

Teacher Support Materials

Maths GCE

Paper Reference MS03

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1 As part of an investigation into the starting salaries of graduates in a European country, the following information was collected.

		Starting salary (€)				
	Sample size	Sample mean	Sample standard deviation			
Science graduates	175	19 268	7321			
Arts graduates	225	17 896	8205			

- (a) Stating a necessary assumption about the samples, construct a 98% confidence interval for the difference between the mean starting salary of science graduates and that of arts graduates.
 (6 marks)
- (b) What can be concluded from your confidence interval?

(2 marks)

Student Response

Question number $Assumption$ 4 C V 98% = 2:3263	BC Leave blank Bl
$(X-Y) \pm 0^{\frac{2}{3}} \pm 0^{\frac{2}{3}} + \frac{0^{2}}{n^{2}}$ = (19268-17896) ± 2.3263 (1321) ² ± (8205) ² 175 ± 225	MI
$= 1372 \pm 2.3263 306268.8007 \pm 294209$ = 1372 \pm 180.15 = -438.15, 3182.15	AI
b. (It) is within in C.I. It is supported that Science graduates X earn higher salaries than Arts graduates	60 60
	6

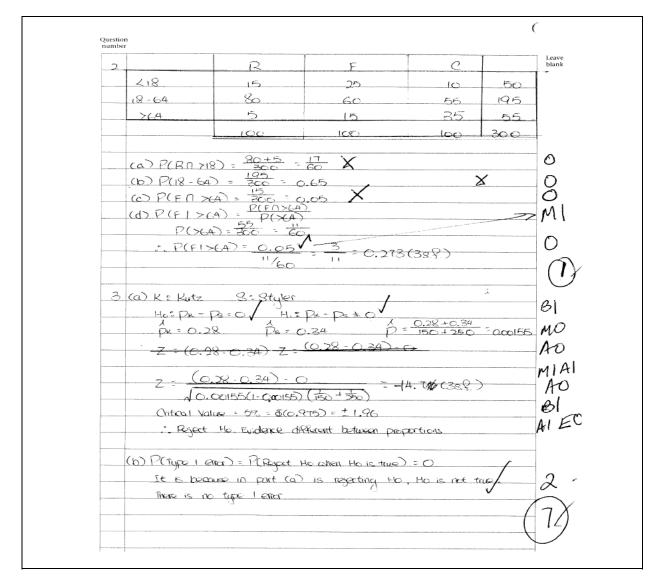
The candidate clearly knew the method required to determine the confidence interval correctly. However the candidate appeared unaware of the necessary assumption and also that when a (98%) confidence interval includes the value zero then it can concluded that there is no evidence (at the 2% significance level) of a difference between the two population means.

Candidates must be aware of assumptions and be able to make deductions.

Q	Solution	Marks	Total	Comments
1(a)	Samples are independent or random	B1		
	$98\% \Rightarrow z = 2.3263$	B1		AWFW 2.32 to 2.33
	CI for $\mu_1 - \mu_2$ is:			
	$(\overline{x}_s - \overline{x}_A) \pm z \times \sqrt{\frac{s_s^2}{n_s} + \frac{s_A}{n_A}}$	M1		Form
	$(x_s - x_A) \pm 2 \times \sqrt{n_s} + \frac{n_A}{n_A}$	A1		Allow: sigmas, $A\&B$ or $1\&2$ and $n-1$ Correct
	(19268–17896)			on z only
	$\pm 2.3263 \times \sqrt{\frac{7321^2}{175} + \frac{8205^2}{225}}$	A1√		$s_p = 7830 \text{ to } 7850$
	ie 1372 ± (1805 to 1820)			1372 ± (1830 to 1845)
	or	A1	6	
	(-450 to -430, 3170 to 3200)			AWFW
(b)	Confidence interval includes zero	B1√		√ on CI; OE
	so (at 5% level)	îdepî		
	Mean starting salaries may be equal	B1√	2	√ on CI; OE
	Total		8	

Pe	Percentage of visitors using				
Road	Funicular railway	Cable car			
Under 18 15	25	10	1		
Age (years) 18 to 64 80	60	55			
Over 64 5	15	35			

Student response



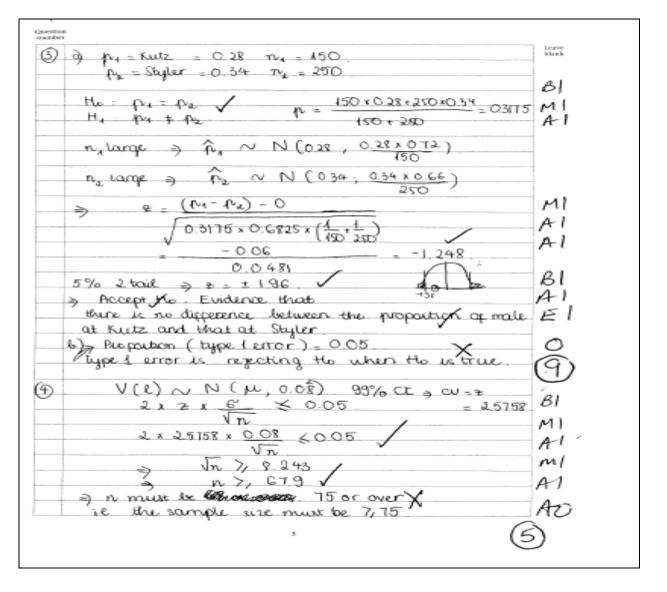
The candidate appeared not to have read the question carefully and so missed the aid in explanation contained in the second paragraph. As a result, the candidate has just treated the given information as a simple 2-way frequency table as would be examined in MS/SS1B.

Candidates must read questions carefully.

MS03 (cont)				I
Q	Solution	Marks	Total	Comments
2(a)	$P(\geq 18 \mid Road) = 0.85$	B1	1	CAO; OE; not 85
(b)	$\begin{array}{l} P(18 \text{ to } 64) = \\ P(\text{Route}) \times P(18 \text{ to } 64 \mid \text{Route}) = \end{array}$	M1		Use of 3 possibilities, each the product of 2 probabilities
	(0.25×0.80) + (0.60×0.35) + (0.55×0.40)	A1		At least 1 term correct
	= 0.20 + 0.21 + 0.22 = 0.63	A1	3	CAO; OE
(c)	$P(FR \cap {>}64) = P(FR) \times P({>}64 \mid FR)$			
	= 0.35 × 0.15	B1		Correct expression
	= 0.052 to 0.053	B1	2	AWFW (0.0525)
(d)	$P(FR \mid > 64) = \frac{(c)}{P(> 64)} =$	M1 M1		$\frac{\text{answer(c)}}{\sum(3\times2) \text{ probabilities}}$
	$\frac{0.0525}{(0.25\times0.05)+(0.35\times0.15)+(0.40\times0.35)}$	A1		At least 2 terms correct
	$= \frac{0.0525}{0.0125 + 0.0525 + 0.1400} = \frac{0.0525}{0.205}$	A1		CAO
	$= 0.256 \text{ or } \frac{21}{82}$	A1	5	AWRT/CAO; OE
	Total		11	

- **3** Kutz and Styler are two unisex hair salons. An analysis of a random sample of 150 customers at Kutz shows that 28 per cent are male. An analysis of an independent random sample of 250 customers at Styler shows that 34 per cent are male.
 - (a) Test, at the 5% level of significance, the hypothesis that there is no difference between the proportion of male customers at Kutz and that at Styler. (9 marks)
 - (b) State, with a reason, the probability of making a Type I error in the test in part (a) if, in fact, the actual difference between the two proportions is 0.05. *(2 marks)*

Student Response



The candidate has scored full marks in part (a) but the conclusion in context is very close to being too definitive. In part (b), the candidate has simply stated the definition of a Type I error and not applied it to the given context where H_0 was false.

Conclusions to hypothesis tests must not be definitive but should be qualified (some/strong evidence) or quantified (by the significance level) and quoted definitions without reference to the context rarely score marks.

MS03 (cont)				
Q	Solution	Marks	Total	Comments
3(a)	$H_0: p_{\rm K} = p_{\rm S}$ $H_1: p_{\rm K} \neq p_{\rm S}$	B1		Both; OE; allow A&B or 1&2
	SL $\alpha = 0.05$ CV $ z = 1.96$	B1		CAO
	$\hat{p} = \frac{(150 \times 0.28) + (250 \times 0.34)}{400}$	M1		Used
	$=\frac{127}{400}$ or 0.317 to 0.318	A1		CAO/AWFW (0.3175)
	$z = \frac{(\hat{p}_{\rm K} - \hat{p}_{\rm S}) - 0}{\sqrt{\hat{p}(1 - \hat{p}) \left(\frac{1}{n_{\rm K}} + \frac{1}{n_{\rm S}}\right)}}$	M1		Used; accept unpooled denominator
	$ z = \frac{ 0.28 - 0.34 }{\sqrt{0.3175 \times 0.6825 \left(\frac{1}{150} + \frac{1}{250}\right)}}$	A1√		\checkmark on \hat{p} ; accept no pooling
	= 1.24 to 1.25	A1		AWFW; 1.26 to 1.27
	Thus accept H ₀ as $ z < 1.96$	A1		\checkmark on <i>z</i> and CV with same sign
	Thus no evidence, at 5% level, of a difference between two proportions of male customers in two salons	E1√	9	on z and CV with same sign In context and qualified
(b)	Zero since	B1		CAO
	Cannot make a Type I error when H ₀ is false	B1	2	OE
	Total		11	

4 A machine is used to fill 5-litre plastic containers with vinegar. The volume, in litres, of vinegar in a container filled by the machine may be assumed to be normally distributed with mean μ and standard deviation 0.08.

A quality control inspector requires a 99% confidence interval for μ to be constructed such that it has a width of at most 0.05 litres.

Calculate, to the nearest 5, the sample size necessary in order to achieve the inspector's requirement. (6 marks)

Student Response

4) Carticle ce interval for per given by $\overline{x} \pm 2.5758 \sqrt{\frac{0.08^2}{n}} \sqrt{\frac{2}{5758} \sqrt{\frac{0.08^2}{n}}}$ Width is given by $\frac{4}{5} \left(2.5758 \sqrt{\frac{0.08^2}{n}} \right) \sqrt{\frac{1}{5}}$	
$2 \times 2.5758 \sqrt{\frac{0.03^{2}}{n}} \leq 0.05 \sqrt{\frac{1}{n}}$ $\Rightarrow \sqrt{n} \geq \frac{2 \times 2.5758 \times 0.08}{0.05} = 8.243 / \frac{1}{0.05}$ $\Rightarrow n \geq 67.9$	er, R
Huee to rearest 5, n > 70.	6

Commentary

The candidate has used knowledge of the form of a confidence interval to deduce an expression for the width (including multiplier of 2) and then solved the resulting inequality (equality would have sufficed) for n.

Those candidates who tried to remember the formula for n sometimes made errors.

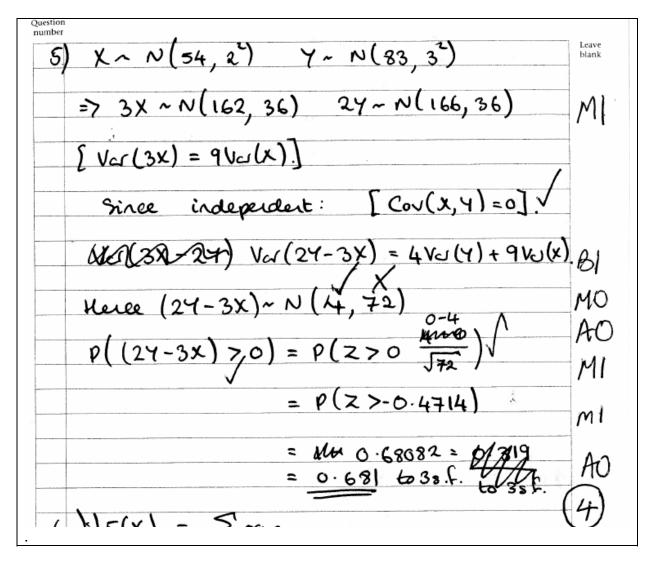
MS03 (cont)				
Q	Solution	Marks	Total	Comments
4	$98\% \implies z = 2.5758$	B1		AWFW 2.57 to 2.58
	CI width is $2 \times \frac{z\sigma}{\sqrt{n}}$	M1		Used; allow $\frac{z\sigma}{\sqrt{n}}$
	Thus $2 \times \frac{2.5758 \times 0.08}{\sqrt{n}} = 0.05$	A1√		OE; \checkmark on z; allow no '2 ×'
	Thus $\sqrt{n} = 8.24256$	m1		Solving for \sqrt{n} or n
	Thus $n = 67.9 \implies 68$	A1		AWRT; \checkmark on z
	Thus, to nearest 5, $n = 70$	A1	6	CAO
	Total		6	

5 The duration, *X* minutes, of a timetabled 1-hour lesson may be assumed to be normally distributed with mean 54 and standard deviation 2.

The duration, Y minutes, of a timetabled $1\frac{1}{2}$ -hour lesson may be assumed to be normally distributed with mean 83 and standard deviation 3.

Assuming the durations of lessons to be independent, determine the probability that the total duration of a random sample of three 1-hour lessons is less than the total duration of a random sample of two $1\frac{1}{2}$ -hour lessons. (7 marks)

Student Response



Commentary

The candidate has used an expression involving 3X and 2Y rather than $\sum X$ and $\sum Y$. As the variance of the difference is then 72 rather than 30 (mean is the same), the final answer is incorrect. Candidates must be aware of the difference is use between nX and $\sum_{n=1}^{n} X$.

5	$D = \sum_{i=1}^{3} X_i - \sum_{i=1}^{2} Y_i$ or $D' = \sum_{i=1}^{2} Y_i - \sum_{i=1}^{3} X_i$	M1		Used or implied
	have means $\mu = 162 - 166 = -4$ $\mu = 166 - 162 = +4$	B1		CAO either
	and variance $\sigma^2 = (3 \times 2^2) + (2 \times 3^2) = 12 + 18$ = 30	M1 A1		Use of $[a \times Var(Z)]$; implied CAO
	$\mathbb{P}\left(\sum_{i=1}^{3} X_{i} < \sum_{i=1}^{2} Y_{i}\right) =$			
	P(D < 0) or $P(D' > 0) =$	M1		Used or implied
	$P\left(Z > \frac{0 - (-4)}{\sqrt{30}}\right)$ or $P\left(Z > \frac{0 - (+4)}{\sqrt{30}}\right) =$	ml		Standardising 0 using μ and $\sqrt{\sigma^2}$
	P(Z < +0.73) or $P(Z > -0.73) =$			
	0.767 to 0.768	Al	7	AWFW
	Total		7	

- 6 (a) The random variable X has a binomial distribution with parameters n and p.
 - (i) Prove that E(X) = np. (4 marks)
 - (ii) Given that $E(X^2) E(X) = n(n-1)p^2$, show that Var(X) = np(1-p). (3 marks)
 - (iii) Given that X is found to have a mean of 3 and a variance of 2.97, find values for n and p.(3 marks)
 - (iv) Hence use a distributional approximation to estimate P(X > 2). (3 marks)
 - (b) Dressher is a nationwide chain of stores selling women's clothes. It claims that the probability that a customer who buys clothes from its stores uses a Dressher store card is 0.45.

Assuming this claim to be correct, use a distributional approximation to estimate the probability that, in a random sample of 500 customers who buy clothes from Dressher stores, at least half of them use a Dressher store card. (7 marks)

Student Response (contd on next page)

Ô	$a_{\gamma}(i) \times N Bin(n, p) \qquad $	
	$\Rightarrow E(X) = \sum x P(X = x) = \sum x n! \qquad \text{for} (1 - 0)^{n-\chi}$	MI
	$\frac{1}{100} = \frac{1}{1000} \frac{1}{10000000000000000000000000000000000$	ΛΛ I
	$= np \frac{(n-1)!}{x=1} p^{x-1} (1-p)^{n-1x+1}$	MI
	$= n\rho \sum_{x} \sum_{x} P(x = x) = n\rho \times L = n\rho$	r ~11
	1 tal	MO

Leave (ii) $Var(X) = E(X^2) - [E(X)]^2$ blank $= [n(n-1)p^{2} + E(X)] - [E(X)]^{2}$ = $n(n-1)p^{2} + np - (np)^{2}$ $n\rho[\rho(n-1)+1-n\rho]$ 3 = np(1-p)(ii) $E(x) = 3' \to nt$ los & p-90 p-: Var(X) = 297-> np(1-p) = 2,97 => 1-p'= 2.97:3 = = 0.01 = 300 3 (iv) X~ Bin (300, 0.01)~ P. (3) 3 $P(X \neq 2) = 1 + P(X \leq 2) = 1 - 0.4232 = 0.5768$ $b_{X} = \frac{1}{1500, 0.45} \times \frac{1}{1500, 0.45} \times \frac{1}{1500, 0.45} \times \frac{1}{1500, 0.45}$ 500 H => Itat statistics ; P(X), 05) = P(X) 0.5-0.45 Q45×0 オモ 500 = # = 7 2.247 $X \sim Bun (500, 0.45) \sim N(500 \times 0.45, \frac{12}{500} \times 0.45 \times 0.55)$ $\sim N(225, 123.75)$ $\Rightarrow P(X > \frac{500}{2}) = P(X > 250)$ BI BI BI BO $\Rightarrow P(X > 2505) = P(z > \frac{250.5 - 225}{\sqrt{123.75}}) = \frac{2}{50}.5 - \frac{2}{50}$ M = P(z > 2.29) = 1 - P(z < 2.29) = 1 \mathcal{M} -1 - 0.98899 = 0.01101AO

In part (a)(i), the candidate clearly knows the principles of a method for proving that E(X) = np but has not produced a fully convincing derivation. In part (b), the use of an incorrect continuity correction has produced an inaccurate final answer. Otherwise a correct solution to the question.

Proofs require a derivation that is fully convincing and care must be taken in applying correctly continuity corrections.

Mark Scheme

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathbf{E}(X) = \sum_{x=0}^{n} x \times {\binom{n}{x}} p^{x} (1-p)^{n-x}$	M1		Use of $\sum x \times P(X = x)$
	$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^{x} (1-p)^{n-x}$	M1		Expansion of ${}^{n}C_{x}$; cancelling of x (Ignore limits)
	$= np \times \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x}$	M1		Factors of n and p (Ignore limits)
	$= np \times \sum P(X = x) B(n-1, p) = np$	M1	4	AG; must be convincing

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)			13	
(b)	$Y \sim B(500, 0.45)$			
	or			
	$Y \sim$ (normal) with mean $\mu = 225$	B1		PI
	and			
	variance $\sigma^2 = 123.75$			AWFW 123 to 124
	or	B1		
	standard deviation $\sigma = 11.124$			AWFW 11.05 to 11.15
	(At least) half \Rightarrow (\geq) 250	B1		CAO
	$P(Y_B \ge 250) = P(Y_N > 249.5) =$	B1		CAO
	p(7, 249.5-225)			Standardising 249.5, 250 or 250.5 with
	$P\left(Z > \frac{249.5 - 225}{\sqrt{123.75}}\right) =$	M1		c's μ and $\sqrt{\sigma^2}$
	P(Z > 2.20) = 1 - P(Z < 2.20)	m1		Area change
	= 0.0138 to 0.014	A1	7	
	Note:			
	Use of $\frac{0.5 - 0.45}{\sqrt{0.000495}} \Rightarrow \max \text{ of } 5 \text{ marks}$			Use of distribution of \hat{p}
				_
	Use of $\frac{0.499 - 0.45}{\sqrt{0.000495}} \Rightarrow \text{max of 7 marks}$			Use of distribution of \hat{p}
	-		20	with continuity correction
	Total		20	

7 In a town, the total number, R, of houses sold during a week by estate agents may be modelled by a Poisson distribution with a mean of 13.

A new housing development is completed in the town. During the first week in which houses on this development are offered for sale by the developer, the estate agents sell a total of 10 houses.

- (a) Using the 10% level of significance, investigate whether the offer for sale of houses by the developer has resulted in a reduction in the mean value of R. (6 marks)
- (b) Determine, for your test in part (a), the critical region for *R*. (2 marks)
- (c) Assuming that the offer for sale of houses on the new housing development has reduced the mean value of R to 6.5, determine, for a test at the 10% level of significance, the probability of a Type II error. (4 marks)

Student Response

BI ٦ RUB(3) 1a). H. B value Sell after the 01 MO for the Havet liousos - and ΑO 10-13 832 Normal JB Fa 10% -1.2816 МI Accept Horo S Ho Alerak 21 Jours Al reduction balve man √b). #f Oritical report for R the whether S less thon evidence that tton is reduction Also, significant land 10% Ο probability of the popresent naking some error Ó 0.1 ₿١ CCP. C). Type I enor ₽ŧ PLACEDT HO MI sch 2 10% test) 063 = 8. if 38 0 bs > 8.38, we accept Ho mI P(Accept Ho) = X >8.38 R= 6.5) P(ΑO 1-DRC0.74) 0.737) 0.77035- 0.230 6

. The candidate has used the normal approximation for Po(13) in part (a) and for Po(6.5) in part (c). In neither case can the mean be reasonably considered as large, particularly as cumulative probabilities for both are given in Table 2 of the supplied booklet. The answer to part (b) does not give a critical region for R although the value 8.38 used in part (c) does suggest a value but again using the normal approximation.

For binomial and Poisson distributions, candidates should not opt for approximations when exact tabled values are available in the supplied booklet unless advised so to do in the question.

Candidates cannot in general expect to have marks awarded in a part of a question for an implied answer in a later part of the question.

Q	Solution	Marks	Total	Comments	
7(a)	$H_0: \lambda = 13$	B1		CAO; OE	
	$H_1: \lambda < 13$	B1		CAO; OE	
	$P(R \le 10 \mid Po(13))$	M1		Used or implied	
	= 0.2517	A1		AWFW 0.251 to 0.252	
	Prob of $0.2517 > 0.10 (10\%)$ z = -0.83 to $-0.70 > -1.28$	M1		Comparison of prob with 0.10 Comparison of z with -1.28	
	Thus no evidence, at 10% level, of a reduction in the mean value of R	A1√	6	✓ on probability or z In 'context' and qualified	
(b)	Require $P(R \le r Po(13)) \approx 0.10$	M1		Stated or implied	
	Critical Region is $R \le 8$ or $R < 9$	A1	2	Accept $R = 8$ May be scored in (a)	
(c)	Require $P(accept H_0 H_0 false)$	B1		OE; PI	
	= P(R > 8 Po(6.5))	M1		Use of Po(6.5)	
	$= 1 - P(R \le 8 Po(6.5))$	m1			
	= 1 - 0.7916				
	= 0.208 to 0.209	Al	4	AWFW	(0.2084
	Total		12		
	TOTAL		75		