Teacher Support Materials

Maths GCE

Paper Reference MD01

Question 1

1 Six people, A, B, C, D, E and F, are to be matched to six tasks, 1, 2, 3, 4, 5 and 6. The following adjacency matrix shows the possible matching of people to tasks.

	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	0	1	0	1	1
D	0	0	0	1	0	0
E	0	1	0	0	0	1
F	0	0	0	1	1	0

(a) Show this information on a bipartite graph. *(2 marks)*

(b) At first F insists on being matched to task 4. Explain why, in this case, a complete matching is impossible. *(1 mark)*

(c) To find a complete matching F agrees to be assigned to either task 4 or task 5.

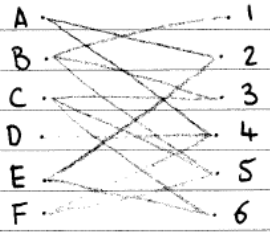
Initially B is matched to task 3, C to task 6, E to task 2 and F to task 4.

From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(6 marks)*

Student Response

Leave blank

1. a.

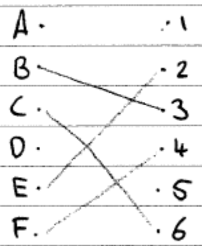


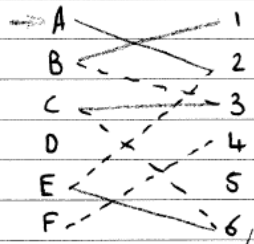
2

b. As D only can be assigned to ~~b~~ task 4, therefore if F takes task 4 a Complete matching is impossible.

1

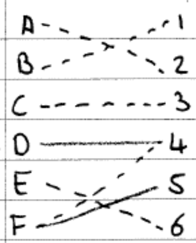
c. initial matching





A - 2 + E - 6 + C - 3 + B - 1

New initial



D - 4 + F - 5

Complete matching

A-2
B-1
C-3
D-4
E-6
F-5

6

9

Commentary

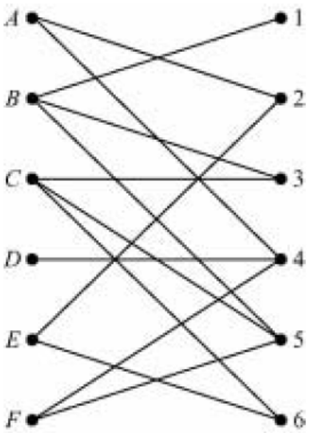
Many candidates fail to score full marks on this type of question due to poor notation. Examiners reports in the past have recommended candidates writing down their alternating path and using a diagram.

It is also essential that whenever 2 paths are required they are shown on separate diagrams.

This candidate has followed all instructions carefully and has shown his first alternating path on his diagram AND written down this alternating path. On the diagram as an edge has been added to the match it is drawn as a solid line and as an edge is removed from the match a dotted line has been used. A separate diagram has been used for each path and the solution is clear and easy to follow.

Mark scheme

MD01

Q	Solution	Marks	Total	Comments
1(a)		M1 A1	2	
(b)	D can only do 4	E1	1	Cannot be matched to task
(c)	$A-2+E-6+C-5$ $D-4+F-5+C-3+B-1$ Match $A2$, $B1$, $C3$, $D4$, $E6$, $F5$	 M1A1 M1A1 A1 B1	 6	Starting with A, D, 5, 1 First pass Second pass All Correct **Alt:1** $A-4+F-5$ $D-4+A-2+E-6+C-3+B-1$ **Alt: 2** $D-4+F-5$ $A-2+E-6+C-3+B-1$
	Total		**9**	

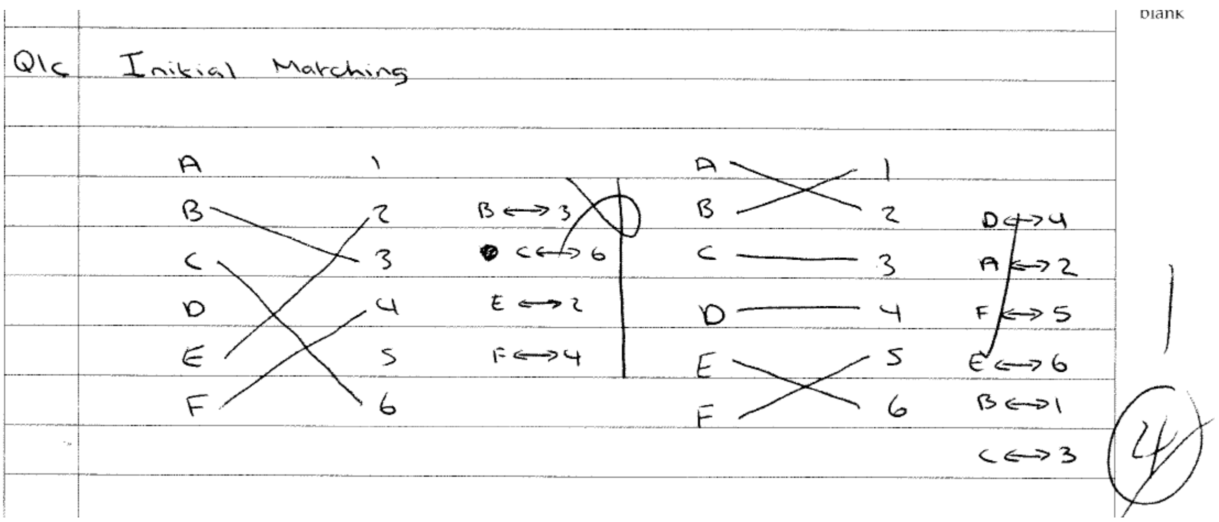
Question 1c

(c) To find a complete matching F agrees to be assigned to either task 4 or task 5.

Initially B is matched to task 3, C to task 6, E to task 2 and F to task 4.

From this initial matching, use the maximum matching algorithm to obtain a complete matching. List your complete matching. *(6 marks)*

Student response



Commentary

Many candidates think that getting a complete match is all that matters to score the marks in an exam question – it isn't!

As there are only six items to be matched to another six items this problem can be solved by inspection BUT the purpose of this module is for the students to have an understanding for the necessity for algorithms. Although this example could be solved by inspection but a similar problem of matching 100 items to 100 items could not be solved without a method.

This candidate has shown no method but has merely written down the final match.

In consequence this script has only scored one mark of the six available for this part of the question.

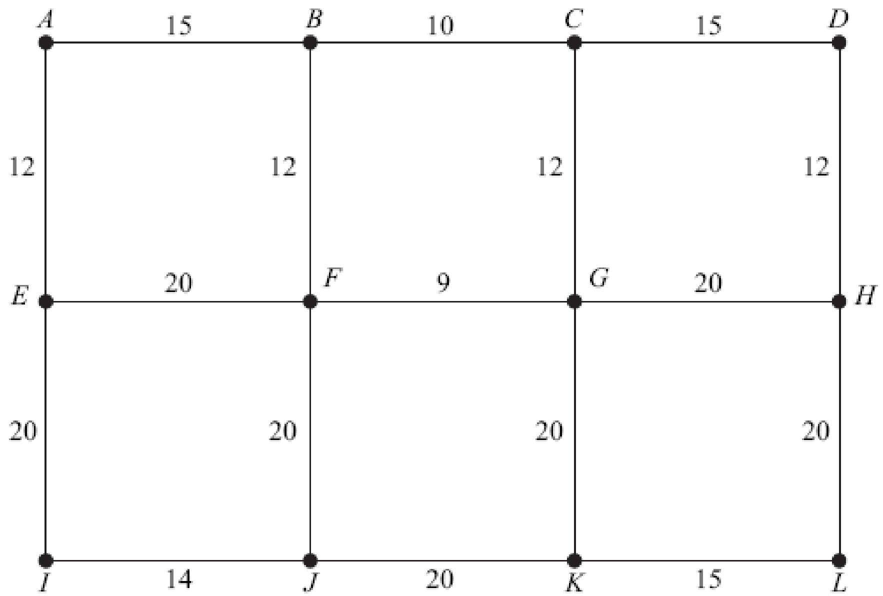
Mark Scheme

(c)	$A-2+E-6+C-5$			Starting with A, D, 5, 1
		M1A1		First pass
	$D-4+F-5+C-3+B-1$	M1A1		Second pass
		A1		All Correct
				Alt:1 $A-4+F-5$ $D-4+A-2+E-6+C-3+B-1$
	Match $A2$, $B1$, $C3$, $D4$, $E6$, $F5$	B1	6	**Alt: 2** $D-4+F-5$ $A-2+E-6+C-3+B-1$

Question 3

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The following network represents the footpaths connecting 12 buildings on a university campus. The number on each edge represents the time taken, in minutes, to walk along a footpath.



(a) (i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to walk from A to L. *(7 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) A new footpath is to be constructed. There are two possibilities:

from A to D, with a walking time of 30 minutes; or

from A to I, with a walking time of 20 minutes.

Determine which of the two alternative new footpaths would reduce the walking time from A to L by the greater amount. *(3 marks)*

Student Response

Q3

7

ai 71min

1

aii LKGFBA

ABFGKL

3

b. The footpath A to D would reduce the time by 9 min but the footpath A would only reduce the time by 2min so the path A to D would reduce the time greater.

11

Commentary

Many candidates lose marks on a Dijkstra's algorithm question due to not following the algorithm precisely.

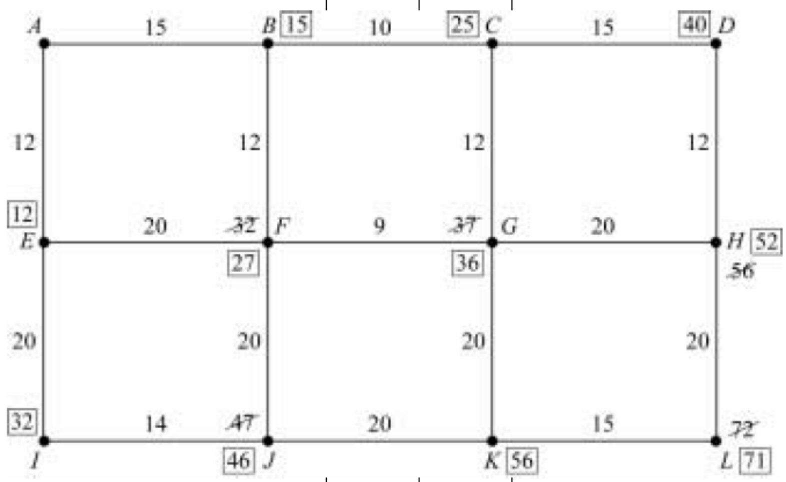
K becomes boxed with a value of 56 from G. The next vertex to be boxed is the 46 at J. From J the distance to K is 66 but this is greater than the current temporary label and as such it SHOULD NOT be recorded.

This candidate also used the notation in which the previous vertex is included. This is good practise as retracing the optimum route becomes simple.

In part (b) of the question candidates were required to amend their previous answer. A significant number of candidates failed to realise the implications of the new routes. This script clearly shows the new routes giving the new figures of 69 and 62, which meant that in the body of the script full marks were obtained. Too many candidates will 'work in their head' and write down the best answer without any justification.

Mark Scheme

MD01 (cont)

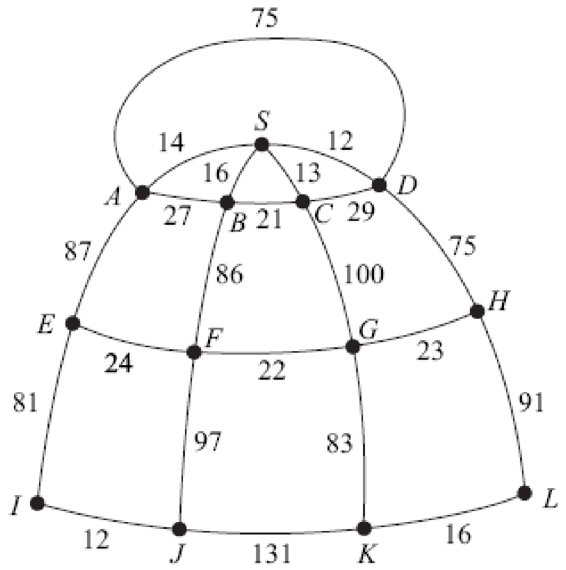
Q	Solution	Marks	Total	Comments
3(a)(i)				
		M1		SCA
		A1		Correct at F
		M1		2 values at G
		M1		2 values at J
		M1		2 values at H
		A1		All correct
	71	B1	7	
	OR			
(a)(i)	Working back from L 35 at G 47 at C 44 at F 49 at I 56 at B 64 at E 71 at A } B1 × 7			
(ii)	$A\ B\ F\ G\ K\ L$	B1	1	
(b)	ADL gives 62	M1		OE
	AIL gives 69	A1		OE Either, considering routes ADL or AIL
	$\therefore A$ to D	A1	3	CSO
	Total		**11**	

MD01

Question 4a

4 The diagram shows the various ski-runs at a ski resort. There is a shop at S. The manager of the ski resort intends to install a floodlighting system by placing a floodlight at each of the 12 points A, B,...,L and at the shop at S.

The number on each edge represents the distance, in metres, between two points.



Total of all edges $= 1135$

(a) The manager wishes to use the minimum amount of cabling, which must be laid along the ski-runs, to connect the 12 points A, B,...,L and the shop at S.

(i) Starting from the shop, and showing your working at each stage, use Prim's algorithm to find the minimum amount of cabling needed to connect the shop and the 12 points. *(5 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(3 marks)*

(iv) The manager used Kruskal's algorithm to find the same minimum spanning tree. Find the seventh and the eighth edges that the manager added to his spanning tree. *(2 marks)*

Student Response

11

(i)

SD = 12	12
DC = 29	41
CB = 21	62
BA = 27	89
AE = 87	176
EF = 24	200
FG = 22	222
GH = 23	245
HL = 91	336
LK = 16	352
KJ = 131	483
JI = 12	495

SDCBAEFGHLKJI

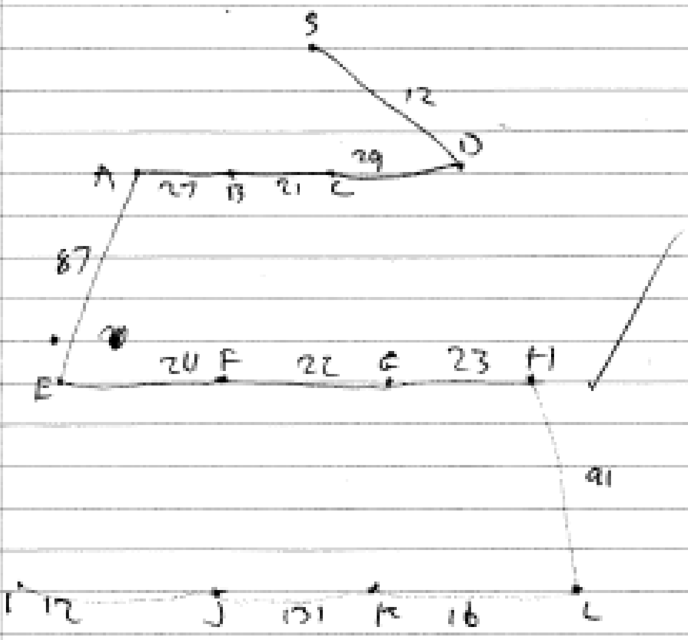
B1

1

(ii) 495 metres

0

(iii)



2

Commentary

Candidates must know the difference between all the algorithms that relate to networks.

Many candidates produced identical solutions to this script.

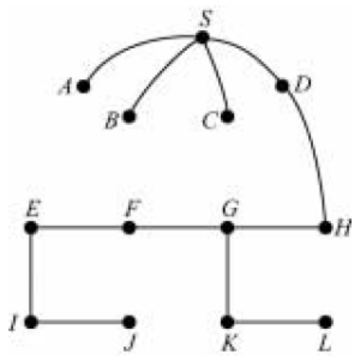
The candidate knows that 'cycles' are not allowed but hasn't understood Prim's algorithm and has produced a path starting at vertex S and finishing at vertex I.

The script did score 1 mark in part (i) for having the correct number of edges, and 2 marks in part (iii) for having a spanning tree again with the correct number of edges.

Overall a return of 3 marks out of a possible 9 was a poor return. Centres must ensure that all candidates have a good knowledge of all algorithms and when they are to be applied.

Mark Scheme

D01 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	*SD* 12	M1		Prim's (first 4 edges, allow 1 slip)
	SC 13			
	SA 14	B1		12 edges
	SB 16			
	DH 75			
	HG 23	A1		*HG* 6th
	GF 22			
	FE 24			
	EI 81	A1		*EI* 9th
	IJ 12			
	GK 83			
	KL 16	B1	5	All correct
(ii)	391	B1	1	
(iii)		M1		MST (10 + edges)
		A1		12 edges
		A1	3	All correct
(iv)	*GF* 7th (22)	B1		
	HG 8th (23)	B1	2	

Question 4b

(b) At the end of each day a snow plough has to drive at least once along each edge shown in the diagram in preparation for the following day's skiing. The snow plough must start and finish at the point L.

Use the Chinese Postman algorithm to find the minimum distance that the snow plough must travel. *(6 marks)*

Student Response

b) Odd vertices ✓ E1

HE = 69 EJ = 93 HJ = 142 M0

JK = 131 HK = ~~107~~ 106 KE = 129 A0

minimum distance = 1135 + 69 + 131 + 93 + 106 + 142 + 129

= 1805 B0

Commentary

Candidates have, in general, made great improvements in answering Chinese postman questions. There are some who are still not providing a detailed solution. The specification states that the maximum number of odd vertices in a problem will be 4, and there are 3 ways of pairing these vertices.

Candidates must list the 3 possible pairings and find the TOTAL of each of these pairings otherwise full marks cannot be obtained.

This candidate realises that the problem is to do with odd vertices and has listed the 6 edges that pair the 'odds', however the candidate has not realised the implication of the vertices.

The script scored 1 mark for use of odd edges but has not scored the method mark for attempt at correct pairings and in total has only scored 1 of the 6 marks available.

Mark Scheme

(b)	Odd vertices (E, H, J, K) $EH + JK = 69 + 131 = (200)$ $EJ + HK = 93 + 106 = (199)$ $EK + JH = 129 + 142 = (271)$ Repeat $EJ + HK$ Total 1135 + 199 = 1334	E1 M1 A3,2, 1,0 B1	 6	PI 2 correct sets of pairings
	Total		**17**	

Question 5b

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

The Jolly Company sells two types of party pack: excellent and luxury.

Each excellent pack has five balloons and each luxury pack has ten balloons.

Each excellent pack has 32 sweets and each luxury pack has 8 sweets.

The company has 1500 balloons and 4000 sweets available.

The company sells at least 50 of each type of pack and at least 140 packs in total.

The company sells x excellent packs and y luxury packs.

(a) Show that the above information can be modelled by the following inequalities.

$x + 2y \leqslant 300$, $4x + y \leqslant 500$, $x \geqslant 50$, $y \geqslant 50$, $x + y \geqslant 140$ *(4 marks)*

(b) The company sells each excellent pack for 80p and each luxury pack for £1.20. The company needs to find its minimum and maximum total income.

(i) On **Figure 2**, draw a suitable diagram to enable this linear programming problem to be solved graphically, indicating the feasible region and an objective line. *(8 marks)*

(ii) Find the company's maximum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

(iii) Find the company's minimum total income and state the corresponding number of each type of pack that needs to be sold. *(2 marks)*

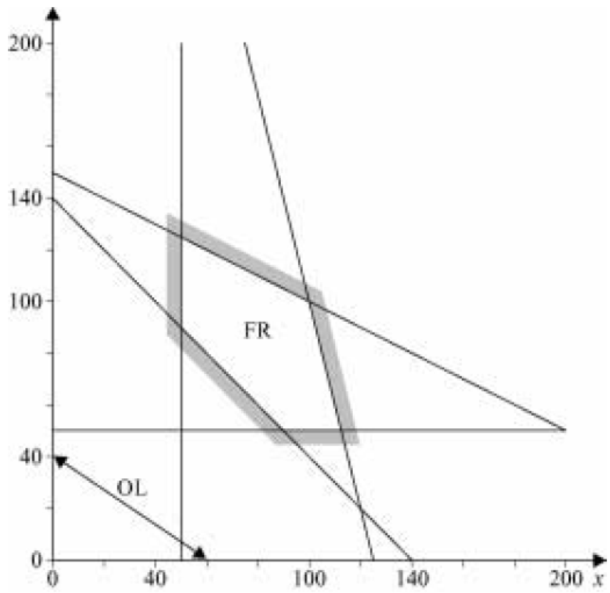
Student Response

We are unable to include the Student Response here due to copyright reasons.

Commentary

A surprising number of candidates made this mistake when squaring the equation. They had obviously been drilled that a square root produces 2 answers and applied the same principle to squaring.
This leads to 2 solutions; the correct one and one extra spurious solution, hence this candidate gained the method mark for squaring but lost both accuracy marks.

Mark Scheme

(b)(i)				
		B1		$x = 50, y = 50$
		B1		$x + y = 140$
		M1		Negative gradient (either)
		A1		$4x + y = 500$
		A1		$x + 2y = 300$
		B1		Feasible region
		M1		Objective line drawn
		A1	8	
(ii)	Maximum$(100,100)$	M1		Considering extreme point on their region
	$=£200$	A1	2	
(iii)	Minimum $(90,50)$	M1		Considering extreme minimum point on their region
	$= £132$	A1	2	

Question 6a

6 (a) Mark is staying at the Grand Hotel (G) in Oslo. He is going to visit four famous places in Oslo: Aker Brygge (A), the National Theatre (N), Parliament House (P) and the Royal Palace (R).

The figures in the table represent the walking times, in seconds, between the places.

	Grand Hotel (G)	**Aker Brygge (A)**	**National Theatre (N)**	**Parliament House (P)**	**Royal Palace (R)**
Grand Hotel (G)	–	165	185	65	160
Aker Brygge (A)	165	–	155	115	275
National Theatre (N)	185	155	–	205	125
Parliament House (P)	65	115	205	–	225
Royal Palace (R)	160	275	125	225	–

Mark is to start his tour from the Grand Hotel, visiting each place once before returning to the Grand Hotel. Mark wishes to keep his walking time to a minimum.

(i) Use the nearest neighbour algorithm, starting from the Grand Hotel, to find an upper bound for the walking time for Mark's tour. *(4 marks)*

(ii) By deleting the Grand Hotel, find a lower bound for the walking time for Mark's tour. *(5 marks)*

(iii) The walking time for an optimal tour is T seconds. Use your answers to parts (a)(i) and (a)(ii) to write down a conclusion about T. *(1 mark)*

Student Response

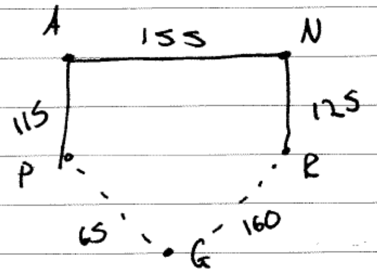
Leave blank

6a i: G, P, A, N, R, G

= 65 + 115 + 155 + 125 + 160

= 620 secs 4

ii



lower bound = 115 + 125 + 155 + 65 + 160

= 620 secs 5

iii lower bound ≤ optimal tour T ≤ upper bound

∴ 620 ≤ T ≤ 620

∴ T = 620 secs

As the values for the lower and the upper bands are the same, then T must equal 620 secs. 1

Commentary

When students are required to use the nearest neighbour algorithm many 'forget' that a tour MUST return to the start vertex.

Also when finding a lower bound by deleting a vertex many candidates fail to understand the significance of the method. i.e. that no tour can be found lower than this value BUT that the answer MAY NOT be a tour.

This candidate has produced a perfect solution that is clear and simple and shows good practise.

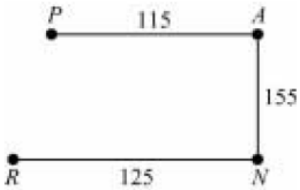
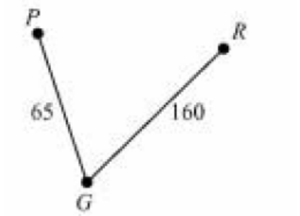
In part (i) the order of the vertices is listed together with their values.

In part (ii) the candidate has shown the minimum spanning tree after G has been deleted and has then shown the 2 shortest edges from G being added to the diagram. The significance is then obvious.

The conclusions have been written clearly.

Mark Scheme

MD01 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$G \to P \to A \to N \to R \to G$	M1		Tour
	65 115 155 125 160	M1		Visits all places
		A1		Correct order
	Total = 620	B1	4	
(ii)		M1		SCA $\big(\text{MST + extra edge(s)}\big)$
		m1		MST
		A1		
		m1		2 edges from G
	$LB = 395 + 225 = 620$	A1	5	
(iii)	$T = 620$	E1F		Their (a)(ii) $\le$ T $\le$ their (a)(i) where (a)(i) $\ge$ (a)(ii)

Question 6b

(b) Mark then intends to start from the Grand Hotel (G), visit three museums, Ibsen (I), Munch (M) and Viking (V), and return to the Grand Hotel. He uses public transport. The table shows the minimum travelling times, in minutes, between the places.

From \ To	Grand Hotel (G)	Ibsen (I)	Munch (M)	Viking (V)
Grand Hotel (G)	–	20	17	30
Ibsen (I)	15	–	32	16
Munch (M)	26	18	–	21
Viking (V)	19	27	24	–

(i) Find the length of the tour $GIMVG$. *(1 mark)*

(ii) Find the length of the tour $GVMIG$. *(1 mark)*

(iii) Find the number of different possible tours for Mark. *(1 mark)*

(iv) Write down the number of different possible tours for Mark if he were to visit n museums, starting and finishing at the Grand Hotel. *(1 mark)*

Student Response

b: GIMVG = 20 + 32 + 21 + 19
= 92 minutes

ii GVMIG = 30 + 24 + 18 + 15
= 87 minutes

2

iii directed network, number of tours = n!
= 4 × 3 × 2 × 1
= 24

0

iv ~~n! to~~ number of tours = ~~n!~~ (n-1)!

2

Commentary

Two of the main topics on this module are calculus and working with natural logs.

This question brought both topics in one question.

This script had the correct answer for the first derivative and knew that for turning points the gradient had to be zero.

Also the candidate knew that the exponential function had to be dealt with. Many candidates were unsure as to how to proceed and used logs without realising the implications.

This solution showed a lack of understanding of questions involving natural logs and their inverses.

Mark Scheme

(b)(i)	92	B1	1		
(ii)	87	B1	1		
(iii)	6	B1	1		
(iv)	$n!$	B1	1		