

General Certificate of Education
June 2007
Advanced Level Examination



MATHEMATICS
Unit Pure Core 3

MPC3

Monday 11 June 2007 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 4 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information

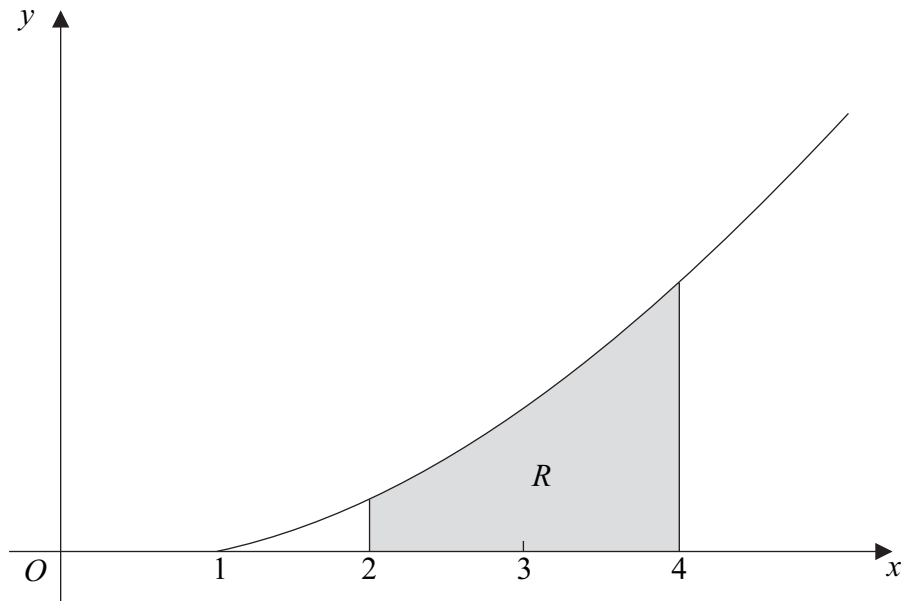
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Differentiate $\ln x$ with respect to x . (1 mark)
- (b) Given that $y = (x + 1) \ln x$, find $\frac{dy}{dx}$. (2 marks)
- (c) Find an equation of the normal to the curve $y = (x + 1) \ln x$ at the point where $x = 1$. (4 marks)
- 2 (a) Differentiate $(x - 1)^4$ with respect to x . (1 mark)
- (b) The diagram shows the curve with equation $y = 2\sqrt{(x - 1)^3}$ for $x \geq 1$.



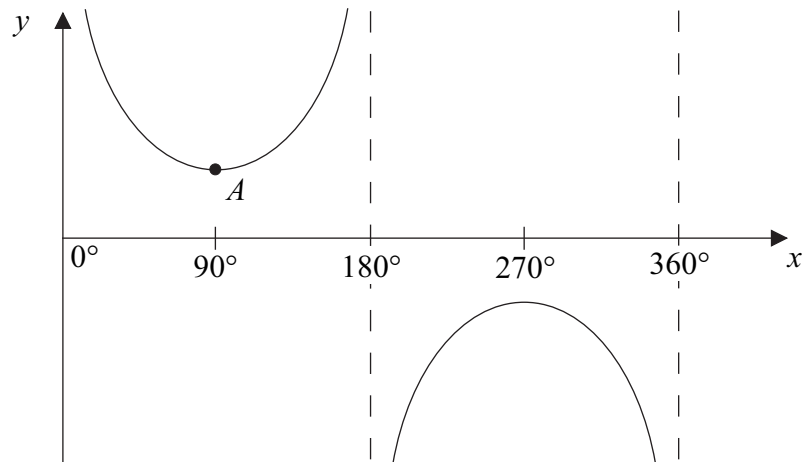
The shaded region R is bounded by the curve $y = 2\sqrt{(x - 1)^3}$, the lines $x = 2$ and $x = 4$, and the x -axis.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x -axis. (4 marks)

- (c) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sqrt{x^3}$ onto the graph of $y = 2\sqrt{(x - 1)^3}$. (4 marks)

- 3 (a) Solve the equation $\operatorname{cosec} x = 2$, giving all values of x in the interval $0^\circ < x < 360^\circ$.
(2 marks)

- (b) The diagram shows the graph of $y = \operatorname{cosec} x$ for $0^\circ < x < 360^\circ$.



- (i) The point A on the curve is where $x = 90^\circ$. State the y -coordinate of A .
(1 mark)
- (ii) Sketch the graph of $y = |\operatorname{cosec} x|$ for $0^\circ < x < 360^\circ$.
(2 marks)
- (c) Solve the equation $|\operatorname{cosec} x| = 2$, giving all values of x in the interval $0^\circ < x < 360^\circ$.
(2 marks)

Turn over for the next question

Turn over ►

4 [Figure 1, printed on the insert, is provided for use in this question.]

(a) Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^2 3^x dx$, giving your answer to three significant figures. (4 marks)

(b) The curve $y = 3^x$ intersects the line $y = x + 3$ at the point where $x = \alpha$.

(i) Show that α lies between 0.5 and 1.5. (2 marks)

(ii) Show that the equation $3^x = x + 3$ can be rearranged into the form

$$x = \frac{\ln(x + 3)}{\ln 3} \quad (2 \text{ marks})$$

(iii) Use the iteration $x_{n+1} = \frac{\ln(x_n + 3)}{\ln 3}$ with $x_1 = 0.5$ to find x_3 to two significant figures. (2 marks)

(iv) The sketch on **Figure 1** shows part of the graphs of $y = \frac{\ln(x + 3)}{\ln 3}$ and $y = x$, and the position of x_1 .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x -axis. (2 marks)

5 The functions f and g are defined with their respective domains by

$$f(x) = \sqrt{x - 2} \quad \text{for } x \geq 2$$

$$g(x) = \frac{1}{x} \quad \text{for real values of } x, \quad x \neq 0$$

(a) State the range of f . (2 marks)

(b) (i) Find $fg(x)$. (1 mark)

(ii) Solve the equation $fg(x) = 1$. (3 marks)

(c) The inverse of f is f^{-1} . Find $f^{-1}(x)$. (3 marks)

6 (a) Use integration by parts to find $\int xe^{5x} dx$. (4 marks)

(b) (i) Use the substitution $u = \sqrt{x}$ to show that

$$\int \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx = \int \frac{2}{1 + u} du \quad (2 \text{ marks})$$

(ii) Find the exact value of $\int_1^9 \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx$. (3 marks)

7 (a) A curve has equation $y = (x^2 - 3)e^x$.

(i) Find $\frac{dy}{dx}$. (2 marks)

(ii) Find $\frac{d^2y}{dx^2}$. (2 marks)

(b) (i) Find the x -coordinate of each of the stationary points of the curve. (4 marks)

(ii) Using your answer to part (a)(ii), determine the nature of each of the stationary points. (2 marks)

8 (a) Write down $\int \sec^2 x dx$. (1 mark)

(b) Given that $y = \frac{\cos x}{\sin x}$, use the quotient rule to show that $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. (4 marks)

(c) Prove the identity $(\tan x + \cot x)^2 = \sec^2 x + \operatorname{cosec}^2 x$. (3 marks)

(d) Hence find $\int_{0.5}^1 (\tan x + \cot x)^2 dx$, giving your answer to two significant figures. (4 marks)

END OF QUESTIONS

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Surname					Other Names				
Centre Number						Candidate Number			
Candidate Signature									

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Insert

Insert for use in **Question 4**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ►

Figure 1 (for use in Question 4)