General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS **MS2A/W**
Unit Statistics 2A

Friday 11 January 2008 9.00 am to 10.15 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS2A/W.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Statistics 2A has a **written paper and coursework**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 David claims that customers have to queue at a supermarket checkout for more than 5 minutes, on average.

The queuing times, x minutes, of 40 randomly selected customers result in $\bar{x} = 5.5$ and $s^2 = 1.31$.

Investigate, at the 1% level of significance, David's claim. *(6 marks)*

2 A 24-hour helpline is staffed each day in sessions of 3 hours.

The number of calls, X, to the helpline each session may be assumed to be modelled by a Poisson distribution with a mean of 12.0.

(a) (i) Calculate the probability that during any session at least 13 calls to the helpline are received. *(2 marks)*

(ii) Hence calculate the probability that at least 13 calls to the helpline are received during exactly 5 out of 8 consecutive sessions. *(3 marks)*

(b) Give a reason why a Poisson distribution may not be a suitable model for X. *(1 mark)*

3 A speed camera was used to measure the speed, V mph, of John's serves during a tennis singles championship.

For 10 randomly selected serves,

$$\sum v = 1179 \qquad \text{and} \qquad \sum (v - \bar{v})^2 = 1014.9$$

where $\bar{v}$ is the sample mean.

(a) Construct a 99% confidence interval for the mean speed of John's serves at this tennis championship, stating any assumption that you make. *(7 marks)*

(b) Hence comment on John's claim that, at this championship, he consistently served at speeds in excess of 130 mph. *(1 mark)*

4 A discrete random variable X has the probability distribution

$$P(X = x) = \begin{cases} \dfrac{x}{20} & x = 1, 2, 3, 4, 5 \\[2ex] \dfrac{x}{24} & x = 6 \\[2ex] 0 & \text{otherwise} \end{cases}$$

(a) Calculate $P(X \geqslant 5)$. *(2 marks)*

(b) (i) Show that $E\left(\dfrac{1}{X}\right) = \dfrac{7}{24}$. *(2 marks)*

(ii) Hence, or otherwise, show that $\operatorname{Var}\left(\dfrac{1}{X}\right) = 0.036$, correct to three decimal places. *(3 marks)*

(c) Calculate the mean and the variance of A, the area of rectangles having sides of length $X + 3$ and $\dfrac{1}{X}$. *(5 marks)*

(d) The mean of C, the circumference of circles having radii of length $1 + \dfrac{3}{X}$, is $k\pi$.

Find the numerical value of k. *(3 marks)*

Turn over for the next question

5 A survey is carried out in an attempt to determine whether the salary achieved by the age of 30 is associated with having had a university education.

The results of this survey are given in the table.

	Salary $<$ £30 000	**Salary $\geqslant$ £30 000**	**Total**
University education	52	78	130
No university education	63	57	120
Total	115	135	250

(a) Use a χ^2 test, at the 10% level of significance, to determine whether the salary achieved by the age of 30 is associated with having had a university education. *(9 marks)*

(b) What do you understand by a Type I error in this context? *(2 marks)*

6 The continuous random variable X has probability density function defined by

$$f(x) = \begin{cases} k(x^2 + 1) & 0 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) (i) Show that the value of k is $\frac{1}{12}$. *(3 marks)*

(ii) Find the distribution function, $F(x)$, for all x. *(3 marks)*

(iii) Sketch the graph of F. *(3 marks)*

(iv) Find $P(X \geqslant 2)$. *(2 marks)*

(b) Calculate the **exact** value of $E(X)$. *(3 marks)*

END OF QUESTIONS