

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Pure Core 4

MPC4

Monday 18 June 2007 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Find the remainder when $2x^2 + x - 3$ is divided by $2x + 1$. (2 marks)
- (b) Simplify the algebraic fraction $\frac{2x^2 + x - 3}{x^2 - 1}$. (3 marks)
- 2 (a) (i) Find the binomial expansion of $(1 + x)^{-1}$ up to the term in x^3 . (2 marks)
- (ii) Hence, or otherwise, obtain the binomial expansion of $\frac{1}{1 + 3x}$ up to the term in x^3 . (2 marks)
- (b) Express $\frac{1 + 4x}{(1 + x)(1 + 3x)}$ in partial fractions. (3 marks)
- (c) (i) Find the binomial expansion of $\frac{1 + 4x}{(1 + x)(1 + 3x)}$ up to the term in x^3 . (3 marks)
- (ii) Find the range of values of x for which the binomial expansion of $\frac{1 + 4x}{(1 + x)(1 + 3x)}$ is valid. (2 marks)
- 3 (a) Express $4 \cos x + 3 \sin x$ in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 360^\circ$, giving your value for α to the nearest 0.1° . (3 marks)
- (b) Hence solve the equation $4 \cos x + 3 \sin x = 2$ in the interval $0^\circ < x < 360^\circ$, giving all solutions to the nearest 0.1° . (4 marks)
- (c) Write down the minimum value of $4 \cos x + 3 \sin x$ and find the value of x in the interval $0^\circ < x < 360^\circ$ at which this minimum value occurs. (3 marks)

- 4 A biologist is researching the growth of a certain species of hamster. She proposes that the length, x cm, of a hamster t days after its birth is given by

$$x = 15 - 12e^{-\frac{t}{14}}$$

- (a) Use this model to find:

- (i) the length of a hamster when it is born; *(1 mark)*
- (ii) the length of a hamster after 14 days, giving your answer to three significant figures. *(2 marks)*

- (b) (i) Show that the time for a hamster to grow to 10 cm in length is given by $t = 14 \ln\left(\frac{a}{b}\right)$, where a and b are integers. *(3 marks)*

- (ii) Find this time to the nearest day. *(1 mark)*

- (c) (i) Show that

$$\frac{dx}{dt} = \frac{1}{14}(15 - x) \quad \text{(*3 marks*)}$$

- (ii) Find the rate of growth of the hamster, in cm per day, when its length is 8 cm. *(1 mark)*

- 5 The point $P(1, a)$, where $a > 0$, lies on the curve $y + 4x = 5x^2y^2$.

- (a) Show that $a = 1$. *(2 marks)*

- (b) Find the gradient of the curve at P . *(7 marks)*

- (c) Find an equation of the tangent to the curve at P . *(1 mark)*

Turn over for the next question

Turn over ►

6 A curve is given by the parametric equations

$$x = \cos \theta \quad y = \sin 2\theta$$

(a) (i) Find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$. (2 marks)

(ii) Find the gradient of the curve at the point where $\theta = \frac{\pi}{6}$. (2 marks)

(b) Show that the cartesian equation of the curve can be written as

$$y^2 = kx^2(1 - x^2)$$

where k is an integer. (4 marks)

7 The lines l_1 and l_2 have equations $\mathbf{r} = \begin{bmatrix} 8 \\ 6 \\ -9 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -3 \\ -1 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} -4 \\ 0 \\ 11 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ respectively.

(a) Show that l_1 and l_2 are perpendicular. (2 marks)

(b) Show that l_1 and l_2 intersect and find the coordinates of the point of intersection, P . (5 marks)

(c) The point $A(-4, 0, 11)$ lies on l_2 . The point B on l_1 is such that $AP = BP$.

Find the length of AB . (4 marks)

8 (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1+2y}}{x^2}$$

given that $y = 4$ when $x = 1$. (6 marks)

(b) Show that the solution can be written as $y = \frac{1}{2} \left(15 - \frac{8}{x} + \frac{1}{x^2} \right)$. (2 marks)

END OF QUESTIONS