

General Certificate of Education
June 2006
Advanced Level Examination



MATHEMATICS
Unit Mechanics 2B

MM2B

Tuesday 6 June 2006 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the **blue** AQA booklet of formulae and statistical tables
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1 A particle moves in a horizontal plane, in which the unit vectors \mathbf{i} and \mathbf{j} are directed east and north respectively. At time t seconds, its position vector, \mathbf{r} metres, is given by

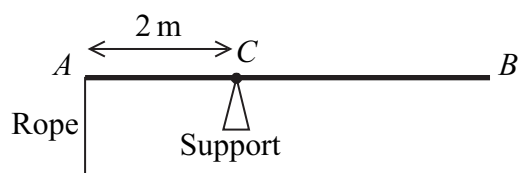
$$\mathbf{r} = (2t^3 - t^2 + 6)\mathbf{i} + (8 - 4t^3 + t)\mathbf{j}$$

- (a) Find an expression for the velocity of the particle at time t . *(3 marks)*
- (b) (i) Find the velocity of the particle when $t = \frac{1}{3}$. *(2 marks)*
- (ii) State the direction in which the particle is travelling at this time. *(1 mark)*
- (c) Find the acceleration of the particle when $t = 4$. *(3 marks)*
- (d) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 4$. *(3 marks)*

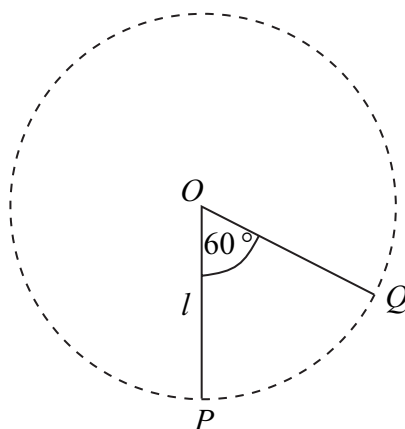
- 2 A ball of mass 0.6 kg is thrown vertically upwards from ground level with an initial speed of 14 m s^{-1} .

- (a) Calculate the initial kinetic energy of the ball. *(2 marks)*
- (b) Assuming that no resistance forces act on the ball, use an energy method to find the maximum height reached by the ball. *(3 marks)*
- (c) An experiment is conducted to confirm the maximum height for the ball calculated in part (b). In this experiment the ball rises to a height of only 8 metres.
- (i) Find the work done against the air resistance force that acts on the ball as it moves. *(3 marks)*
- (ii) Assuming that the air resistance force is constant, find its magnitude. *(2 marks)*
- (d) Explain why it is **not** realistic to model the air resistance as a constant force. *(1 mark)*

- 3 The diagram shows a uniform rod, AB , of mass 10 kg and length 5 metres . The rod is held in equilibrium in a horizontal position, by a support at C and a light vertical rope attached to A , where AC is 2 metres .



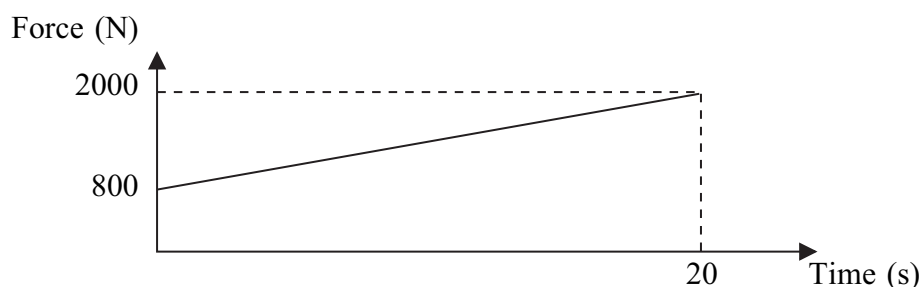
- (a) Draw and label a diagram to show the forces acting on the rod. (1 mark)
- (b) Show that the tension in the rope is 24.5 N . (3 marks)
- (c) A package of mass $m\text{ kg}$ is suspended from B . The tension in the rope has to be doubled to maintain equilibrium.
- (i) Find m . (4 marks)
- (ii) Find the magnitude of the force exerted on the rod by the support. (3 marks)
- (d) Explain how you have used the fact that the rod is uniform in your solution. (1 mark)
- 4 A particle of mass m is suspended from a fixed point O by a light inextensible string of length l . The particle hangs in equilibrium at the point P vertically below O . The particle is then set into motion with a horizontal velocity U so that it moves in a complete vertical circle with centre O . The point Q on the circle is such that $\angle POQ = 60^\circ$, as shown in the diagram.



- (a) Find, in terms of g , l and U , the speed of the particle at Q . (4 marks)
- (b) Find, in terms of g , l , m and U , the tension in the string when the particle is at Q . (5 marks)
- (c) Find, in terms of g , l , m and U , the tension in the string when the particle returns to P . (2 marks)

Turn over ►

- 5 The graph shows a model for the resultant horizontal force on a car, which varies as it accelerates from rest for 20 seconds. The mass of the car is 1200 kg.



- (a) The acceleration of the car at time t seconds is $a \text{ m s}^{-2}$. Show that

$$a = \frac{2}{3} + \frac{t}{20}, \quad \text{for } 0 \leq t \leq 20 \quad (5 \text{ marks})$$

- (b) Find an expression for the velocity of the car at time t . (3 marks)
- (c) Find the distance travelled by the car in the 20 seconds. (4 marks)
- (d) An alternative model assumes that the resultant force increases uniformly from 900 to 2100 newtons during the 20 seconds. Which term in your expression for the velocity would change as a result of this modification? Explain why. (2 marks)

- 6 A car of mass 1200 kg travels round a roundabout on a horizontal, circular path at a constant speed of 14 m s^{-1} . The radius of the circle is 50 metres. Assume that there is no resistance to the motion of the car and that the car can be modelled as a particle.

- (a) A friction force, directed towards the centre of the roundabout, acts on the car as it moves. Show that the magnitude of this friction force is 4704 N. (4 marks)
- (b) The coefficient of friction between the car and the road is μ . Show that $\mu \geq 0.4$. (3 marks)

- 7 A particle of mass 20 kg moves along a straight horizontal line. At time t seconds the velocity of the particle is $v \text{ m s}^{-1}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is 25 m s^{-1} .

- (a) Show that, at time t

$$v = \left(\frac{20 - t}{4} \right)^2 \quad (7 \text{ marks})$$

- (b) State the value of t when the particle comes to rest. (1 mark)

END OF QUESTIONS