General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MFP2**
Unit Further Pure 2

Tuesday 26 June 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Given that $f(r) = (r-1)r^2$, show that

$$f(r+1) - f(r) = r(3r+1)$$ *(3 marks)*

(b) Use the method of differences to find the value of

$$\sum_{r=50}^{99} r(3r+1)$$ *(4 marks)*

2 The cubic equation

$$z^3 + pz^2 + 6z + q = 0$$

has roots α, β and γ.

(a) Write down the value of $\alpha\beta + \beta\gamma + \gamma\alpha$. *(1 mark)*

(b) Given that p and q are real and that $\alpha^2 + \beta^2 + \gamma^2 = -12$:

(i) explain why the cubic equation has two non-real roots and one real root; *(2 marks)*

(ii) find the value of p. *(4 marks)*

(c) One root of the cubic equation is $-1 + 3\mathrm{i}$.

Find:

(i) the other two roots; *(3 marks)*

(ii) the value of q. *(2 marks)*

3 Use De Moivre's Theorem to find the smallest positive angle θ for which

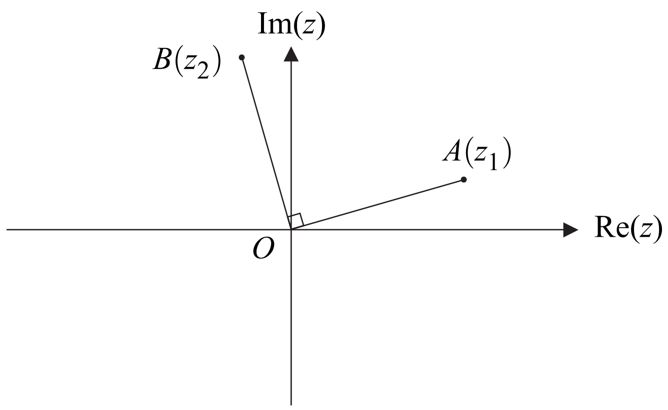
$$(\cos\theta + \mathrm{i}\sin\theta)^{15} = -\mathrm{i}$$ *(5 marks)*

4 (a) Differentiate $x \tan^{-1} x$ with respect to x. *(2 marks)*

(b) Show that

$$\int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \ln \sqrt{2}$$ *(5 marks)*

5 The sketch shows an Argand diagram. The points A and B represent the complex numbers z_1 and z_2 respectively. The angle $AOB = 90°$ and $OA = OB$.



(a) Explain why $z_2 = \mathrm{i}z_1$. *(2 marks)*

(b) On a **single** copy of the diagram, draw:

(i) the locus L_1 of points satisfying $|z - z_2| = |z - z_1|$; *(2 marks)*

(ii) the locus L_2 of points satisfying $\arg(z - z_2) = \arg z_1$. *(3 marks)*

(c) Find, in terms of z_1, the complex number representing the point of intersection of L_1 and L_2. *(2 marks)*

6 (a) Show that

$$\left(1 - \frac{1}{(k+1)^2}\right) \times \frac{k+1}{2k} = \frac{k+2}{2(k+1)}$$ *(3 marks)*

(b) Prove by induction that for all integers $n \geqslant 2$

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\dots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$$ *(4 marks)*

Turn over for the next question

7 A curve has equation $y = 4\sqrt{x}$.

(a) Show that the length of arc s of the curve between the points where $x = 0$ and $x = 1$ is given by

$$s = \int_0^1 \sqrt{\frac{x+4}{x}}\, \mathrm{d}x$$ *(4 marks)*

(b) (i) Use the substitution $x = 4\sinh^2\theta$ to show that

$$\int \sqrt{\frac{x+4}{x}}\, \mathrm{d}x = \int 8\cosh^2\theta\ \mathrm{d}\theta$$ *(5 marks)*

(ii) Hence show that

$$s = 4\sinh^{-1} 0.5 + \sqrt{5}$$ *(6 marks)*

8 (a) (i) Given that $z^6 - 4z^3 + 8 = 0$, show that $z^3 = 2 \pm 2\mathrm{i}$. *(2 marks)*

(ii) Hence solve the equation

$$z^6 - 4z^3 + 8 = 0$$

giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(6 marks)*

(b) Show that, for any real values of k and θ,

$$(z - k\mathrm{e}^{\mathrm{i}\theta})(z - k\mathrm{e}^{-\mathrm{i}\theta}) = z^2 - 2kz\cos\theta + k^2$$ *(2 marks)*

(c) Express $z^6 - 4z^3 + 8$ as the product of three quadratic factors with real coefficients. *(3 marks)*

END OF QUESTIONS