General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS
Unit Mechanics 2A

MM2A/W

Tuesday 15 January 2008 9.00 am to 10.15 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2A/W.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Mechanics 2A has a **written paper and coursework**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A ball is thrown vertically upwards from ground level with an initial speed of $15\,\text{m}\,\text{s}^{-1}$. The ball has a mass of 0.6 kg. Assume that the only force acting on the ball after it is thrown is its weight.

(a) Calculate the initial kinetic energy of the ball. *(2 marks)*

(b) By using conservation of energy, find the maximum height above ground level reached by the ball. *(3 marks)*

(c) By using conservation of energy, find the kinetic energy and the speed of the ball when it is at a height of 3 m above ground level. *(4 marks)*

2 A particle moves in a horizontal plane under the action of a single force, **F** newtons. The unit vectors **i** and **j** are directed east and north respectively. At time t seconds, the position vector, **r** metres, of the particle is given by

$$\mathbf{r} = (t^3 - 3t^2 + 4)\mathbf{i} + (4t + t^2)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

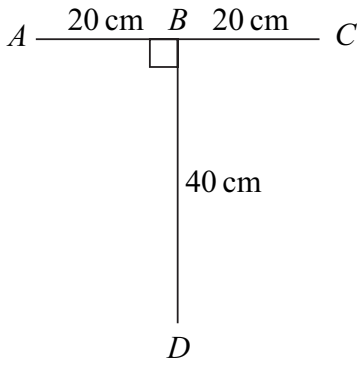
(b) The mass of the particle is 3 kg.

(i) Find an expression for **F** at time t. *(3 marks)*

(ii) Find the magnitude of **F** when $t = 3$. *(2 marks)*

(c) Find the value of t when **F** acts due north. *(2 marks)*

3 Two identical uniform rods, AC and BD, are rigidly joined together to form a letter ⊤, as shown in the diagram. The two rods are perpendicular.



(a) Explain why the centre of mass of the letter ⊤ lies on BD. *(1 mark)*

(b) Find the distance of the centre of mass of the letter ⊤ from AC. *(3 marks)*

(c) The letter ⊤ is freely suspended from A.

Find, to the nearest degree, the angle between AC and the horizontal when the letter ⊤ hangs in equilibrium. *(3 marks)*

4 A light elastic string has natural length 6 metres and modulus of elasticity 300 newtons. It has one end attached to a fixed point, A, on a rough horizontal plane. The other end of the string is attached to a particle of mass 4 kilograms. The particle is pulled along the plane until it is 8 metres from the point A. The particle is then released from rest.

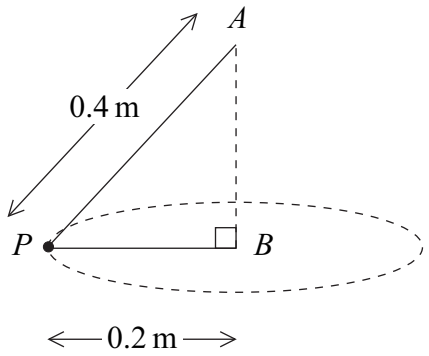
(a) Calculate the elastic potential energy of the string when the particle is 8 metres from the point A. *(2 marks)*

(b) The coefficient of friction between the particle and the plane is 0.3.

Show that the speed of the particle when the string becomes slack is $6.18\,\mathrm{m\,s^{-1}}$, correct to three significant figures. *(6 marks)*

Turn over for the next question

5 Two light inextensible strings, of lengths 0.4 m and 0.2 m, each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the points A and B respectively. The point A is vertically above the point B. The particle moves in a horizontal circle, centre B and radius 0.2 m, at a speed of $2\,\text{m}\,\text{s}^{-1}$. The particle and strings are shown in the diagram.

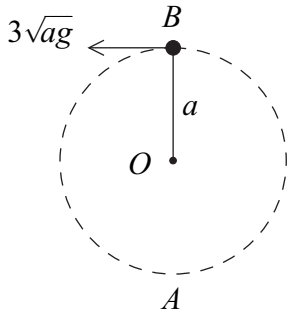


(a) Calculate the magnitude of the acceleration of the particle. *(2 marks)*

(b) Show that the tension in string PA is 45.3 N, correct to three significant figures. *(4 marks)*

(c) Find the tension in string PB. *(3 marks)*

6 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end. The particle is moving in a vertical circle, centre O. When the particle is at B, vertically above O, the string is taut and the particle is moving with speed $3\sqrt{ag}$.



(a) Find, in terms of g and a, the speed of the particle at the lowest point, A, of its path. *(4 marks)*

(b) Find, in terms of g and m, the tension in the string when the particle is at A. *(4 marks)*

7 A car of mass 600 kg is driven along a straight horizontal road. The resistance to motion of the car is kv^2 newtons, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the car at time t seconds and k is a constant.

(a) When the engine of the car has power 8 kW, show that the equation of motion of the car is

$$600\frac{\mathrm{d}v}{\mathrm{d}t} - \frac{8000}{v} + kv^2 = 0 \qquad \textit{(4 marks)}$$

(b) When the velocity of the car is $20\,\mathrm{m\,s^{-1}}$, the engine is turned off.

(i) Show that the equation of motion of the car now becomes

$$600\frac{\mathrm{d}v}{\mathrm{d}t} = -kv^2 \qquad \textit{(1 mark)}$$

(ii) Find, in terms of k, the time taken for the velocity of the car to drop to $10\,\mathrm{m\,s^{-1}}$. *(5 marks)*

END OF QUESTIONS

There are no questions printed on this page

There are no questions printed on this page

There are no questions printed on this page