j)t$ ${}_Ar_B = \left[(40i - 90j) + (3i + 4j)t\right] - \left[(60i + 160j) + (5i - j)t\right]$ m1A1
(c)	${}_Ar_B = (100 - 2t)i + (-250 + 5t)j$ $(100 - 2t) = 0 \quad \Leftrightarrow \quad t = 50$ $(-250 + 5t) = 0 \quad \Leftrightarrow \quad t = 50$ $\therefore$ *A* and *B* would collide.	M1 A1F E1	3	Collecting *i* and *j* terms **ALTERNATIVE:** $\left[(100 - 2t)i + (-250 + 5t)j\right].(-2i + 5j) = 0$ M1 $-200 + 4t - 1250 + 25t = 0 \Rightarrow t = 50$ A1 $\lvert{}_Ar_B\rvert \sqrt{(100 - 2\times 50)^2 + (-250 + 5\times 50)^2} = 0$ $\therefore$ *A* and *B* would collide E1
	Total		**8**	

Question 3

3 A particle of mass 0.2 kg lies at rest on a smooth horizontal table. A horizontal force of magnitude F newtons acts on the particle in a constant direction for 0.1 seconds. At time t seconds,

$$F = 5 \times 10^3 t^2, \quad 0 \leqslant t \leqslant 0.1$$

Find the value of t when the speed of the particle is $2\,\mathrm{m\,s^{-1}}$. *(4 marks)*

Student Response

Question number

Leave blank

2.

a) ${}_BV_A = V_B - V_A = (3\underline{i} + 4\underline{j}) - (5\underline{i} - \underline{j}) = -2\underline{i} + 5\underline{j}$ ✓ 2

b) At t seconds,

$r_A = (-60\underline{i} + 160\underline{j}) + t(5\underline{i} - \underline{j})$ ✓

$r_B = (40\underline{i} - 90\underline{j}) + t(3\underline{i} + 4\underline{j})$ ✓

So ${}_Br_A = r_B - r_A = (100\underline{i} - 250\underline{j}) + t(-2\underline{i} + 5\underline{j})$ ✓

$= (100 - 2t)\underline{i} - (250 - 5t)\underline{j}$ ✓ 3

c) For A and B ✓ to collide, ${}_Br_A = 0$, so

$100 - 2t = 0$ ①

$-250 + 5t = 0$ ② ✓

These are both satisfied by $t = 50s$ ✓

So if they continue running with the same velocities they will collide when $t = 50s$, at the point $(190\underline{i} + 110\underline{j})$ ✓ 3

8

3. $F = 5000t^2$

Impulse $= \int_0^{0.1} F \, dt$

When speed of particle is 2 ms^{-1}, the change in momentum from $t=0$ is

$2 \times 0.2 - 0 = 0.4 Ns$. This is the impulse.

This occurs at T, where

$\int_0^T 5000t^2 \, dt = 0.4 \Rightarrow \left[\frac{5000}{3}t^3\right]_0^T = 0.4$ ✓

$\frac{5000T^3}{3} = 0.4$, $5000T^3 = 1.2$, $T^3 = \frac{1.2 \times 10^{-3}}{5} = 2.4 \times 10^{-4}$ ✓

So $T = 0.0621s$ ✓

4

Commentary

The candidate understands how the impulse of a variable force, as opposed to a constant force, is found. Also, they understand the relationship between impulse and momentum. The candidate calculates the gain in the momentum of the particle and states the equivalence of the momentum and the impulse of the force. The Impulse/momentum equation is written with appropriate limits for the integral. The correct integration and simplification of the equation is followed by taking the cubic root of 2.4×10^{-4}. The candidate states the correct result to three significant figures and gains full marks for this question.

Mark Scheme

Q	Solution	Marks	Total	Comments
3	$\int_0^t 5\times10^3 t^2\,dt = 0.2(2) - 0.2(0)$	M1A1		Impulse-Momentum principle
	$\frac{5\times10^3}{3}t^3 = 0.4$	A1F		
	$t = 0.0621$	A1F	4	At least 3 sig. fig. required
	Total		**4**	

Question 4

4 Two smooth spheres, A and B, have equal radii and masses m and $2m$ respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(4\mathbf{i} + 3\mathbf{j})$ when it collides with the sphere B which has velocity $(-2\mathbf{i} + 2\mathbf{j})$. After the collision, the velocity of B is $(\mathbf{i} + \mathbf{j})$.

(a) Find the velocity of A immediately after the collision. *(3 marks)*

(b) Find the angle between the velocities of A and B immediately after the collision. *(3 marks)*

(c) Find the impulse exerted by B on A. *(3 marks)*

(d) State, as a vector, the direction of the line of centres of A and B when they collide. *(1 mark)*

Student Response

3 $I = \int F\,dt = 5\int 10^3 t^2\,dt \rightarrow$

$\rightarrow \left[5 \times \frac{10^3}{3} t^3\right]_0^{0.1} = 16\frac{2}{3}$

Impulse = $0.2 \times 2 - 0 = 0.4$ ✓ M1

$0.4 = 5 \times \frac{10^3}{3} t^3$ ✓ A1

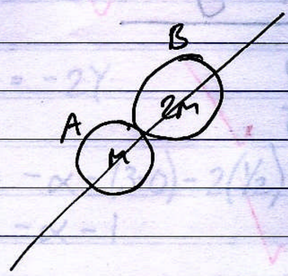
A1

A0

$t = 0.0288\ s\ (3\ s.f)$ ✗

(3)

4



a) $(4i + 3j)m + (-2i + 2j)2m = V_A m + (i + j)2m$ ✓

$4i + 3j - 4i + 4j = V_A + 2i + 2j$ ✓

$7j = V_A + 2i + 2j$ ✓

$V_A = (5j - 2i)\ ms^{-1}$ ✓ 3

Commentary

(a) The candidate is familiar with the principle of conservation of linear momentum in two dimensions and applies it correctly to write the necessary equation. Both sides of the equation are divided by *m* and then the equation is simplified and solved to give the velocity of the sphere *A* immediately after the collision.

(b) The candidate uses effective angle diagrams for guidance. The inverse tangents are used to find the angles which the velocities of *A* and *B* make with the ***j*** direction. The angles are then added to give the correct answer for this part of the question.

(c) Evidently, the candidate understands that the impulse exerted by *B* on *A* is equal to the decrease in the momentum of *B*. This is indicated in the correct impulse/momentum equation written by the candidate. The equation is then simplified to give the requested impulse. The magnitude of the impulse was not required in this question.

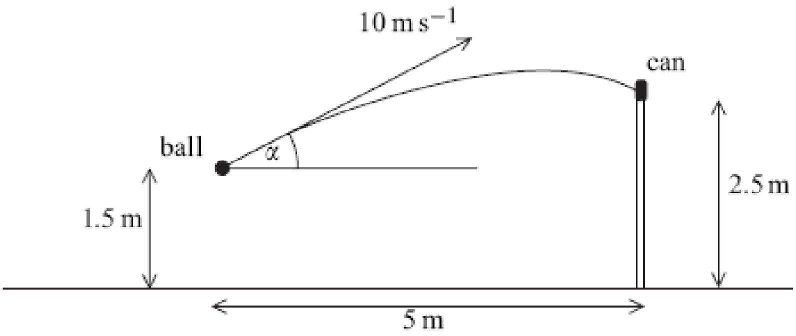
(d) The candidate understands that the direction of the line of centres of the spheres is the same as the direction of the impulse and gives a correct direction vector.

Mark Scheme

4(a)	C.L.M. $m(4\mathbf{i}+3\mathbf{j})+2m(-2\mathbf{i}+2\mathbf{j})=mv+2m(\mathbf{i}+\mathbf{j})$ $7\mathbf{j}=v+(2\mathbf{i}+2\mathbf{j})$ $v=-2\mathbf{i}+5\mathbf{j}$	M1 A2,1,0	3	A1 for one slip
(b)	The angle with **j** direction : A: $\tan^{-1}\frac{2}{5}=21.8^\circ$ B: $\tan^{-1}\frac{1}{1}=45^\circ$ The angle $=21.8^\circ+45^\circ=67^\circ$	M1 A1F	3	OE. in **i** direction M1 for two inverse tan and addition of angles AWRT. **Alternative (not in the specification)** $(-2\mathbf{i}+5\mathbf{j}).(\mathbf{i}+\mathbf{j})=\sqrt{29}\times\sqrt{2}\cos\theta$ (M1) $\cos\theta=\frac{3}{\sqrt{58}}$ (A1) $\theta=67^\circ$ (A1F) awrt
(c)	The impulse = Gain in momentum of *A* $=m(-2\mathbf{i}+5\mathbf{j})-m(4\mathbf{i}+3\mathbf{j})$ $=-6m\mathbf{i}+2m\mathbf{j}$	M1 A1F A1F	3	
(d)	$-3\mathbf{i}+\mathbf{j}$ or any scalar multiple of $-3\mathbf{i}+\mathbf{j}$	B1	1	
	Total		**10**	

Question 5

5 A boy throws a small ball from a height of 1.5 m above horizontal ground with initial velocity $10\,\text{m}\,\text{s}^{-1}$ at an angle α above the horizontal. The ball hits a small can placed on a vertical wall of height 2.5 m, which is at a horizontal distance of 5 m from the initial position of the ball, as shown in the diagram.



(a) Show that α satisfies the equation

$$49\tan^2\alpha - 200\tan\alpha + 89 = 0$$ *(7 marks)*

(b) Find the **two** possible values of α, giving your answers to the nearest 0.1°. *(3 marks)*

(c) (i) To knock the can off the wall, the horizontal component of the velocity of the ball must be greater than $8\,\text{m}\,\text{s}^{-1}$.

Show that, for one of the possible values of α found in part (b), the can will be knocked off the wall, and for the other, it will **not** be knocked off the wall. *(3 marks)*

(ii) Given that the can is knocked off the wall, find the direction in which the ball is moving as it hits the can. *(4 marks)*

Student Response

5. a) $s = ut + \frac{1}{2}at^2$ (↑)

$1 = 10\sin\alpha t - \frac{1}{2}gt^2$ ①

$s = ut + \frac{1}{2}at^2$ (→)

$5 = 10\cos\alpha t$

$t = \frac{5}{10\cos\alpha} = \frac{1}{2\cos\alpha}$ ②

② → ① $1 = \frac{10\sin\alpha}{2\cos\alpha} - \frac{g}{2 \times 4\cos^2\alpha}$

$1 = 5\tan\alpha - \frac{g}{8}(\sec^2\alpha)$

$1 = 5\tan\alpha - \frac{g}{8}(1+\tan^2\alpha)$

$c^2+s^2=1$
$1+\tan^2=\sec^2$

$8 = 40\tan\alpha - 9.8 - 9.8\tan^2\alpha$

$9.8\tan^2\alpha - 40\tan\alpha + 17.8 = 0$

$49\tan^2\alpha - 200\tan\alpha + 89 = 0$

b) $\tan\alpha = \frac{200 \pm \sqrt{22556}}{98} = 3.57$ or 0.508

$\alpha = 74.4°$ or $26.9°$

c) i) $v = u + at$ (→)

$\alpha = 74.4$ $v = 10\cos 74.4 = 2.69$ ms^{-1}

$\alpha = 26.9$ $v = 10\cos 26.9 = 8.91$ ms^{-1}

∴ when $\alpha = 26.9°$, it will be knocked off but when $\alpha = 74.4°$ it won't be.

Leave blank

ii) $s = ut + \frac{1}{2}at^2$ $(\rightarrow)$

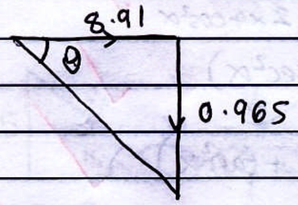
$5 = 10\cos 26.9\, t$

$t = \underline{0.561\, s}$

$v = u + at$ (↓)

$v = 10\sin 26.9 - g \times 0.561$

$v = \underline{-0.965}\ ms^{-1}$



$\tan\theta = \frac{0.965}{8.91}$

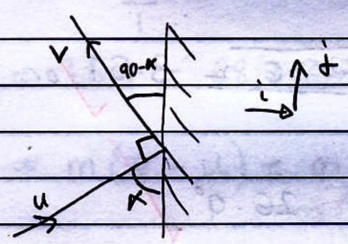
$\theta = 6.18°$

∴ travelling at an angle 6.18° below the horizontal

4

17

6



a) $e = \frac{v_i}{u_i}$ $\frac{3}{4} = \frac{v\sin(90-\alpha)}{u\sin\alpha}$ ①

$u\cos\alpha = v\cos(90-\alpha)$

$v = \frac{u\cos\alpha}{\cos(90-\alpha)}$ ②

② → ① $\frac{3}{4} = \frac{u\cos\alpha\sin(90-\alpha)}{u\cos(90-\alpha)\sin\alpha} = \frac{\cos\alpha(\sin 90\cos\alpha - \cos 90\sin\alpha)}{\sin\alpha(\cos 90\cos\alpha + \sin 90\sin\alpha)}$

Commentary

(a) The candidate considers the vertical and the horizontal motion of the ball using the appropriate constant acceleration formula and realising that the ball has zero acceleration horizontally. The correct values for the horizontal and vertical distances travelled by the ball are substituted. The time in the equation for vertical motion is eliminated by using the equation for horizontal motion. The candidate manipulates the resulting equation effectively and uses the appropriate trigonometric identity to show the required result.

(b) The candidate solves the quadratic equation in $\tan\alpha$ and finds the two possible values of α correct to one decimal place as requested.

(c) (i) The candidate calculates the horizontal component of the velocity of the ball for each of the values of α accurately to three significant figures. The correct conclusions are then stated for both results.

(c)(ii) The candidate finds the time taken by the ball to reach the can from its point of projection accurately to three significant figures. The candidate then uses the time to determine the vertical component of the velocity of the ball as it hits the can. The candidate clarifies the situation with a diagram showing the horizontal and vertical components of the velocity. This diagram is then used to find the required angle by using the tangent ratio. The candidate states the direction unambiguously.

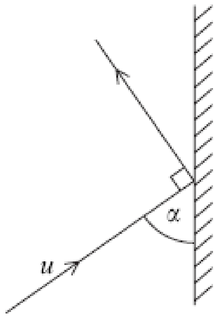
Mark Scheme

Q	Solution	Marks	Total	Comments
5(a)	$5 = 10\cos\alpha . t$	M1		
	$t = \dfrac{5}{10\cos\alpha}$	A1		
	$1 = -\dfrac{1}{2}(9.8)t^2 + 10\sin\alpha . t$	M1A1		
	$1 = -\dfrac{1}{2}(9.8)\dfrac{25}{100\cos^2\alpha} + 10\sin\alpha\dfrac{5}{10\cos\alpha}$	m1		Dependent on both M1s
	$1 = -\dfrac{1}{2}(9.8)\dfrac{25}{100}(1+\tan^2\alpha) + 10\sin\alpha\dfrac{5}{10\cos\alpha}$	A1		
	$49\tan^2\alpha - 200\tan\alpha + 89 = 0$	A1	7	Answer given
(b)	$\tan\alpha = \dfrac{200 \pm \sqrt{40000 - 4(49)(89)}}{2 \times 49}$	M1		
	$= 3.57,\ 0.508$	A1		AWRT
	$\alpha = 74.4^\circ, 26.9^\circ$	A1F	3	
(c)(i)	$10\cos 26.9^\circ = 8.92$ (or 8.91) > 8			
	$\Rightarrow$ The can will be knocked off the wall	M1		Both values checked
	$10\cos 74.4^\circ = 2.69 < 8$	A1F		Acc. of both results
	$\Rightarrow$ The can will not be knocked off the wall	E1	3	Correct conclusions
		ALTERNATIVE The can will be knocked off the wall if $10\cos\alpha > 8$ $\cos\alpha > 0.8$ $\alpha < 36.9°$ M1A1 So, for $\alpha = 26.9°$ the can will be knocked off and for $\alpha = 74.4°$, the can will not be knocked off E1		
5(c)(ii)	$x = ut$			
	$t = \dfrac{5}{10\cos 26.9^\circ}$			
	$v = 10\sin 26.9^\circ - 9.8(\dfrac{5}{10\cos 26.9^\circ})$	M1		Any correct use of equations
	$v = -0.970$	A1F		
	$\tan\theta = \dfrac{-0.970}{8.92}$	M1		
	$\theta = -6.2^\circ$			
	At an angle of depression of 6.2°	A1F	4	AWRT 6°
	Total		**17**	

Question 6

6 A small smooth ball of mass m, moving on a smooth horizontal surface, hits a smooth vertical wall and rebounds. The coefficient of restitution between the wall and the ball is $\frac{3}{4}$.

Immediately before the collision, the ball has velocity u and the angle between the ball's direction of motion and the wall is α. The ball's direction of motion immediately after the collision is at right angles to its direction of motion before the collision, as shown in the diagram.



(a) Show that $\tan\alpha = \dfrac{2}{\sqrt{3}}$. *(5 marks)*

(b) Find, in terms of u, the speed of the ball immediately after the collision. *(2 marks)*

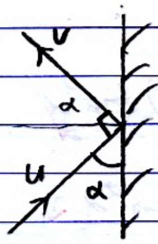
(c) The force exerted on the ball by the wall acts for 0.1 seconds.

Given that $m = 0.2\,\text{kg}$ and $u = 4\,\text{m}\,\text{s}^{-1}$, find the average force exerted by the wall on the ball. *(6 marks)*

Student Response

Question number | Leave blank

6a)



$e = \frac{3}{4}$

components of velocities parallel to wall unchanged

$u\cos\alpha = V\sin\alpha$ ✓

$u = V\tan\alpha$ ① ✓

perpendicular to wall $e = \frac{\text{speed of separation}}{\text{speed of approach}}$

$\frac{3}{4} = \frac{V\cos\alpha}{u\sin\alpha}$ ✓

$3u\sin\alpha = 4V\cos\alpha$ ✓

$\frac{3}{4}\frac{u\sin\alpha}{\cos\alpha} = V$ ✓

$V = \frac{3u\tan\alpha}{4}$ ② ✓

put ② into ① $\quad u = \frac{3u}{4}\tan^2\alpha$ ✓

$\frac{4}{3} = \tan^2\alpha$ ✓

$\tan\alpha = \frac{2}{\sqrt{3}}$ ✓

5

6b) $v = \frac{3u}{4}\tan\alpha \qquad v = \frac{3u}{4} \times \frac{2}{\sqrt{3}} \qquad v = \frac{1u}{2} \times \frac{3}{\sqrt{3}}$ ✓

$v = \frac{\sqrt{3}}{2}u$ ✓

2

c) $I = mv - mu$ perpendicular to the wall

$I = 0.2(V\cos\alpha) - 0.2(-u\sin\alpha)$ ✓

$= 0.2 \times \frac{\sqrt{3}}{2}u\cos\alpha + 0.2u\sin\alpha$ ✓

Question number

$\tan\alpha = \frac{2}{\sqrt{3}} \quad \alpha = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right) = 49.1066$ ✓

$I = \frac{0.2\sqrt{3}}{2} \times 4\cos(49.1066) + (0.2 \times 4\sin(49.1066))$ ✓

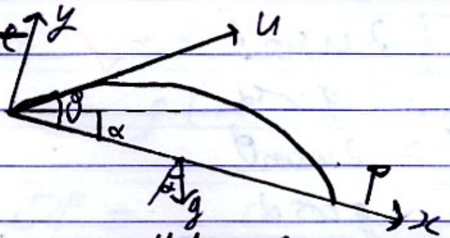
$I = 1.058\ Ns$ ✓

$I = Ft \quad 1.058 = F \times 0.1$ ✓

average Force $F = \frac{1.058}{0.1}$ ✓

$F = 10.6\ N$ to 3s.f. ✓

7a) ~~resolve~~



~~resolve parallel to the plane s=~~

~~resolve normal to the plane $s = ut + \frac{1}{2}at^2$ at P~~

~~$0 = u\sin\theta t - \frac{g\cos\alpha}{2} t^2$~~

~~$0 = t\left(u\sin\theta - \frac{g\cos\alpha}{2} t\right)$~~ WR

~~$t = 0$ (trivial) or $t = \frac{2u\sin\theta}{g\cos\alpha}$~~

~~resolve parallel to the plane $s = ut + \frac{1}{2}at^2$~~

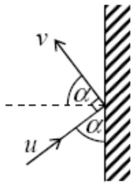
Commentary

(a) The candidate states that the components of the velocities of the ball parallel to the wall are unchanged and forms an equation by equating these components. Newton's experimental law is used for the components of the velocities perpendicular to the wall to write another equation involving the velocities. The two equations are then manipulated effectively to show the required result.

(b) The candidate uses the equation arising from the application of the experimental law and the result from part (a) to find the speed of the ball in terms of u immediately after collision. The result is given in surd form.

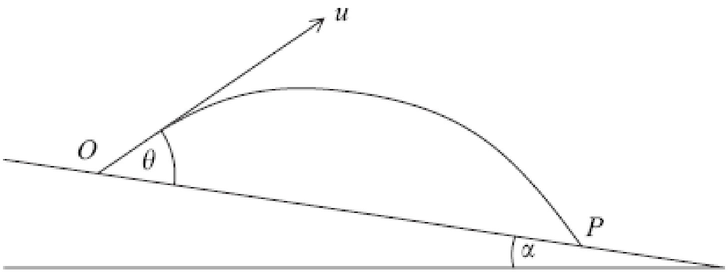
(c) The angle α is calculated accurately. The candidate shows understanding that the impulse exerted by the wall on the ball is equal to the change of momentum of the ball in the direction perpendicular to the wall. Also, the candidate is able to find the change in momentum without making any sign errors. The components of the velocities used are correct and resulting impulse/momentum equation written is free from any errors. The candidate finds the impulse accurately. The average force exerted by the wall on the wall is found correctly to three significant figures.

Mark Scheme

Q	Solution	Marks	Total	Comments
6(a)				
	Parallel to the wall : velocity is unchanged $u\cos\alpha = v\sin\alpha$	M1		
	Perpendicular to the wall : Law of Restitution $\frac{v\cos\alpha}{u\sin\alpha} = \frac{3}{4}$	M1		
	$\frac{v\cos\alpha}{v\tan\alpha\sin\alpha} = \frac{3}{4}$	m1		Dependent on both M1s
	$\frac{\cos^2\alpha}{\sin^2\alpha} = \frac{3}{4}$	m1		Dependent on both M1s
	$\tan^2\alpha = \frac{4}{3}$			
	$\tan\alpha = \frac{2}{\sqrt{3}}$	A1	5	Answer given
(b)	$v = \frac{u}{\tan\alpha}$	M1		
	$v = \frac{\sqrt{3}}{2}u$ or $0.866u$	A1	2	
(c)	Magnitude of Impulse = Change in momentum perpendicular to the wall	M1		
	$= 0.2\times v\cos\alpha - (-0.2\times 4\sin\alpha)$	A1 A1		
	$= 0.2\times\frac{\sqrt{3}}{2}\times 4\cos\alpha + 0.2\times 4\sin\alpha$	m1		
	$= 1.06$ Ns	A1F		
	Average Force = $\frac{1.06}{0.1} = 10.6$ N	A1F	6	
	Total		**13**	

Question 7

7 A projectile is fired with speed u from a point O on a plane which is inclined at an angle α to the horizontal. The projectile is fired at an angle θ to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point P, lower down the inclined plane, as shown in the diagram.



(a) Find, in terms of u, g, θ and α, the greatest perpendicular distance of the projectile from the plane. *(4 marks)*

(b) (i) Find, in terms of u, g, θ and α, the time of flight from O to P. *(2 marks)*

(ii) By using the identity $\cos A \cos B + \sin A \sin B = \cos(A - B)$, show that the distance OP is given by $\dfrac{2u^2 \sin\theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$. *(6 marks)*

(iii) Hence, by using the identity $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$ or otherwise, show that, as θ varies, the maximum possible distance OP is $\dfrac{u^2}{g(1 - \sin\alpha)}$. *(5 marks)*

Student Response

Question number

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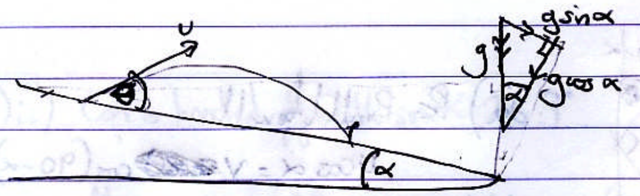
(7)



Parallel to plane: $x = (u\cos\theta)t + \frac{1}{2}(g\sin\alpha)t^2$.
Perp to plane: $y = (u\sin\theta)t + \frac{1}{2}(-g\cos\alpha)t^2$. ✓

a) Perp: $u = u\sin\theta$
$v = 0$ (since greatest distance is where $v=0$)
$a = -g\cos\alpha$
$s = ?$

$v^2 = u^2 + 2as$
$0 = u^2\sin^2\theta - (2g\cos\alpha)s$ ✓
$\therefore s = \dfrac{u^2\sin^2\theta}{2g\cos\alpha}$ ✓ 4

b)i) Lands at P where $y = 0$.

$\therefore 0 = (u\sin\theta)t - \left(\frac{g}{2}\cos\alpha\right)t^2$ ✓
$0 = u\sin\theta - \left(\frac{g}{2}\cos\alpha\right)t$
$\therefore t = \dfrac{u\sin\theta}{g/2\cos\alpha}$ or $t = \dfrac{2u\sin\theta}{g\cos\alpha}$ ✓ 2

(ii) Range: $x = (u\cos\theta)t + \frac{g}{2}\sin\alpha(t)^2$ ✓
$= (u\cos\theta)\left(\dfrac{2u\sin\theta}{g\cos\alpha}\right) + \dfrac{g}{2}\sin\alpha\left(\dfrac{2u\sin\theta}{g\cos\alpha}\right)^2$ ✓

$= \dfrac{2u^2\sin\theta\cos\theta}{g\cos\alpha} + \dfrac{2u^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$ ✓

$= \dfrac{2u^2\sin\theta\cos\theta\cos\alpha}{g\cos^2\alpha} + \dfrac{2u^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$ ✓

$= \dfrac{2u^2\sin\theta}{g\cos^2\alpha}(\cos\theta\cos\alpha + \sin\alpha\sin\theta)$ ✓

From formula, $A = \theta, B = \alpha$, $\therefore x = \dfrac{2u^2\sin\theta}{g\cos^2\alpha}(\cos(\theta - \alpha))$ ✓ 6

Question number

iii) $R = \frac{u^2}{g\cos^2\alpha}\left(\sin(2\theta - \alpha) + \sin\alpha\right)$

For R_{MAX}, only θ can vary, so $\sin(2\theta - \alpha) = 1$ for maximum

$$R_{MAX} = \frac{u^2(1 + \sin\alpha)}{g\cos^2\alpha}$$

$$= \frac{u^2(1 + \sin\alpha)}{g(1 - \sin^2\alpha)}$$

$$= \frac{u^2(1 + \sin\alpha)}{g(1 + \sin\alpha)(1 - \sin\alpha)}$$

$$= \frac{u^2}{g(1 - \sin\alpha)}$$

Commentary

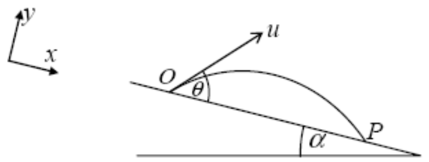
The candidate writes the kinematic equations of the motion of the projectile parallel and perpendicular to the plane. The equations are correct and they are possibly informed by the candidate's diagram which include the components of the acceleration along and perpendicular to the inclined plane.

(a) The candidate makes the efficient choice of using the constant acceleration formula $v^2 = u^2 + 2as$ to find the greatest perpendicular distance of the projectile from the plane. The component of the initial velocity and the component of the acceleration used are both correct and the latter is free from sign error. The candidate uses the fact that for the maximum perpendicular distance $v = 0$ and finds the correct expression.

(b) (i) The candidate understands that when the projectile lands at *P* its perpendicular distance from the plane is zero. The value of zero is substituted for *y* in the equation of motion perpendicular to the plane. The resulting quadratic equation for *t* is solved to give the time of flight from *O* to *P* in simplified form.

(c) (ii) The candidate has changed $2\sin\vartheta\cos(\vartheta-\alpha)$ in the result for part b(ii) to $\sin(2\vartheta-\alpha)+\sin\alpha$. The candidate recognises and states that, as ϑ varies, the distance *OP* is a maximum for $\sin(2\vartheta-\alpha)=1$. This condition is used to give the expression $\dfrac{u^2(1+\sin\alpha)}{g\cos^2\alpha}$. The candidate then shows two stages of algebraic manipulation to give the printed result.

Mark Scheme

Q	Solution	Marks	Total	Comments
7				
(a)	$v_y{}^2 = u^2\sin^2\theta$ - $2g\cos\alpha.y$	M1 A1		
	$0 \quad = u^2\sin^2\theta$ - $2g\cos\alpha.y_{\max}$	m1		
	$y_{\max} = \dfrac{u^2\sin^2\theta}{2g\cos\alpha}$	A1F	4	
(b)(i)	$u\sin\theta\ t - \dfrac{1}{2}g\cos(\alpha)t^2 = 0$	M1		
	$t = \dfrac{2u\sin\theta}{g\cos\alpha}$	A1	2	
(ii)	$x = u\cos\theta t - \dfrac{1}{2}g\sin(-\alpha)t^2$	M1 A1		
	$R = u\cos\theta(\dfrac{2u\sin\theta}{g\cos\alpha}) + \dfrac{1}{2}g\sin\alpha(\dfrac{2u\sin\theta}{g\cos\alpha})^2$	M1		
	$= \dfrac{2u^2\cos\theta\sin\theta\cos\alpha + 2u^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$	m1		Dependent on both M1s
	$= \dfrac{2u^2\sin\theta(\cos\theta\cos\alpha + \sin\theta\sin\alpha)}{g\cos^2\alpha}$	A1F		
	$= \dfrac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$	A1	6	Answer given
(iii)	$\overline{OP} = \dfrac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$			
	$= \dfrac{2u^2\frac{1}{2}\left[\sin(2\theta-\alpha)+\sin\alpha\right]}{g\cos^2\alpha}$	M1A1		
	$\overline{OP}$ is max when $\sin(2\theta-\alpha) = 1$	M1		
	$\overline{OP}_{\max} = \dfrac{u^2(1+\sin\alpha)}{g\cos^2\alpha}$	A1F		
	$\overline{OP}_{\max} = \dfrac{u^2(1+\sin\alpha)}{g(1-\sin^2\alpha)}$			
	$\overline{OP}_{\max} = \dfrac{u^2}{g(1-\sin\alpha)}$	A1	5	Answer given
	Total		**17**	

Q	Solution	Marks	Total	Comments
7(a)	**ALTERNATIVE**			
	$0 = u\sin\theta - g\cos a\ t$	M1		
	$t = \dfrac{u\sin\theta}{g\cos a}$	A1		
	$y_{max} = u\sin\theta\left(\dfrac{u\sin\theta}{g\cos a}\right) - \dfrac{1}{2}g\cos a\left(\dfrac{u\sin\theta}{g\cos a}\right)^2$	m1		
	$y_{max} = \dfrac{u^2\sin^2\theta}{2g\cos a}$	A1F	4	
	Total		**4**	