Teacher Support Materials 2008

Maths GCE

Paper Reference MFP1

Question 1

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β .

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Show that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{9}{5}$. *(2 marks)*

(d) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. *(2 marks)*

Student Response

Leave blank

1) a) $x^2 + x + 5 = 0$

$\alpha + \beta = -1$

$\alpha\beta = 5$ ✓ 2

b) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-1)^2 - 2(5)$

$= -9$ ✓ 2

Leave blank

c) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{-9}{5}$ ✓ 2

d) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{-9}{5}$

$\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = \frac{\alpha\beta}{\alpha\beta} = \frac{5}{5} = 1$ ✓ 1

$= x^2 + \frac{9}{5}x + 1 = 0$ 0

(7)

Commentary

Most candidates answered this question well. Common errors occurred in part (d), where, as in this example, candidates failed to give integer coefficients. Others failed to write "equals zero" in their equation.

Mark scheme

MFP1

Q	Solution	Marks	Total	Comments
1(a)	$\alpha + \beta = -1$, $\alpha\beta = 5$	B1B1	2	
(b)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$	M1		with numbers substituted
	$\ldots = 1 - 10 = -9$	A1F	2	ft sign error(s) in (a)
(c)	$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$	M1		
	$\ldots = -\frac{9}{5}$	A1	2	AG: A0 if $\alpha + \beta = 1$ used
(d)	Product of new roots is 1	B1		PI by constant term 1 or 5
	Eqn is $5x^2 + 9x + 5 = 0$	B1F	2	ft wrong value for product
	Total		**8**	

Question 2

2 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$3\mathrm{i}z + 2z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$3\mathrm{i}z + 2z^* = 7 + 8\mathrm{i}$$ *(3 marks)*

Student response

Leave blank

2

a) $3i\,z + 2(x - iy)$

$3i(x+iy) + 2(x - iy)$ ✓ M 1

$3i(x) + 3i^2y + 2x - 2iy$ ✗

M 1

$3i - 3y + 2x - 2iy$

A 0

real: $2x - 3y$ ✓ imaginary: $3i - 2iy$ ✓

b) $2x - 3y = 7$ ✓ $3i - 2iy = 8i$

erased $3 - 2y = 8$ M 1

$2x - (3 \times -\frac{5}{2}) = 7$ $-2y = 5$

$2x - (-7.5) = 7$ $2y = -5$ m 0

$2x + 7.5 = 7$ $y = -\frac{5}{2}$

$2x = -0.5$

$x = \frac{1}{4}$

$z = \frac{1}{4} + -\frac{5}{2}i$ ✗

(3)

Commentary

Most candidates attempted to expand and simplify $3i(x + iy) + 2(x - iy)$. Unfortunately this led to many algebraic errors; in this script $3i(x + iy)$ is found to be $3i + 3i^2y$.

Mark Scheme

2(a)	Use of $z^* = x - iy$ Use of $i^2 = -1$ $3iz + 2z^* = (2x - 3y) + i(3x - 2y)$	M1 M1 A1	3	Condone inclusion of i in I part
(b)	Equating R and I parts $2x - 3y = 7,\ 3x - 2y = 8$ $z = 2 - i$	M1 m1 A1	3	with attempt to solve Allow $x = 2, y = -1$
	Total		**6**	

Question 3

3 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\int_9^\infty \frac{1}{\sqrt{x}}\,dx$; *(3 marks)*

(b) $\int_9^\infty \frac{1}{x\sqrt{x}}\,dx$. *(4 marks)*

Student Response

3a $\int_9^\infty \frac{1}{\sqrt{x}}\,dx$

$\int_9^\infty x^{-1/2}\,dx$

$\left[2x^{1/2}+c\right]_9^\infty$ ✓ ∞ Problem limit

let ∞ = k

$\left[2x^{1/2}+c\right]_9^k$

$2k^{1/2}-6$

As $k \to \infty$, $2k^{1/2}-6 \to \infty$. ✓ 3

b) $\int_9^\infty \frac{1}{x\sqrt{x}}$

$\int_9^\infty x^{-3/2}$

$\left[-\frac{2}{3}x^{-1/2}+c\right]_9^\infty$ ✓ ~~Problem limit ∞~~ — M 1, A 0

Problem limit of ∞

let K = ∞

$-\frac{2}{3\sqrt{K}} + \frac{2}{3\sqrt{9}}$

$-\frac{2}{3\sqrt{K}} + \frac{2}{27}$

As K -> ∞ $\quad -\frac{2}{3\sqrt{K}} \to 0$ ✓ — E 1

So $-\frac{2}{3\sqrt{K}} + \frac{2}{27} \to \frac{2}{\ }$ ✗ No FT — A 0

Commentary

Many algebraic errors occurred in the evaluation of $\int \frac{1}{x\sqrt{x}}\,dx$ in part (b). In this example, the candidate correctly simplified $\frac{1}{x\sqrt{x}}$ to $\frac{1}{x^{\frac{3}{2}}}$, but then after raising the power by one divides by the old power rather than the new power.

Mark Scheme

3(a)	$\int x^{-1/2}\,dx = 2x^{1/2}(+c)$	M1A1		M1 for correct power in integral
	$x^{1/2} \to \infty$ as $x \to \infty$, so no value	E1	3	
(b)	$\int x^{-3/2}\,dx = -2x^{-1/2}(+c)$	M1A1		M1 for correct power in integral
	$x^{-1/2} \to 0$ as $x \to \infty$	E1		PI
	$\int_9^\infty x^{-3/2}\,dx = -2(0-\frac{1}{3}) = \frac{2}{3}$	A1	4	Allow A1 for correct answer even if not fully explained
	Total		7	

Question 4

4 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are related by an equation of the form

$$y = ax + \frac{b}{x+2}$$

where a and b are constants.

(a) The variables X and Y are defined by $X = x(x+2)$, $Y = y(x+2)$.

Show that $Y = aX + b$. *(2 marks)*

(b) The following approximate values of x and y have been found:

x	1	2	3	4
y	0.40	1.43	2.40	3.35

(i) Complete the table in **Figure 1**, showing values of X and Y. *(2 marks)*

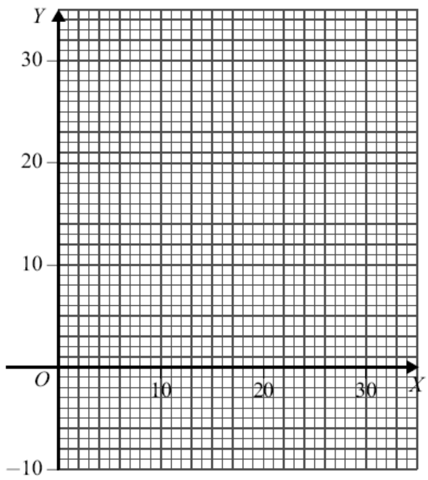
(ii) Draw on **Figure 2** a linear graph relating X and Y. *(2 marks)*

(iii) Estimate the values of a and b. *(3 marks)*

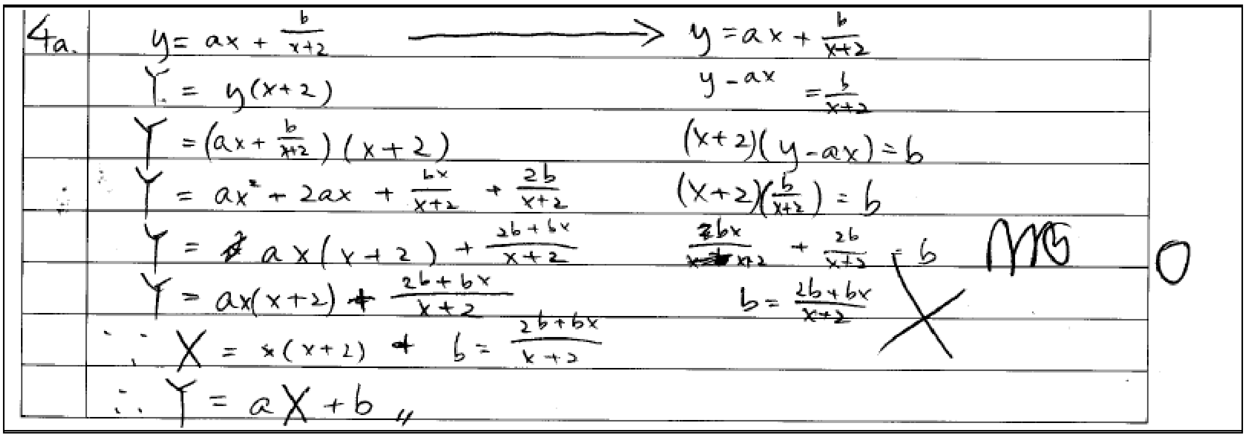
Figure 1 (for use in Question 4)

x	1	2	3	4
y	0.40	1.43	2.40	3.35
X	3			
Y	1.20			

Figure 2 (for use in Question 4)



Student Response



Commentary

Many candidates appreciated that the first step should be to multiply the equation $y = ax + \dfrac{b}{x+2}$ by $x + 2$.

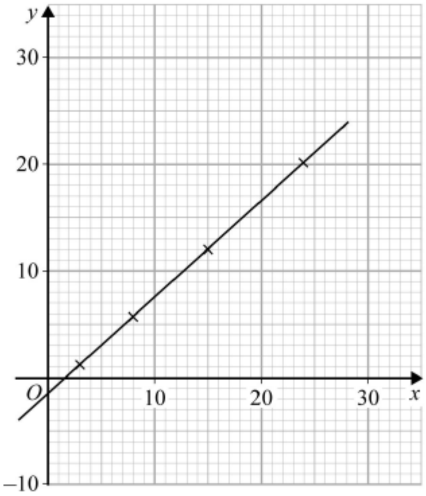
The equation would then become $y\,(x+ 2) = ax\,(x+ 2) + b$,

or, $Y = aX + b$

This script shows a typical candidate who struggled with the required algebraic multiplication by $x + 2$.

Mark Scheme

4(a)	Multiplication by $x + 2$	M1		applied to all 3 terms
	$Y = aX + b$ convincingly shown	A1	2	AG
(b)(i)	$X = 8, 15, 24$ in table	B1		
	$Y = 5.72, 12, 20.1$ in table	B1	2	Allow correct to 2SF

Q	Solution	Marks	Total	Comments
4(b)(ii)				
	Four points plotted	B1F		ft incorrect values in table
	Reasonable line drawn	B1F	2	ft incorrect points
(iii)	Method for gradient	M1		or algebraic method for *a* or *b*
	a = gradient ≈ 0.9	A1		Allow from 0.88 to 0.93 incl
	b = *Y*-intercept ≈ −1.5	B1F	3	Allow from −2 to −1 inclusive; ft incorrect points/line **NMS** B1 for *a*, B1 for *b*
	Total		**9**	

Question 5

5 (a) Find, in **radians**, the general solution of the equation

$$\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$$

giving your answer in terms of π. *(5 marks)*

(b) Hence find the smallest **positive** value of x which satisfies this equation. *(2 marks)*

Student Response

Leave blank

5) a) $\cos\left(\frac{x}{2}+\frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$ $2\pi n + \alpha$

$\alpha = \frac{\pi}{4}$ $\frac{x}{2}+\frac{\pi}{3} = \frac{\pi}{4} + 2\pi n$ ✓   B 1

M 1

$\frac{x}{2} = 2\pi n - \frac{\pi}{12}$

$x = 4\pi n - \frac{\pi}{6}$ ✓   m 1

±B 0

A 0

b) $\underline{\underline{0.524}}$ ✗   M 0

Commentary

Many candidates made good progress in this question. Some omitted any term containing $n\pi$ or $2n\pi$; this candidate showed a typical error and wrote the solution of

$\cos\theta = \frac{1}{\sqrt{2}}$

as $\theta = \frac{\pi}{4}$ rather than $\pm\frac{\pi}{4}$.

The terms $\frac{\pi}{4} - \frac{\pi}{3}$ gave $-\frac{\pi}{12}$, which was much simpler than the required term $\pm\frac{\pi}{4} - \frac{\pi}{3}$

Mark Scheme

5(a)	$\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ stated or used	B1		Degrees or decimals penalised in 5th mark only
	Appropriate use of ±	B1		OE
	Introduction of $2n\pi$	M1		OE
	Subtraction of $\frac{\pi}{3}$ and multiplication by 2	m1		All terms multiplied by 2
	$x=-\frac{2\pi}{3}\pm\frac{\pi}{2}+4n\pi$	A1	5	OE
5(b)	$n = 1$ gives min pos $x = \frac{17\pi}{6}$	M1A1	2	**NMS** 1/2 provided (a) correct
	Total		7	

Question 6

6 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

(a) Calculate the matrix $\mathbf{AB}$. *(2 marks)*

(b) Show that $\mathbf{A}^2$ is of the form $k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(c) Show that $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$. *(3 marks)*

Student Response

6a) $\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ ✗ (BA) M 0

$\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}$ ✗

b) $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

$\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 4I \quad k=4$ ✓ 2

c $B^2 = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$ ✓ B 1

$(AB)^2 = \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}\begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} \quad \begin{bmatrix} -16 & 0 \\ 0 & -16 \end{bmatrix}$ ✗ FIW B 0

A^2B^2

$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix}$ ✓ B 1

$\begin{pmatrix} 16 & 0 \\ 0 & 16 \end{pmatrix} \neq \begin{pmatrix} -16 & 0 \\ 0 & -16 \end{pmatrix}$ ✓

3

4

Commentary

This question was answered well by the majority of candidates. This script shows a common error, finding BA rather than AB (forgetting that matrix multiplication is not commutative). Numerical errors were frequently seen in the multiplication of two vectors.

Mark Scheme

6(a)	$\mathbf{AB} = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$	M1A1	2	M1A0 if 3 entries correct
(b)	$\mathbf{A}^2 = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$	B1		
	$\ldots = 4\mathbf{I}$	B1	2	
(c)	$(\mathbf{AB})^2 = -16\mathbf{I}$	B1		
	$\mathbf{B}^2 = 4\mathbf{I}$	B1		PI
	so $\mathbf{A}^2\,\mathbf{B}^2 = 16\mathbf{I}$ (hence result)	B1	3	Condone absence of conclusion
	Total		7	

Question 7

7 A curve C has equation

$$y = 7 + \frac{1}{x+1}$$

(a) Define the translation which transforms the curve with equation $y = \frac{1}{x}$ onto the curve C. *(2 marks)*

(b) (i) Write down the equations of the two asymptotes of C. *(2 marks)*

(ii) Find the coordinates of the points where the curve C intersects the coordinate axes. *(3 marks)*

(c) Sketch the curve C and its two asymptotes. *(3 marks)*

Student Response

Leave blank

7a) Add 7 on the y axis. (✓)
Minus -1 on the x axis. (✓) Allow 1/2 — 1

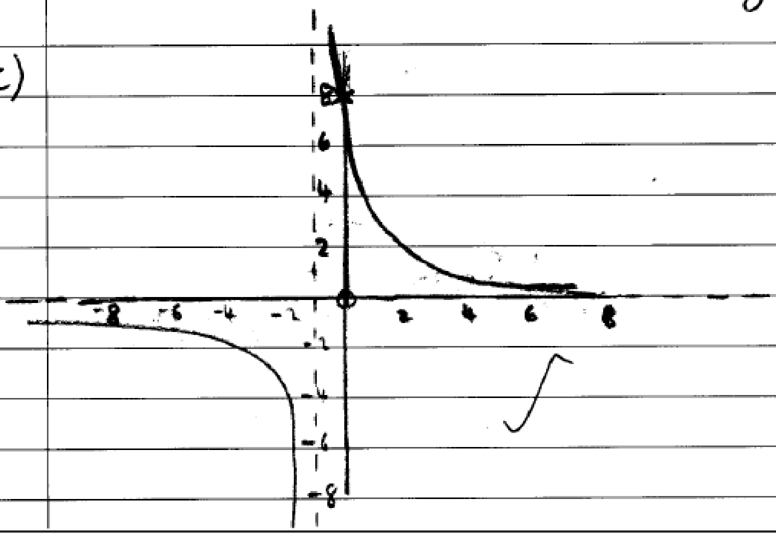
b i) Horizontal asymptote = $\frac{1}{x}$ $x \to \infty$ = 0 ✗ — 0

Vertical asymptote = $x+1=0$ ✓ $x=-1$ ✓ = -1 — 1

ii) Y int. $7 + \frac{1}{0+1}$ = 8 ✓ — 1

x int. $1 \neq 0$: not ~~defined~~ ~~defind~~ dfined ✗ M0

c)



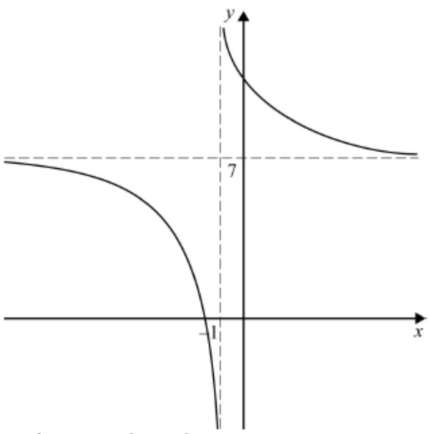
2

(5)

Commentary

Candidates were required to use standard mathematical terminology. In part (a), this candidate's description of "add" and "minus" was not adequate.
In part (b) (i), most candidates found the vertical asymptote to be $x + 1 = 0$, or $x = -1$. The identification of the horizontal asymptote proved more challenging, as in this script, where
$\frac{1}{x}$ x $\pm$ ∞ $= 0$, did not identify the equation of a line.

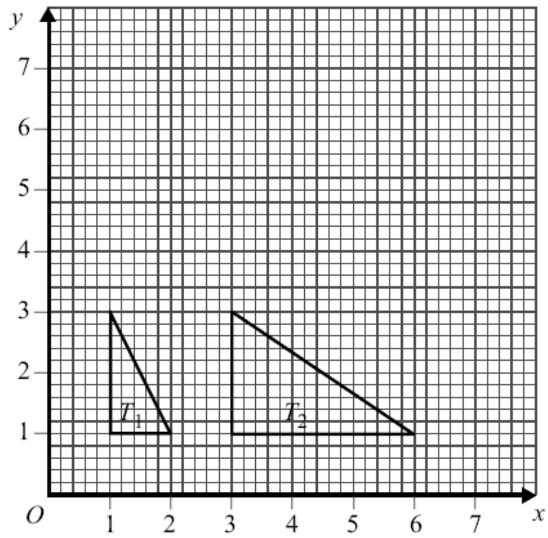
Mark Scheme

Q	Solution	Marks	Total	Comments
7(a)	Curve translated 7 in y direction ... and 1 in negative x direction	B1 B1	2	or answer in vector form
(b)(i)	Asymptotes $x = -1$ and $y = 7$	B1B1	2	
(ii)	Intersections at (0, 8) ... ... and $(-\frac{8}{7}, 0)$	B1 M1A1	3	Allow AWRT –1.14; NMS 1/2
(c)				
	At least one branch Complete graph All correct including asymptotes	B1 B1 B1	3	of correct shape translation of $y = 1/x$ in roughly correct positions
	Total		**10**	

Question 8

8 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows two triangles, T_1 and T_2.



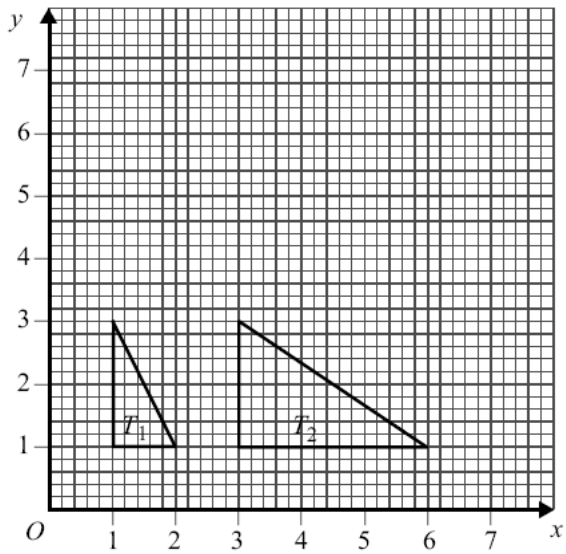
(a) Find the matrix of the stretch which maps T_1 to T_2. *(2 marks)*

(b) The triangle T_2 is reflected in the line $y = x$ to give a third triangle, T_3.

On **Figure 3**, draw the triangle T_3. *(2 marks)*

(c) Find the matrix of the transformation which maps T_1 to T_3. *(3 marks)*

Figure 3 (for use in Question 8)



Student Response

8 a) $\begin{pmatrix}1\\3\end{pmatrix} \rightarrow \begin{pmatrix}3\\3\end{pmatrix}$

$\begin{pmatrix}1\\1\end{pmatrix} \rightarrow \begin{pmatrix}3\\1\end{pmatrix}$

$\begin{pmatrix}2\\1\end{pmatrix} \rightarrow \begin{pmatrix}6\\1\end{pmatrix}$ $\begin{pmatrix}3 & 0\\0 & 1\end{pmatrix}$ ✓ 2

G 2

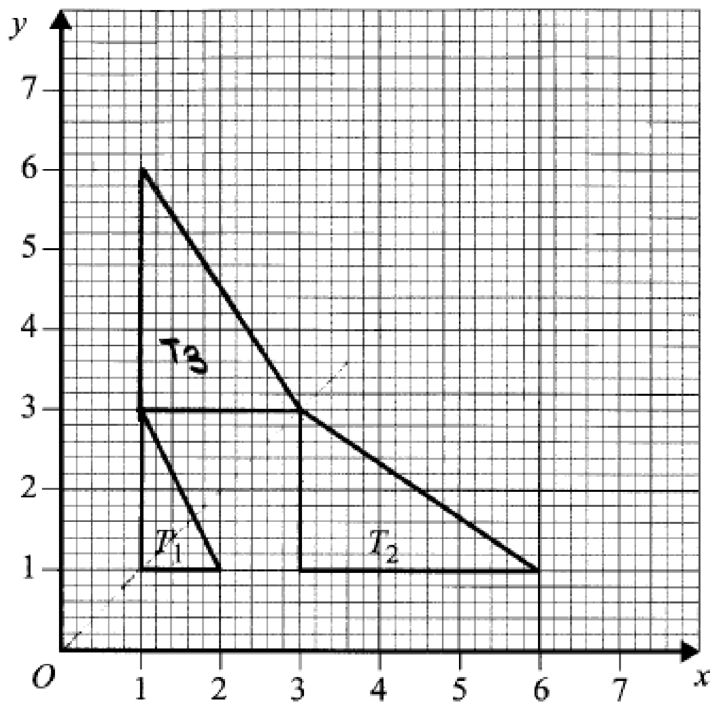
c) $\begin{pmatrix}1\\1\end{pmatrix} \rightarrow \begin{pmatrix}1\\3\end{pmatrix}$

$\begin{pmatrix}1\\3\end{pmatrix} \rightarrow \begin{pmatrix}1\\6\end{pmatrix}$

$\begin{pmatrix}2\\1\end{pmatrix} \rightarrow \begin{pmatrix}3\\3\end{pmatrix}$ ✗ $\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}$ ✗ 0

(4)

Figure 3 (for use in Question 8)

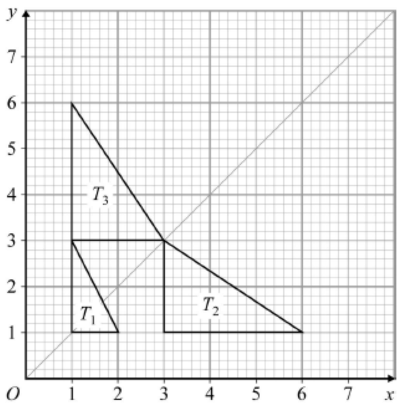


✓

Commentary

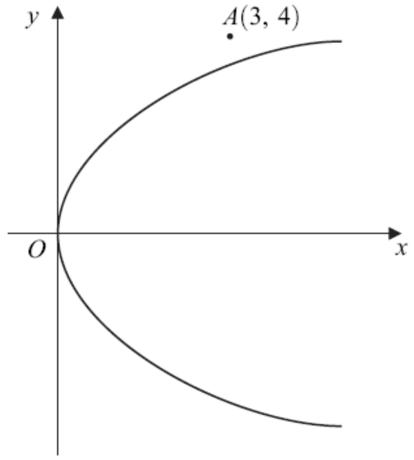
Many candidates found the matrix in part (a) and in part (b) drew the triangle T_3. As shown in this script, some candidates assumed that T_1 moved "up" into T_3 and did not check the transformation of the points. However, the point (2, 1) on triangleT_1 was transformed into the point (6, 1) in triangle T_2. This point was reflected into the point (1, 6) in triangle T_3. Thus the combined transformation did not transform (2, 1) into (3, 3) as this candidate assumed.

Mark Scheme

8(a)	Matrix is $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$	M1A1	2	M1 if zeros in correct positions; allow NMS
(b)	 Third triangle shown correctly	M1A1	2	M1A0 if one point wrong
8(c)	Matrix of reflection is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	B1		**Alt**: calculating matrix from the coordinates: M1 A2,1
	Multiplication of above matrices	M1		in correct order
	Answer is $\begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$	A1F	3	ft wrong answer to (a); **NMS** 1/3
	Total		7	

Question 9

9 The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.



(a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$. *(2 marks)*

(b) Show that, if this straight line intersects the parabola, then the y-coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0$$ *(3 marks)*

(c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

(d) Find the coordinates of the points at which these tangents touch the parabola. *(4 marks)*

Student Response

blank

4. $y^2 = 4x$ A(3, 4)

a) $y - y_1 = m(x - x_1)$
$y - 4 = m(x - 3)$ ✓ 2
$y = mx - 3m + 4$

b) $y^2 = (mx^?)^2 + 9m^2 + 16$ ✗ 0
$4x = m^2x^2 + 9m^2 + 16$
$= m^2x^2 - 4x + 9m^2 + 16$

c) $a = m$ $b^2 - 4ac$
$b = -4$ $-4^2 - 4(m)(16 - 12m)$
$c = 16 - 12m$ $16 - 64m + 48m^2 = 0$ ✓
$48m^2 - 64m + 16 = 0$
∴ tangents = $12m^2 - 16m + 4 = 0$
$y = x + 1$ $3m^2 - 4m + 1 = 0$ ✓
$3y = x + 9$ ✓ $(m - 1)(3m - 1) = 0$
$m = 1$ $m = \frac{1}{3}$ ✓ 3

2

d) $\frac{1}{3}y^2 - 4y + (16 - 12(\frac{1}{3})) = 0$
$\frac{1}{3}y^2 - 4y + 12 = 0$
$y = 6$

$y^2 - 4y + (16 - 12(1)) = 0$ ✓ M 2
$y^2 - 4y + 4 = 0$
$y = 2$

blank

$y = mx - 3m + 4$

$6 = \frac{1}{3}x - 3\left(\frac{1}{3}\right) + 4$

$6 = \frac{1}{3}x - 5$ ✗

$1 = \frac{1}{3}x$

$3 = x$ (3, 6) A 0

$2 = 1x - 3(1) + 4$

$2 = x + 1$

$1 = x$ (1, 2) ✓ A 1

(10)

Commentary

Part (a) was answered well. Instead of using the simple substitution in part (b), whereby $y = mx - 3m + 4 \Rightarrow 4y = 4mx - 12m + 16$

$4y = m\,y^2 - 12m + 16$,

some candidates, as shown, assumed that they must eliminate y^2 and hence attempted to square y, but rarely did this correctly. Then in part (c), most candidates, as seen in this script, correctly equated the discriminant to zero, and so found the two values of m.

Mark Scheme

9(a)	Equation is $y - 4 = m(x - 3)$	M1A1	2	OE; M1A0 if one small error
(b)	Elimination of x $4y - 16 = m(y^2 - 12)$ Hence result	M1 A1 A1	 3	 OE (no fractions) convincingly shown (AG)
(c)	Discriminant equated to zero $(3m - 1)(m - 1) = 0$ Tangents $y = x + 1$, $y = \frac{1}{3}x + 3$	M1 m1A1 A1A1	 5	 OE; m1 for attempt at solving OE
(d)	$m = 1 \Rightarrow y^2 - 4y + 4 = 0$ so point of contact is (1, 2) $m = \frac{1}{3} \Rightarrow \frac{1}{3}y^2 - 4y + 12 = 0$ so point of contact is (9, 6)	M1 A1 M1 A1	 4	OE; $m = 1$ needed for this OE; $m = \frac{1}{3}$ needed for this
	Total		**14**	
	TOTAL		**75**	