Teacher Support Materials

Maths GCE

Paper Reference MFP1

Question 1

1 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. *(2 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement of scale factor p and a reflection in a line L. State the value of p and write down the equation of L. *(2 marks)*

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

Student Response

1a) $A - 2B = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} = 3\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ ✓

b) $p=3$ ✓ $L =$ reflection along line $y = x$. ✗

c) $M^2 = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}\begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

$\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

Commentary

This question was generally very well answered. The most common mistake was in part (b), as exemplified in the chosen script, where the mirror line was given as $y = x$ rather than $y = -x$. It was, however, acceptable to give this reflection in conjunction with an enlargement with scale factor -3.

Mark scheme

Q	Solution	Mark	Total	Comments
1(a)	$\mathbf{M} = \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}$	B2,1	2	B1 if subtracted the wrong way round
(b)	$p = 3$	B1F		ft after B1 in (a)
	L is $y = -x$	B1	2	Allow $p = -3$, L is $y = x$
(c)	$\mathbf{M}^2 = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$	B1F		Or by geometrical reasoning; ft as before
	$\ldots = 9\mathbf{I}$	B1F	2	ft as before
	Total		**6**	

Question 2

2 (a) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8. *(3 marks)*

(b) Use interval bisection **twice**, starting with the interval in part (a), to give this root to one decimal place. *(4 marks)*

Student response

Leave blank

2(a) Let $f(X) = X^3 + X - 7$

~~$f(1.6) = 1.6^3 + 1.6 - 7$~~

$f(1.6) = 1.6^3 + 1.6 - 7 = -1.304 < 0$ ✓

$f(1.8) = 1.8^3 + 1.8 - 7 = 0.632 > 0$ ✓

change ~~of~~ of sign ∴

the ~~equan~~ equation $X^3 + X - 7$ has a root between 1.6 and 1.8 ✓ 3

(b) $\frac{1.6 + 1.8}{2} = 1.7$

$f(1.7) = 1.7^3 + 1.7 - 7 = -0.387$ ✓

$-0.387 < 0$ ✓

$\frac{1.7 + 1.8}{2} = 1.75$

$f(1.75) = 1.75^3 + 1.75 - 7$ ✓ 3

$= 0.109375$ ✓

≈ 0.1 (1d.p.) ∴ ✗ 0

(6)

Commentary

Almost all the candidates were able to make a good start to this question, though some failed to draw a proper conclusion in part (a). In part (b) the response was again very good. The most common error was a failure to grasp what exactly was being asked for at the end of the question. If it is known that the root lies between 1.7 and 1.75, then it must be 1.7 to one place of decimals. But many candidates gave no value to 1 DP, or, like this candidate, gave a value of the function rather than a value of *x*.

Mark Scheme

2(a)	$f(1.6) = -1.304$, $f(1.8) = 0.632$ Sign change, so root between	B1,B1 E1	 3	Allow 1 dp throughout
(b)	$f(1.7)$ considered first $f(1.7) = -0.387$, so root > 1.7 $f(1.75) = 0.109375$, so root ≈ 1.7	M1 A1 m1A1	 4	 m1 for $f(1.65)$ after error
	Total		7	

Question 3

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$z - 3\mathrm{i}z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$z - 3\mathrm{i}z^* = 16$$ *(3 marks)*

Student Response

3a) $z - 3iz^*$

$x + iy - 3i(x - iy)$

$x + iy - 3ix \ominus 3i^2y$ ✗

$x + iy - 3ix + 3y$

Real $= x + 3y$ ✓

imaginary $= y - 3x$ 2

b) $z - 3iz^* = 16$

$x + iy - 3ix + 3y = 16$

$x + 3y = 16$

$-9x + 3y = 0$

$10x = 16$ ✓ $z = 1{\cdot}6 + 4{\cdot}8i$ ✓

$x = 1{\cdot}6$ 3

$y = 4{\cdot}8$ (5)

Commentary

Although most of the candidates who take this paper are good at algebra, there are still quite a number who make elementary sign errors. This candidate is typical of many who made the same error, affecting the real part of the given complex number. The candidate then uses the incorrect expression in part (b), but does correct work to arrive at a value of z. Full marks were awarded in part (b) for candidates who had lost a mark in part (a) for this particular error.

Mark Scheme

3(a)	Use of $z^* = x - \mathrm{i}y$ $z - 3\mathrm{i}z^* = x + \mathrm{i}y - 3\mathrm{i}x - 3y$ $\mathrm{R} = x - 3y$, $\mathrm{I} = -3x + y$	M1 m1 A1	 3	 Condone sign error here Condone inclusion of i in I Allow if correct in (b)
(b)	$x - 3y = 16,\ -3x + y = 0$ Elimination of x or y $z = -2 - 6\mathrm{i}$	M1 m1 A1F	 3	 Accept $x = -2$, $y = -6$; ft $x + 3y$ for $x - 3y$
	Total		**6**	