

General Certificate of Education
June 2006
Advanced Level Examination



MATHEMATICS
Unit Pure Core 3

MPC3

Thursday 15 June 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 The curve $y = x^3 - x - 7$ intersects the x -axis at the point where $x = \alpha$.
- (a) Show that α lies between 2.0 and 2.1. *(2 marks)*
- (b) Show that the equation $x^3 - x - 7 = 0$ can be rearranged in the form $x = \sqrt[3]{x+7}$.
(1 mark)
- (c) Use the iteration $x_{n+1} = \sqrt[3]{x_n+7}$ with $x_1 = 2$ to find the values of x_2 , x_3 and x_4 , giving your answers to three significant figures. *(3 marks)*
- 2 (a) Find $\frac{dy}{dx}$ when $y = (3x - 1)^{10}$. *(2 marks)*
- (b) Use the substitution $u = 2x + 1$ to find $\int x(2x + 1)^8 dx$, giving your answer in terms of x . *(4 marks)*
- 3 (a) Solve the equation $\sec x = 5$, giving all the values of x in the interval $0 \leq x \leq 2\pi$ in radians to two decimal places. *(3 marks)*
- (b) Show that the equation $\tan^2 x = 3 \sec x + 9$ can be written as
- $$\sec^2 x - 3 \sec x - 10 = 0 \quad \text{span style="float: right;">*(2 marks)*$$
- (c) Solve the equation $\tan^2 x = 3 \sec x + 9$, giving all the values of x in the interval $0 \leq x \leq 2\pi$ in radians to two decimal places. *(4 marks)*
- 4 (a) Sketch and label on the same set of axes the graphs of:
- (i) $y = |x|$; *(1 mark)*
- (ii) $y = |2x - 4|$. *(2 marks)*
- (b) (i) Solve the equation $|x| = |2x - 4|$. *(3 marks)*
- (ii) Hence, or otherwise, solve the inequality $|x| > |2x - 4|$. *(2 marks)*

5 (a) A curve has equation $y = e^{2x} - 10e^x + 12x$.

(i) Find $\frac{dy}{dx}$. (2 marks)

(ii) Find $\frac{d^2y}{dx^2}$. (1 mark)

(b) The points P and Q are the stationary points of the curve.

(i) Show that the x -coordinates of P and Q are given by the solutions of the equation

$$e^{2x} - 5e^x + 6 = 0 \quad (1 \text{ mark})$$

(ii) By using the substitution $z = e^x$, or otherwise, show that the x -coordinates of P and Q are $\ln 2$ and $\ln 3$. (3 marks)

(iii) Find the y -coordinates of P and Q , giving each of your answers in the form $m + 12 \ln n$, where m and n are integers. (3 marks)

(iv) Using the answer to part (a)(ii), determine the nature of each stationary point. (3 marks)

6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_1^5 \ln x \, dx$, giving your answer to three significant figures. (3 marks)

(b) (i) Given that $y = x \ln x$, find $\frac{dy}{dx}$. (2 marks)

(ii) Hence, or otherwise, find $\int \ln x \, dx$. (2 marks)

(iii) Find the exact value of $\int_1^5 \ln x \, dx$. (2 marks)

7 (a) Given that $z = \frac{\sin x}{\cos x}$, use the quotient rule to show that $\frac{dz}{dx} = \sec^2 x$. (3 marks)

(b) Sketch the curve with equation $y = \sec x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. (2 marks)

(c) The region R is bounded by the curve $y = \sec x$, the x -axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x -axis, giving your answer to three significant figures. (3 marks)

Turn over ►

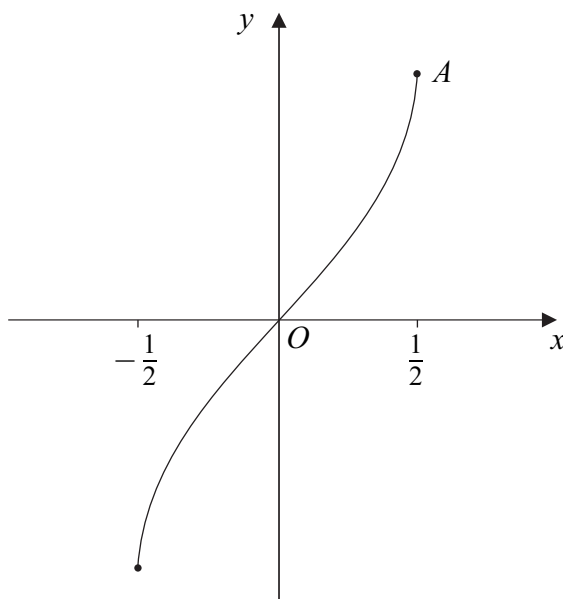
8 A function f is defined by $f(x) = 2e^{3x} - 1$ for all real values of x .

(a) Find the range of f . (2 marks)

(b) Show that $f^{-1}(x) = \frac{1}{3} \ln \left(\frac{x+1}{2} \right)$. (3 marks)

(c) Find the gradient of the curve $y = f^{-1}(x)$ when $x = 0$. (4 marks)

9 The diagram shows the curve with equation $y = \sin^{-1} 2x$, where $-\frac{1}{2} \leq x \leq \frac{1}{2}$.



(a) Find the y -coordinate of the point A , where $x = \frac{1}{2}$. (1 mark)

(b) (i) Given that $y = \sin^{-1} 2x$, show that $x = \frac{1}{2} \sin y$. (1 mark)

(ii) Given that $x = \frac{1}{2} \sin y$, find $\frac{dx}{dy}$ in terms of y . (1 mark)

(c) Using the answers to part (b) and a suitable trigonometrical identity, show that

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} \quad (4 \text{ marks})$$

END OF QUESTIONS