Teacher Support Materials

Maths GCE

Paper Reference MM05

Question 1

1 A particle moves with simple harmonic motion along a straight line. Its maximum speed is $4\,\text{m s}^{-1}$ and its maximum acceleration is $100\,\text{m s}^{-2}$.

(a) Show that the period of motion is $\frac{2\pi}{25}$ seconds. *(4 marks)*

(b) Find the amplitude of the motion. *(1 mark)*

Student Response

Script not available

Commentary

This script is typical of those who did not use simple algebra to find ω.
Instead of using $v = \omega a$ to obtain $4 = \omega a$,
and maximum acceleration = $\omega^2 a$ to obtain $100 = \omega^2 a$, then dividing to find $\omega = 25$, many calculations were used before eventually achieving $\omega = 25$.

Mark Scheme

MM05

Q	Solution	Marks	Total	Comments
1(a)	Maximum speed $\Rightarrow \omega a = 4$	B1		
	Maximum acceleration $\Rightarrow \omega^2 a = 100$	B1		
	$\omega = 25$	M1		
	Period is $\frac{2\pi}{\omega}$			
	$= \frac{2\pi}{25}$	A1	4	AG; needs to use a justified $\omega = 25$
(b)	Amplitude is $\frac{4}{25}$ m	B1	1	
	Total		**5**	

Question 2

2 A simple pendulum consists of a particle, of mass m, fixed to one end of a light, inextensible string of length l. The other end of the string is attached to a fixed point. When the pendulum is in motion, the angle between the string and the downward vertical is θ at time t. The motion takes place in a vertical plane.

(a) Show, using a trigonometrical approximation, that for small angles of oscillation the motion of the pendulum can be modelled by the differential equation

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$ *(4 marks)*

(b) The pendulum has length 0.5 metres. The pendulum is released from rest with the string taut and at an angle of $\frac{\pi}{400}$ to the vertical.

(i) Given that $\theta = A\cos\omega t$, find the values of A and ω. *(3 marks)*

(ii) Find the maximum speed of the particle in the subsequent motion. *(3 marks)*

Student response

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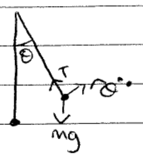
2a



N2L ↗

~~m~~$r\ddot{\theta}$ = -~~m~~g sinθ

$l\ddot{\theta} = -g\sin\theta$ ✓

For small angles, $\theta \approx \sin\theta$ ✓

so $\ddot{\theta} = \frac{-g\theta}{l}$

$\Rightarrow \frac{d^2\theta}{dt^2} = \frac{-g\theta}{l}$ ✓ 4

bi $l = 0.5$ $\theta = A\cos\omega t$ $\dot{\theta} = -A\omega\sin\omega t$

$a = \frac{\pi}{400}$ ✓ $\ddot{\theta} = -A\omega^2\cos\omega t = -\theta\omega^2$

From (a) → $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{g}{0.5}} \approx 4.43$ rads⁻¹ (3sf) ✓ 3

$t = 0 \Rightarrow \theta = A$ ~~$\ddot{\theta} = -\frac{g\theta}{l}$~~

$\Leftrightarrow A = \frac{\pi}{400}$ ~~$\Rightarrow \theta\omega^2 = \frac{g\theta}{l}$~~

~~$\omega^2 = \sqrt{\frac{g}{l}}$~~

ii $V_{max} = a\omega$

$= \frac{\pi}{400}\omega$ = ~~0.0348 ms⁻¹ (3sf)~~ ✗ 0

7

Commentary

This script shows parts (a) and (b) (i) answered correctly.

In part (b) (ii), the candidate found the maximum **angular velocity**, not the maximum **speed** which the question required.

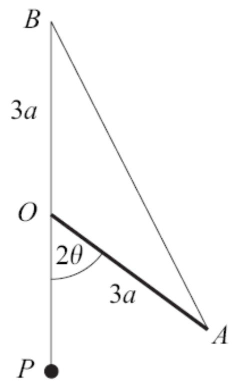
The formula used, $v_{max} = a\omega$ needed a to be the maximum displacement for the pendulum (where $a = Al$) rather than the maximum angular displacement which was A.

Mark Scheme

	Total		5	
2(a)	Using transverse component of acceleration is $r\frac{d^2\theta}{dt^2}$	B1		
	$ml\frac{d^2\theta}{dt^2} = -mg\sin\theta$	M1		
	$\frac{d^2\theta}{dt^2} = -\frac{g\sin\theta}{l}$			
	For small angles of θ, $\sin\theta \approx \theta$	B1		
	$\frac{d^2\theta}{dt^2} = -\frac{g\theta}{l}$	A1	4	AG
(b)(i)	$A = \frac{\pi}{400}$	B1		
	$\omega = \sqrt{\frac{g}{l}}$	M1		
	$= \sqrt{\frac{9.8}{0.5}} = \frac{7\sqrt{10}}{5}$ or 4.43	A1	3	
(ii)	Maximum speed is $a\omega$			
	$= \frac{7\sqrt{10}}{5} \times 0.5 \times \frac{\pi}{400}$	M1A1		Needs 0.5 term $\sqrt{\frac{g}{2}} \times \frac{\pi}{400}$
	= 0.0174	A1	3	
	Total		**10**	

Question 3

3 A uniform rod, OA, of length $3a$ and mass $2m$, is freely pivoted at O. A light, inextensible string, of length $10a$, is attached to the rod at A and passes over a fixed, smooth peg at B, a distance $3a$ vertically above O. A particle, P, of mass m, is attached to the other end of the string. The angle between the rod and the vertical is 2θ, as shown in the diagram.



(a) Show that the total potential energy of the system, V, is given by

$$V = 6mga\cos\theta - 7mga - 3mga\cos 2\theta$$

where gravitational potential energy is taken to be zero at O. *(5 marks)*

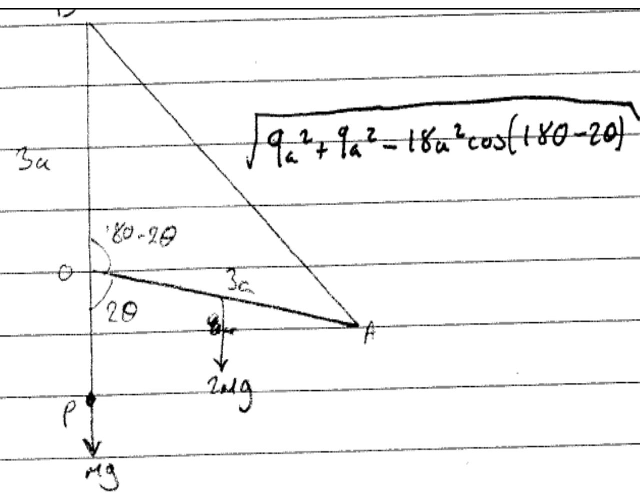
(b) Find the **two** values of θ, $0 \leqslant \theta < \frac{\pi}{2}$, for which the system is in equilibrium. *(6 marks)*

(c) Determine the stability of each position of equilibrium. *(4 marks)*

Student Response

Leave blank

3a)



$PE = 2mg\left(-\frac{3a}{2}\cos 2\theta\right)$

$= -3mga\cos 2\theta$ ✓

Length $AB = \sqrt{9a^2+9a^2-18a^2\cos(180-2\theta)}$

$= \sqrt{18a^2-18a^2[\cos 180\cos(-2\theta)+\sin 180\sin(-2\theta)]}$

$= \sqrt{18a^2-18a^2[-1\cos 2\theta+0]}$

$= \sqrt{18a^2+18a^2\cos 2\theta}$

$= a\sqrt{18+18\cos 2\theta}$ ✓

Length $OP = 10a - \sqrt{18+18\cos 2\theta}\,a - 3a$

$PE = mga\left(7 - \sqrt{18+18\cos 2\theta}\right)$

$s^2+c^2=1$

$+\quad c^2-s^2=\cos 2A$

$2\cos^2\theta = 1+\cos 2\theta$ ✓

$= mga\left(7-\sqrt{18}\sqrt{1+\cos 2\theta}\right)$

$= mga\left(7-\sqrt{18}\sqrt{2\cos^2\theta}\right)$

$= mga\left(7-\sqrt{18\times 2}\cos\theta\right)$

$= mga\left(7-\sqrt{36}\cos\theta\right)$

$= -mag\left(7-6\cos\theta\right)$

$V = -3mga\cos 2\theta - 7mga + 6mga\cos\theta$ Ans. ✓

5

Leave blank

b) $V = 6mga\cos\theta - 7mga - 3mga\cos 2\theta$

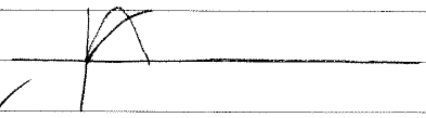
$\frac{dV}{d\theta} = -6mga\sin\theta + 6mga\sin 2\theta$

$= 6mga(\sin 2\theta - \sin\theta)$

$0 = 6mga(\sin 2\theta - \sin\theta)$

$\sin 2\theta = \sin\theta$

$\sin 2\theta = 2\cos\theta\sin\theta$



$2\cos\theta\sin\theta = \sin\theta$ ✓

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$ 6

$\theta = \cos^{-1}\frac{1}{2}$

$= \pi/3$

$\therefore\ \theta = 0$ and $\theta = \pi/3$ Ans. ✓

c) $\frac{d^2V}{d\theta^2} = -6mga\cos\theta + 12mga\cos 2\theta$ ✓

$\theta = 0$ $\frac{d^2V}{d\theta^2} = -6mga + 12mga$

$= 6mga$ +ve $\therefore$ stable Ans. ✓

$\theta = \pi/3$ $\frac{d^2V}{d\theta^2} = -6mga \times \frac{1}{2} + 12mga \times -\frac{1}{2}$

$= -3mga - 6mga$ 4

$= -9mga$ −ve $\therefore$ unstable Ans. ✓

15

Commentary

This example shows a candidate spending considerable time and effort instead of thinking and then using a more suitable approach as outlined in the Examiners Report.

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In order to find *AB*, the candidate used the cosine rule in triangle *OAB*, and then took more time to simplify $\sqrt{18 + 18\cos 2\theta}$ into $6\cos\theta$.

Mark Scheme

MM05 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$AB = 6a\cos\theta$	M1A1		
	Potential energy, below O, of rod is $-2mga\frac{3}{2}\cos 2\theta = -3mga\cos 2\theta$	B1		
	Potential energy, below O, of particle is $-mg(7a - 6a\cos\theta)$	B1		
	$= 6mga\cos\theta - 7mga$			
	$V = 6mga\cos\theta - 7mga - 3mga\cos 2\theta$	A1	5	AG
(b)	At equilibrium, $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$	M1		
	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 6mga\sin 2\theta - 6mga\sin\theta$	M1A1		
	$= 6mga\sin\theta(2\cos\theta - 1)$			
	$= 0$ when			
	$\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$	A1		
	$\therefore$ system is in equilibrium when			
	$\theta = 0$ and $\frac{\pi}{3}$	A1,A1	6	
(c)	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 12mga\cos 2\theta - 6mga\cos\theta$	M1		
	When $\theta = 0$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 6mga$	A1		
	This is positive $\Rightarrow$ minimum PE Position is stable equilibrium	E1		
	When $\theta = \frac{\pi}{3}$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -9mga$			
	$\Rightarrow$ maximum PE Position is unstable equilibrium	E1	4	
	Total		**15**	

Question 4

4 A particle of mass m is moving along a smooth wire that is fixed in a plane. The polar equation of the wire is $r = ae^{3\theta}$. The particle moves with a constant angular velocity of 6.

At time $t = 0$, the particle is at the point with polar coordinates $(a, 0)$.

(a) Find the transverse and radial components of the acceleration of the particle in terms of a and t. *(10 marks)*

(b) The resultant force on the particle is **F**.

Show that the magnitude of **F**, at time t, is $360mae^{18t}$. *(4 marks)*

Student Response

(4) $r = ae^{3\theta}$ $\quad \dot{\theta} = 6 \quad \ddot{\theta} = 0$

($t = 0 \quad r = a \quad \theta = 0$)

a) radial component $= \ddot{r} - r\dot{\theta}^2$

transverse $= r\ddot{\theta} + 2\dot{r}\dot{\theta}$

~~$\dot{r} = \dot{\theta}3ae^{3\theta} = 3ae^{18t}(6)$ (angular velocity is a constant $\therefore \ddot{\theta} = 0$)~~

~~$\ddot{r} = 9ae^{3\theta} = 9ae^{18t}$ $\quad$ and $\therefore \theta = 6t$~~

~~(submitting) radial component $= 9ae^{18t} - (ae^{18t})(36)$~~

~~$= -27ae^{18t}$~~

~~transverse $= 0 + (2)(3ae^{18t})(6)$~~

~~$= 36ae^{18t}$~~

~~b) $F = m\sqrt{(\text{radial acceleration})^2 + (\text{transverse})^2}$~~

~~$= m\sqrt{(27ae^{18t})^2 + (36ae^{18t})^2} = 45mae^{18t}$~~

a) cont.

$r = ae^{18t}$ $\quad$ ($\theta = 6t \quad \dot{\theta} = 6$, $\ddot{\theta} = 0$) ✓

$\dot{r} = 18ae^{18t}$

$\ddot{r} = 324ae^{18t}$ ✓

(submitting) $\quad$ radial $= 324ae^{18t} - (ae^{18t})(36)$

$= 288ae^{18t}$ ✓ $\quad$ 10

transverse $= 0 + (2)(18ae^{18t})(6)$

$= 216ae^{18t}$ ✓

b) $F = m\sqrt{\text{radial}^2 + \text{transverse}^2} = m\sqrt{(288ae^{18t})^2 + (216ae^{18t})^2}$ ✓ $\quad$ 4

$= m(360ae^{18t}) = 360mae^{18t}$ ✓

(14)

Leave blank

Commentary

This script shows a concise and accurate solution to the problem asked.

Mark Scheme

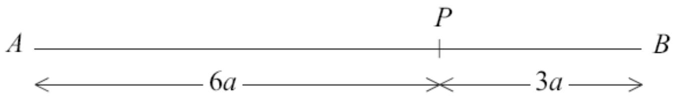
MM05 (cont)

Q		Marks	Total	Comments
4(a)	$r = ae^{3\theta}$			
	$\dot{r} = 3ae^{3\theta}\dot{\theta}$	M1		
	$\ddot{r} = 9ae^{3\theta}\dot{\theta}^2$	M1		
	Since $\ddot{\theta} = 0$,	B1		B1 for $\ddot{\theta} = 0$
	$\dot{r} = 18ae^{3\theta}$	A1		
	$\ddot{r} = 324ae^{3\theta}$	A1		
	Since $\dot{\theta}$ is a constant, $\theta = 6t$ and $\theta = 0$ when $t = 0$	B1		
	Transverse acceleration is $2\dot{r}\dot{\theta} + r\ddot{\theta}$	M1		
	$= 216ae^{18t}$	A1		
	Radial acceleration is $\ddot{r} - r\dot{\theta}^2$			
	$= 324ae^{18t} - 36ae^{18t}$	M1		
	$= 288ae^{18t}$	A1	10	
(b)	Using $F = ma$, $\mathbf{F} = 288mae^{18t}\hat{r} + 216mae^{18t}\hat{\theta}$	M1A1		
	Magnitude is $\{(288mae^{18t})^2 + (216mae^{18t})^2\}^{1/2}$	M1		
	$= 360mae^{18t}$	A1	4	AG
	Total		**14**	

Question 5a

5 The ends of a light, uniform elastic string are fixed to two points, A and B, a distance $9a$ apart on a smooth, horizontal plane. The string is of natural length $6a$ and modulus of elasticity $4mn^2a$, where n is a constant.

A particle of mass m is attached to the string at P, where $AP = 6a$. The natural length of AP is $4a$ and the natural length of BP is $2a$. In this position, the particle is in equilibrium.



The particle is moved a distance $\frac{1}{2}a$ towards B and then released from rest at time $t = 0$. The displacement of the particle from its equilibrium position at time t is x. Hence initially $x = +\frac{1}{2}a$.

The motion of the particle is resisted by a force of magnitude $2mnv$, where v is the speed of the particle at time t.

(a) Show that x satisfies

$$\frac{d^2x}{dt^2} + 2n\frac{dx}{dt} + 3n^2x = 0$$ *(7 marks)*

(b) Given that $n = 1$, find x in terms of a and t. *(8 marks)*

Student Response

Leave blank

5a) $\lambda = 4mn^2a$ $\quad l = 6a$ $\quad AB = 9a$ $\quad$ Force due to spring

$l_{AP} = 4a$ $\quad AP = 6a$ $\quad = \frac{\lambda x}{l_{AP}} - \left(\frac{\lambda x - x}{l_{BP}}\right)$

$l_{BP} = 2a$ $\quad BP = 3a$

$$F = -\left(\frac{4mn^2a\,x}{4a} + \frac{4mn^2a\,x}{2a}\right) - 2mnv$$ ✗

M1

$$F = -(mn^2x + 2mn^2x) - 2mn\dot{x}$$

$$m\ddot{x} = -3mn^2x - 2mn\dot{x}$$

$$\ddot{x} + 3n^2x + 2n\dot{x} = 0$$ ✗

Commentary

A number of candidates 'invented' the answer given to part (a). This example shows a candidate who uses the extensions to be *x* in both strings, rather than the correct 2*a* + *x* in string *AP* and *a* – *x* in string *BP*. The candidate ignored the fact that the two tensions were acting in opposite directions to enable him to arrive at the correct answer as printed. Naturally such methods are not accepted and are penalised.

Mark Scheme

MM05 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Natural length of AP is $4a$ and natural length of BP is $2a$			
	When particle is x from equilibrium position:			
	Tension in AP is $\dfrac{4mn^2a(2a+x)}{4a}$	M1A1		
	Tension in BP is $\dfrac{4mn^2a(a-x)}{2a}$	M1A1		
	In general position, using $F = ma$: $m\dfrac{d^2x}{dt^2} = \dfrac{4mn^2a(a-x)}{2a} - \dfrac{4mn^2a(2a+x)}{4a} - 2mn\dfrac{dx}{dt}$	M1A1		
	$m\ddot{x} =$ $2mn^2a - 2mn^2x - 2mn^2a - mn^2x - 2mn\dot{x}$			
	$\dfrac{d^2x}{dt^2} + 2n\dfrac{dx}{dt} + 3n^2x = 0$	A1	7	AG

Question 6

6 A large snowball, which may be modelled as a uniform sphere of radius r, moves with speed v down a slope inclined at 30° to the horizontal. The snowball picks up snow at a rate proportional to both its speed and its mass, m, and hence it may be assumed that $\frac{\mathrm{d}m}{\mathrm{d}t} = kmv$ at time t, where k is a constant.

You should ignore any rotational motion of the snowball.

(a) Neglecting any resistance forces acting on the snowball, show that

$$2\frac{\mathrm{d}v}{\mathrm{d}t} + 2kv^2 = g$$ *(4 marks)*

(b) Using the identity

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t} \times \frac{\mathrm{d}v}{\mathrm{d}x} = v\frac{\mathrm{d}v}{\mathrm{d}x}$$

where x is the distance travelled by the centre of the snowball, show that the differential equation in part (a) can be written as

$$2v\frac{\mathrm{d}v}{\mathrm{d}x} = g - 2kv^2$$ *(1 mark)*

(c) At time $t = 0$, $v = 0$ and $x = 0$.

Solve the differential equation in part (b) to find v^2 as a function of x. *(6 marks)*

(d) When $t = 0$, $v = 0$ and $x = 0$, the radius of the snowball is $\frac{1}{3}$ metre.

(i) Show that $r^3 = Ce^{kx}$, where C is a constant to be determined. *(3 marks)*

(ii) Find, in terms of g and k, the speed of the snowball when its radius is 1 metre. *(2 marks)*

Student Response

6

b) $\frac{dv}{dt} = \frac{dx}{dt} \times \frac{dv}{dx} = v\frac{dv}{dx}$

$2\frac{dv}{dt} + 2kv^2 = g$

$\frac{dv}{dt} = v\frac{dv}{dx}$

$\therefore 2v\frac{dv}{dx} + 2kv^2 = 2\frac{dv}{dt} + 2kv^2$

$\therefore 2v\frac{dv}{dx} + 2kv^2 = g$

and $2v\frac{dv}{dx} = g - 2kv^2$

✓ 1

c) $\frac{2v}{g-2kv^2}\,dv = dx$

$\int \frac{2v}{g-2kv^2}\,dv = \int dx$

$\frac{1}{2}\int \frac{2v}{\frac{g}{2}-kv^2}\,dv = \int dx$

$\frac{1}{2k}\ln\left(\frac{g}{2}-kv^2\right) = x + c$

$\ln\left(\frac{g}{2}-kv^2\right) = 2x + c$ $\quad t=0 \quad v=0 \quad x=0$

~~$e^{2x} = \frac{g}{2} - kv^2$~~ $\ln\left(\frac{g}{2} - 0\right) = 0 + c$

$c = \ln\frac{g}{2}$

$\frac{1}{2}\ln\left(\frac{g/2 - kv^2}{g/2}\right) = x$

P.T.O

6c)

$\frac{1}{2}\ln\left(\frac{g/2 - kv^2}{g/2}\right) = x$

$\ln\left[\frac{\frac{g}{2} - kv^2}{\frac{g}{2}}\right] = 2x$

$\frac{g}{2} - kv^2 = \frac{g}{2}e^{2x}$

$kv^2 = \frac{g}{2} - \frac{g}{2}e^{2x}$

$v^2 = \frac{g}{2k}\left[1 - e^{\frac{k}{2x}}\right]$

5

d) $\frac{dm}{dt} = kmv$

i) $\frac{dm}{dx}\frac{dx}{dt} = kmv$

$\frac{dm}{dx}\frac{1}{m} = \frac{dt}{dx}kv$

Volume of sphere = $\frac{4\pi r^3}{3}$ $\quad \pi r^2 \times \pi r$ $\quad \pi r^3$

πr^3 $\quad m = \pi r^3 \times \rho$

$\frac{dm}{dt} = kmv$ $\quad m \propto \pi r^3$

$m = Cr^3$ $\quad m = Cr^3$

~~$\frac{dm}{dt} = kr^3$~~

$r^3 = \frac{C}{m}$ $\quad \frac{dm}{dt}\frac{1}{m} = \ln m$

6 d ii)

~~r = 1~~ $\frac{1}{3}^{3} = Ce^{k0}$

$= C$

$C = \frac{1}{9}$

$1 = \frac{1}{9}e^{kx}$

$9 = e^{kx}$

$\frac{\ln 9}{k} = x$

$v^2 = \frac{g}{2k}\left[1 - e^{\frac{2\ln 9}{k}}\right)$ M1

$= \frac{g}{2k}\left[1 - \left[e^{2\ln 9}\right]^{\frac{2}{k}}\right]$

$= \frac{g}{2k}\left[1 - \ln 9^{2/k}\right]$

$= \frac{g}{2k}\left[1 - \frac{2}{k}(\ln 9)^{\frac{2}{k}}\right]$

$v = \sqrt{\frac{g}{2k}\left[1 - (\ln 9)^{\frac{2}{k}}\right]}$

Commentary

This script shows good practice, using of a small element of time, δt, when using work done equals change in momentum in part (a). The use of $\frac{dv}{dt} = v\frac{dv}{dx}$ caused no problems in part (b).

To avoid problems with $\int \frac{2v}{g-2kv^2}$ *dv* the candidate used $\frac{1}{2}\int \frac{2v}{\frac{g}{2}-kv^2}$ *dv* to obtain $\frac{1}{2}\ln(\frac{g}{2}-kv^2)$. The simpler conversion of $\int \frac{2v}{g-2kv^2}$ *dv* = $-\frac{1}{2k}\int \frac{4kv}{g-2kv^2}$ *dv* leading to $-\frac{1}{2k}\ln(g-2kv^2)+c$ was rarely seen.

Many candidates gave the result of $\int \frac{2v}{g-2kv^2}$ *dv* immediately to be $-\frac{1}{2k}\ln(g-2kv^2)+c$.

Mark Scheme

Q	Solution	Marks	Total	Comments
6(a)	Change in linear momentum = work done by external force $(m+\delta m)(v+\delta v)-mv = mg\sin 30\ \delta t$	M1A1		Needs δ terms
	$v\delta m + m\delta v = \frac{1}{2}mg\delta t$ (to first order of δ terms)			
	$\frac{1}{2}mg = m\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}m}{\mathrm{d}t}$	M1		Accept $mg\sin 30 = \frac{1}{2}mg = \frac{m\mathrm{d}v}{\mathrm{d}t} + kmv^2$
	Using $\frac{\mathrm{d}m}{\mathrm{d}t} = kmv$: $m\frac{\mathrm{d}v}{\mathrm{d}t} + v\,kmv = \frac{1}{2}mg$			
	$2\frac{\mathrm{d}v}{\mathrm{d}t} + 2kv^2 = g$	A1	4	AG
(b)	Using the identity $\frac{\mathrm{d}v}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}x}$: $2v\frac{\mathrm{d}v}{\mathrm{d}x} + 2kv^2 = g$			
	$2v\frac{\mathrm{d}v}{\mathrm{d}x} = g - 2kv^2$	B1	1	AG
(c)	$\int \frac{2v}{g-2kv^2}\,\mathrm{d}v = \int \mathrm{d}x$	M1		
	$-\frac{1}{2k}\ln(g-2kv^2) = x + c$	A1		
	When $x = 0$, $v = 0 \Rightarrow c = -\frac{1}{2k}\ln g$	M1		
	$x = \frac{1}{2k}\ln\frac{g}{g-2kv^2}$	M1A1		
	$\frac{g}{g-2kv^2} = \mathrm{e}^{2kx}$ $g\mathrm{e}^{-2kx} = g - 2kv^2$			
	$v^2 = \frac{g(1-\mathrm{e}^{-2kx})}{2k}$	A1	6	

MM05 (cont)

Q	Solution	Marks	Total	Comments
6(d)(i)	Using $m = \frac{4}{3}\pi r^3\rho$: $\frac{\mathrm{d}m}{\mathrm{d}t} = kmv \Rightarrow$ $4\pi r^2\rho\frac{\mathrm{d}r}{\mathrm{d}t} = k\frac{4}{3}\pi r^3\rho v$			
	$3\frac{\mathrm{d}r}{\mathrm{d}t} = krv$	M1		
	$3\int\frac{\mathrm{d}r}{r} = \int kv\,\mathrm{d}t$ $= \int k\,\mathrm{d}x$ $3\ln r = kx + c$ $r^3 = C\mathrm{e}^{kx}$			
	When $x = 0$, $r = \frac{1}{3} \Rightarrow C = \frac{1}{27}$	A1		
	$r^3 = \frac{1}{27}\mathrm{e}^{kx}$	B1	3	
(ii)	When $r = 1$, $\mathrm{e}^{kx} = 27$			
	Using result in (c), $v^2 = \frac{g(1-\frac{1}{729})}{2k}$	M1		
	$v = \sqrt{\frac{364}{729}\frac{g}{k}}$	A1	2	
	Total		**16**	
	TOTAL		**75**	