Teacher Support Materials

Maths GCE

Paper Reference MS04

Question 1

1 The headteacher of a school believes that the standard deviation of the annual number of new pupils joining the school is 10.

A statistician on the staff collects the following data on the number of new pupils joining the school during each of a sample of ten years.

124	123	139	136	128	125	128	133	131	133

Investigate, at the 5% level of significance, the headteacher's belief. Assume that these data may be regarded as a random sample from a normal distribution. *(8 marks)*

Student Response

Leave blank

1. $H_0 \; \sigma^2 = 10^2$

$H_1 : \sigma^2 \neq 10^2$ B 1

$T.S = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ M 1

$S^2 = 28.22 \; n = 10 \; \nu = 9$ A 1

M 1

$T.S = \frac{9 \times 28.22}{10^2} = 2.54$ A 1

B 1

$C.V = 19.023$ B 0

$\therefore$ Accept H_0 since the evidence shows $\sigma = 10$. A 0

6

Commentary

Question 1 was done well by most candidates. The solution selected exhibits a common error, which was to only find one value of χ^2_{crit}, usually the upper one. For this two tail test, the lower value was required, since the value of χ^2_{calc} falls below it and the null hypothesis, H_0, is rejected, showing evidence that the headmaster's belief is incorrect. Conclusions for hypothesis tests in this unit, as in others, should be given in context.

Mark scheme

MS04

Q	Solution	Marks	Total	Comments
1	$H_0 : \sigma = 10 \quad H_1 : \sigma \neq 10$	B1		Both
	$\Sigma(x - \bar{x})^2 = 254$	M1A1		Or $s^2 = 28.\dot{2}$; B1 for 25.4
	Under H_0, $\sigma^2 = 100$			
	Hence $\chi^2_{\text{calc}} = \dfrac{254}{100} = 2.54$	M1A1		$\dfrac{9 \times 28.\dot{2}}{100} = 2.54$
	$\nu = 9$	B1		
	$\chi^2_{\text{crit}}\ (2.7,\ 19.0)$	B1		Both required
	Reject H_0			
	Evidence that headmaster's belief is incorrect	A1√	8	
	Total		**8**	

Question 2

2 The discrete random variable X has a geometric distribution with parameter p.

(a) Given that the value of the mean is 4 times that of the variance, find the value of p. *(3 marks)*

(b) Hence determine $\mathrm{P}(X > 4 \mid X > 2)$. *(4 marks)*

Student response

2 a) $\frac{4}{p} = \frac{1-p}{p^2}$ X

B 1

$4 = \frac{p-p^2}{p^2}$

M 0

$4 = \frac{p}{p^2} - \frac{\cancel{p^2}}{\cancel{p^2}}$ 1

$\frac{1}{p} = 4+1$

$p = \frac{1}{5}$ X

Commentary

In part (a) of this question, the candidate, like numerous others, multiplied the mean by 4 rather than the variance. The question states that the mean is four times the variance. Hence the mark for knowing the values of the mean and the variance was earned (both of which are in the formula booklet), but not the method mark for forming the equation, nor the accuracy mark for solving it.

In part (b) the candidate shows understanding of conditional probability and the 'no memory' property of the geometric distribution, earning both method marks He also demonstrates knowledge of the result $\mathrm{P}(X>r) = q^r$ for the geometric distribution. This earns him an accuracy mark, on a follow through basis, from his incorrect answer to part (a).

In part (b) many candidates were not aware of some, or all, of these points.

2(a)	$\mathrm{E}(X)=\dfrac{1}{p}$ $\quad$ $\mathrm{Var}(X)=\dfrac{1-p}{p^2}$	B1		Both
	$\dfrac{1}{p}=\dfrac{4(1-p)}{p^2}$	M1		
	$\Rightarrow p=4-4p$ $\Rightarrow 5p=4$ $\Rightarrow p=0.8$	A1	3	
(b)	$\mathrm{P}(X>4 \mid X>2)=\dfrac{\mathrm{P}(X>4)}{\mathrm{P}(X>2)}$	M1		Use of
	$=\dfrac{0.2^4}{0.2^2}$	m1		Or $\dfrac{1-0.8\left(1+0.2+0.2^2+0.2^3\right)}{1-0.8(1+0.2)}$
	$=0.2^2$	A1√		√ on (a)
	$=0.04$	A1	4	CAO
	Total		**7**	

Question 3

3 The assessment of a physics course has two components: a written examination and a practical test. Each component has a maximum mark of 75. The marks achieved by 10 students in each component are shown in the table.

Student	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Written Mark	35	47	54	55	43	48	41	59	47	31
Practical Mark	57	63	47	72	73	27	39	60	53	22

(a) Investigate, using a paired t-test and the 5% level of significance, whether the mean mark in the written examination is less than that in the practical test. *(10 marks)*

(b) State **two** assumptions that were necessary in order to carry out the test in part (a). *(2 marks)*

Student Response

3 a. $H_0: \mu d = 0$ where μd = practical mark − written mark

~~$H_1: \mu d < 0$~~ $H_1: \mu d > 0$

Student	A	B	C	D	E	F	G	H	I	J
written mark	35	47	54	55	43	48	41	59	47	31
practical mark	57	63	47	72	73	27	39	60	53	22
d (practical − written)	22	16	−7	17	30	−21	−2	1	6	−9

$\bar{d} = 5.3$

$S_d = 15.84683635$

$n_d = 10$

$v = n_d - 1 = 10 - 1 = 9$

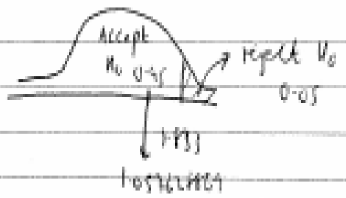
5% level 1 tailed

$t_{crit} = 1.833$

$$t_{calc} = \frac{\bar{d} - \mu_d}{S_d / \sqrt{n_d}} = \frac{5.3 - 0}{\left(\frac{15.84683635}{\sqrt{10}}\right)}$$

$$= 1.057628869$$

Leave blank



$1.057618869 < 1.833$

$\Rightarrow$ Accept H_0 at 5% level.

There is no significant evidence to suggest that mean difference between practical mark and written mark ~~differs from 0~~. is greater than 0. Hence no evidence to suggest mean mark in written exam is less than practical exam.

10

b. assumption that mean differences were normally distributed.

2

assumption that the sample of students was random.

12

Commentary

On this question, many candidates performed a two-sample *t*-test instead of a paired *t*-test as the question demanded. This candidate produced a complete solution to the whole question. He stated his hypotheses correctly, calculated the test statistic correctly, used the appropriate number of degrees of freedom and the corresponding correct critical value and produced a full conclusion, in context.

In part (b) he gained both marks, which was rarely the case. Many candidates pointed out that the samples needed to be random. Few, however, said that the **differences** must be normally distributed. It is possible for the marks in both written and practical exams to be normally distributed and the differences not to be normally distributed. Consider, for example, the case where written mark = practical mark ± constant.

Mark Scheme

Q	Solution	Marks	Total	Comments	
3(a)	Differences				**2-sample *t*-test**
	22, 16, –7, 17, 30, –21, –2, 1, 6, –9	M1		Attempt at differences	
	$\overline{d} = \frac{53}{10} = 5.3$	B1			B1
	$H_0 : \mu_d = 0 \qquad H_1 : \mu_d > 0$	B1,B1		$\overline{d}$ for μ_d B1	B1, B1
	$t_{\text{calc}} = \frac{5.3 - 0}{\left(\frac{15.85}{\sqrt{10}}\right)}$	B1		s or σ	
		M1		Or $\left(\frac{15.03}{\sqrt{9}}\right)$	
	$= 1.06$	A1			
	$\nu = 9$	B1		PI	$\nu = 18$ B1
	$t_{\text{crit}} = 1.833$	B1			$t_{\text{crit}} = 1.734$ B1
	Retain H_0 - No evidence that mean mark is less on written examination	A1√	10	OE	5/10 max
(b)	Random sample	E1			E1
	Differences are normally distributed	E1	2	Differences required	E0 1/2 max
	Total		**12**		

Question 4

4 (a) A continuous random variable X has probability density function

$$f(x) = \lambda e^{-\lambda x} \quad \text{for } x \geqslant 0$$

Prove that the mean value of X is $\frac{1}{\lambda}$. *(4 marks)*

(b) The lifetime of a component in a machine is T hours, where T has probability density function

$$f(t) = \frac{1}{a} e^{-\frac{t}{a}} \quad \text{for } t \geqslant 0$$

The mean lifetime of these components is known to be 62.5 hours.

(i) Find the value of $\frac{1}{a}$. *(2 marks)*

(ii) Calculate the probability that a component will last for at least 80 hours. *(4 marks)*

(iii) Given that a component has lasted for 80 hours, find the probability that it will last for a further 20 hours. *(3 marks)*

Student Response

3. a. $H_0 : \mu_d = 0$ where d = practical mark − written mark

~~$H_1 : \mu_d < 0$~~ $H_1 : \mu_d > 0$

Student	A	B	C	D	E	F	G	H	I	J
Written mark	35	47	54	55	43	48	41	59	47	31
Practical mark	57	63	47	72	73	27	39	60	53	22
d (practical − written)	22	16	−7	17	30	−21	−2	1	6	−9

$\bar{d} = 5.3$

$S_d = 15.84683635$

$n_d = 10$

$v = n_d - 1 = 10 - 1 = 9$

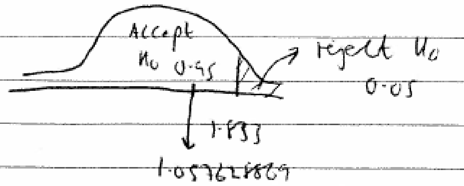
5% level 1 tailed

$t_{crit} = 1.833$

$$t_{calc} = \frac{\bar{d} - \mu_d}{S_d / \sqrt{n_d}} = \frac{5.3 - 0}{\left(\frac{15.84683635}{\sqrt{10}}\right)}$$

$= 1.057628869$

Leave blank



$1.057628869 < 1.833$

$\Rightarrow$ Accept H_0 at 5% level.

There is no significant evidence to suggest that mean difference between practical mark and written mark ~~differs from 0~~. is greater than 0. Hence no evidence to suggest mean mark in written exam is less than practical exam.

10

b. assumption that mean differences were normally distributed.

2

assumption that the sample of students was random.

12

Commentary

As this is not a paper in pure mathematics, a more informal treatment of infinite integrals than this candidate offered, would still earn the first 4 marks in part (a).

In part (b)(i) the question asks for the value of $\frac{1}{a}$ and an expression, which was not evaluated, lost a mark. A number of candidates made this error.

Part (b)(ii) requires knowledge of the cumulative distribution function (cdf), or integration of the probability density function, of the exponential distribution. The cdf is specifically mentioned in the specification.

Like many, this candidate was not able to do the conditional probability in part (b)(iii).

Either $e^{-0.016\times100} \div e^{-0.016\times80}$ or simply $e^{-0.016\times20}$ ('no memory' property) will suffice.

4(a)	$\mathrm{E}(X)=\int_0^\infty \lambda x\mathrm{e}^{-\lambda x}\,\mathrm{d}x$	M1		Use of
	$=\left[-x\mathrm{e}^{-\lambda x}\right]_0^\infty+\int_0^\infty \mathrm{e}^{-\lambda x}\,\mathrm{d}x$	A1		Integrate by parts
	$=\left[\dfrac{-\mathrm{e}^{-\lambda x}}{\lambda}\right]_0^\infty$	A1		Correctly
	$=\dfrac{1}{\lambda}$	A1	4	AG
(b)(i)	$\dfrac{1}{a}=\dfrac{1}{62.5}=0.016$	M1A1	2	
(ii)	$\mathrm{F}(t)=1-\mathrm{e}^{-0.016t}$	M1A1		Or $\int_{80}^\infty 0.016\mathrm{e}^{-0.016t}\,\mathrm{d}t$ Award M1A1 for complement
	$\mathrm{P}(T>80)=\mathrm{e}^{-0.016\times 80}$ $=0.278$	M1 A1	4	
(iii)	**Either** $\mathrm{e}^{-0.016\times 100}\div\mathrm{e}^{-0.016\times 80}$ $=0.726$ **or** $\mathrm{e}^{-0.016\times 20}$ $=0.726$	 M1A1 A1 (M2) (A1)	 3	
	Total		**13**	

Question 5

5 One hundred 1-millilitre samples of water were taken at random and the number of bacteria in each sample was counted. The results are shown in the table.

Number of bacteria	0	1	2	3	4	5	6	7
Frequency	7	15	27	25	11	10	3	2

(a) For these data, show that the mean number of bacteria per 1-millilitre of water is 2.7. *(2 marks)*

(b) Hence, using a χ^2 goodness of fit test with the 10% level of significance, investigate whether the number of bacteria per 1-millilitre of water can be modelled by a Poisson distribution. *(11 marks)*

Student Response

4. a. $f(x) = \lambda e^{-\lambda x} \qquad x \geq 0$

$$\mu = E(x) = \int x f(x)\, dx$$

$$= \int_0^a x \lambda e^{-\lambda x}\, dx$$

$$= \left[(x)\left(-e^{-\lambda x}\right) \right]_0^a - \int_0^a 1\left(-e^{-\lambda x}\right) dx$$

Leave blank

4\. a. $E(X) = \left[-xe^{-\lambda x}\right]_0^a + \int_0^a e^{-\lambda x}\,dx$

$= \left[-xe^{-\lambda x}\right]_0^a + \left[-\frac{1}{\lambda}e^{-\lambda x}\right]_0^a$

$= -ae^{-\lambda a} - \left(-0e^{-0}\right) + \left(-\frac{1}{\lambda}e^{-\lambda a} - \left(-\frac{1}{\lambda}e^{0}\right)\right)$

$= -ae^{-\lambda a} - \frac{1}{\lambda}e^{-\lambda a} + \frac{1}{\lambda}$

$\lim_{a\to\infty} -ae^{-\lambda a} - \frac{1}{\lambda}e^{-\lambda a} + \frac{1}{\lambda}$

4

$= \frac{1}{\lambda}$

$\Rightarrow \mu = \frac{1}{\lambda}$ Q.E.D.

b. $f(t) = \frac{1}{a}e^{-t/a}$ $t \geq 0$

i. mean $= 62.5$

$\frac{1}{a} = \lambda$

$a = \frac{1}{\lambda} = $ mean

$a = 62.5$

$\Rightarrow \frac{1}{a} = \frac{1}{62.5} = ?$

M1
A0

Leave blank

ii. $f(t) = \frac{1}{62.5} e^{-\frac{t}{62.5}} \quad t \geq 0$

$F(t) = \int_0^t f(t)\, dt$

$= \int_0^t \frac{1}{62.5} e^{-t/62.5}\, dt$

$= \left[-e^{-t/62.5} \right]_0^t$

$= -e^{-t/62.5} - (-e^{0})$

$= 1 - e^{-t/62.5}$

$P(t \geq 80) = 1 - F(80)$

$= 1 - \left(1 - e^{-\frac{80}{62.5}}\right)$

$= 0.278$ (3sf) ✓ 4

iii. prob (will last a further 20mins)

$= 0.2780 - (80 < t < 100)$

$= 0.2780 - (F(100) - F(80))$

$= 0.2780 - \left(\left(1 - e^{-\frac{100}{62.5}}\right) - \left(1 - e^{-\frac{80}{62.5}}\right)\right)$

$= 0.202$ (3sf) ✗ M0

9

Commentary

This question produced high marks for most candidates. A number of errors caused a few marks to be lost for some candidates.

In this example the candidate does not combine the last two classes, in order that expected frequencies are more than 5 in each case. Some errors also occur in the values he adds to give χ^2_{calc}. By not combining classes, his degrees of freedom are incorrect, but he can still earn follow through marks for his critical value and conclusion.

Mark Scheme

MS04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Mean $= \frac{\sum fx}{\sum f} = \frac{270}{100} = 2.7$	M1A1	2	AG
(b)	O E			
	7 6.72			
	15 18.15	M1		Probabilities
	27 24.50			
	25 22.05	M1		$\times 100$
	11 14.88			
	10 8.04	M1		≥ 7 Frequency = 2.04
	3 3.62			
	2 2.04	M1		Combine classes
	$H_0 : X \sim \text{Po}$	B1		
	$\chi^2_{\text{calc}} = \frac{0.28^2}{6.72} + \frac{3.15^2}{18.15} + \frac{2.50^2}{24.50} + \frac{2.95^2}{22.05}$	M1		
	$+ \frac{3.88^2}{14.88} + \frac{1.96^2}{8.04} + \frac{0.66^2}{5.66}$	A1		≥ 4 terms correct (0.0117 + 0.5467 + 0.2551 + 0.3947 + 1.012 + 0.4778 + 0.0770)
	$= 2.77$	A1		AWFW (2.75, 2.85)
	$\nu = 7 - 2 = 5$	B1		
	$\chi^2_{\text{crit}} = 9.236$	B1√		√ on $\nu = 6$ only $\left(\chi^2_{\text{crit}} = 10.645\right)$
	$\chi^2_{\text{calc}} << \chi^2_{\text{crit}}$			
	$\Rightarrow$ Accept $X \sim \text{Po}$			
	ie no evidence that it is not a Poisson distribution	A1√	11	√ on χ^2_{calc} and upper $\chi^2_{5 \text{ or } 6}$
	Total		**13**	

Question 6

6 A random variable X is distributed with mean μ and variance σ^2. Three independent observations, X_1, X_2 and X_3, are taken on X.

The combined statistic

$$T = aX_1 + bX_2 + cX_3$$

where a, b and c are constants, is used as an estimator for μ.

(a) Show that, if T is an unbiased estimator for μ, then $a + b + c = 1$. *(3 marks)*

(b) Two unbiased estimators for μ are T_1 and T_2, defined by

$$T_1 = \tfrac{1}{3}X_1 + \tfrac{1}{2}X_2 + \tfrac{1}{6}X_3$$

$$T_2 = \tfrac{2}{3}X_1 + \tfrac{3}{4}X_2 - \tfrac{5}{12}X_3$$

(i) Calculate the relative efficiency of T_1 with respect to T_2. *(5 marks)*

(ii) With reference to your answer to part (b)(i), state, with a reason, which of T_1 and T_2 is the better unbiased estimator for μ. *(2 marks)*

Student Response (below)

6\. (a) $E(T) = \mu$

$E(aX_1 + bX_2 + cX_3) = \mu$

$aE(X_1) + bE(X_2) + cE(X_3) = \mu$

$a\mu + b\mu + c\mu = \mu$

$\mu(a+b+c) = \mu$

$a+b+c = 1$ ✓ 3

(b)(i) $Var(T_1) = Var\left(\frac{1}{3}X_1 + \frac{1}{2}X_2 + \frac{1}{6}X_3\right)$

$= \frac{1}{3^2}Var(X_1) + \frac{1}{2^2}Var(X_2) + \frac{1}{6^2}Var(X_3)$

$= \frac{1}{9}\sigma^2 + \frac{1}{4}\sigma^2 + \frac{1}{36}\sigma^2$ M1

$= \frac{7\sigma^2}{18}$ ✓

$= 0.3\dot{8}\sigma^2$ A1

$Var(T_2) = Var\left(\frac{2}{3}X_1 + \frac{3}{4}X_2 - \frac{5}{12}X_3\right)$

$= \left(\frac{2}{3}\right)^2 Var(X_1) + \left(\frac{3}{4}\right)^2 Var(X_2) - \left(\frac{5}{12}\right)^2 Var(X_3)$

$= \frac{4}{9}\sigma^2 + \frac{9}{16}\sigma^2 - \frac{25}{144}\sigma^2$ A0

$= \frac{5\sigma^2}{6}$

$= 0.8\dot{3}\sigma^2$

(ii) T_1 is the better unbiased estimator for μ because T_1 has a smaller variance therefore more consistent. B1

6

Commentary

The response of this candidate is fairly typical of many candidates.

Part (a) is done well.

In part (b)(i) the result that variance of differences is the sum of the variances was not known. Neither was the result Relative Efficiency of T_1 with respect to T_2 = $\frac{Var(T_2)}{Var(T_1)}$.

In part (b)(ii) one mark was earned for saying that T_1 was more efficient as it had the smaller variance. In order to earn both marks, however, it was necessary to say that the value calculated in part (b)(i) was > 1, hence T_1 was preferred.

Mark Scheme

MS04 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\mathrm{E}(aX_1+bX_2+cX_3)$			
	$=a\mathrm{E}(X_1)+b\mathrm{E}(X_2)+c\mathrm{E}(X_3)$	M1		Can be implied by next line
	$\Rightarrow \mu = a\mu + b\mu + c\mu$	M1		
	$\Rightarrow a+b+c=1$	A1	3	AG
(b)(i)	$\mathrm{Var}(T_1)=$			
	$\frac{1}{9}\mathrm{Var}(X_1)+\frac{1}{4}\mathrm{Var}(X_2)+\frac{1}{36}\mathrm{Var}(X_3)$	M1		Either T_1 or T_2
	$=\frac{7\sigma^2}{18}$	A1		Accept any correct unreduced fraction or $0.389\sigma^2$
	$\mathrm{Var}(T_2)=$			
	$\frac{4}{9}\mathrm{Var}(X_1)+\frac{9}{16}\mathrm{Var}(X_2)+\frac{25}{144}\mathrm{Var}(X_3)$			
	$=\frac{85\sigma^2}{72}$	A1		Any equivalent fraction or $1.18\sigma^2$
	Hence $\mathrm{RE}(T_1 \text{ wrt } T_2)=\frac{\mathrm{Var}\ (T_2)}{\mathrm{Var}\ (T_1)}$	M1		Use of
	$=\frac{85}{72}\times\frac{18}{7}=\frac{85}{28}$	A1√	5	AWFW $[3.03, 3.04]$
(ii)	Since $\frac{85}{28}>1$,	M1		
	T_1 is preferred	A1	2	SC $0.39<1.18 \Rightarrow T_1$ more efficient B1√
	Total		**10**	

Question 7

7 A student at an agricultural college was asked to compare the variability of the weight, X grams, of eggs laid by free-range hens with the weight, Y grams, of eggs laid by battery hens.

The variables X and Y may be assumed to be normally distributed with variances $\sigma_X{}^2$ and $\sigma_Y{}^2$ respectively.

A random sample of 12 values of X resulted in $\sum(x-\bar{x})^2 = 761.2$, where $\bar{x}$ denotes the sample mean.

A random sample of 10 values of Y resulted in $\sum(y-\bar{y})^2 = 386.1$, where $\bar{y}$ denotes the sample mean.

(a) Calculate unbiased estimates of $\sigma_X{}^2$ and $\sigma_Y{}^2$. *(2 marks)*

(b) (i) Hence determine a 90% confidence interval for the ratio $\dfrac{\sigma_X{}^2}{\sigma_Y{}^2}$. *(8 marks)*

(ii) Comment on the suggestion that the weights of eggs laid by free-range hens are more variable than the weights of eggs laid by battery hens. *(2 marks)*

Student Response

Leave blank

7 (a)

$\sigma_x{}^2 = 761.2 / 12-1$

$= 761.2 / 11$

$= 69.2$ ✓

$\sigma_y{}^2 = 386.1 / 10-1$

$= 386.1 / 9$

$= 42.9$ ✓

2

7 (b)(i) 90% CI for σ_x^2 / σ_y^2 two tail

a = upper = $F(v_y, v_x)$

b = Lower = $1/F(v_x, v_y)$

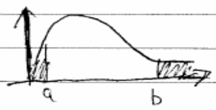
$v_x = 12-1 = 11$ upper = $F(9,11) = 2.896$

$v_y = 10-1 = 9$ Lower = $1/F(11,9) = 1/3.102 = 0.322$

$F(9,11) = 2.896$ $1/F(11,9) = 0.3224$

$0.3224 \dfrac{69.2}{42.9} < \dfrac{\sigma_x^2}{\sigma_y^2} < \dfrac{69.2}{42.9} 2.896$

$0.52 < \dfrac{\sigma_x^2}{\sigma_y^2} < 4.6714$,,



CI is between 0.52 to 4.6714

[0.52 , 4.67] ✓

8

7 (b)(ii) no evidence that the weight of eggs laid by free range hens are more variable than the weights of eggs laid by battery hens.

because CI contains 1 ✓

σ_x^2 / σ_y^2 is in range of 1,,

2

(12)

Commentary

Weak candidates were only able to score the first two marks in part (a) and possibly obtain critical values of *F* in part (b)(i).

Even candidates who knew what to do got into some tangles with reciprocals and their lower and/or upper critical values were inaccurate.

Very few were able to come to the correct conclusion that since 1 $\in$ confidence interval, then the suggestion that the weights of eggs laid by free-range hens were more variable than the weights of eggs laid by battery hens should be rejected.

This candidate shows admirable clarity and presentation of his argument.

Mark Scheme

MS04 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\sigma_X{}^2 = \frac{761.2}{11} = 69.2$	M1		Either
	$\sigma_Y{}^2 = \frac{386.1}{9} = 42.9$	A1	2	Both correct
(b)(i)	$\frac{69.2}{42.9} = 1.613$	M1A1		
	$\nu_1 = 12 - 1 = 11 \qquad \nu_2 = 10 - 1 = 9$	B1		Both
	$\mathrm{F}_{11,9} = 3.102 \qquad \mathrm{F}_{9,11} = 2.896$	B1,B1		
	$\frac{1}{3.102} \le \frac{\left(\frac{\sigma_X{}^2}{\sigma_Y{}^2}\right)}{1.613} \le 2.896$	M1 A1√		
	$\therefore 0.520 \le \frac{\sigma_X{}^2}{\sigma_Y{}^2} \le 4.67$	A1	8	B1 for limit without full working max 2/5
(ii)	Reject suggestion Since 1$\in$ CI	B1√ E1√	2	
	Total		**12**	
	TOTAL		**75**	