General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS
Unit Mechanics 2A

MM2A/W

Tuesday 6 June 2006 1.30 pm to 2.45 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2A/W.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Mechanics 2A has a **written paper and coursework**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves in a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively. At time t seconds, its position vector, **r** metres, is given by

$$\mathbf{r} = (2t^3 - t^2 + 6)\mathbf{i} + (8 - 4t^3 + t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

(b) (i) Find the velocity of the particle when $t = \frac{1}{3}$. *(2 marks)*

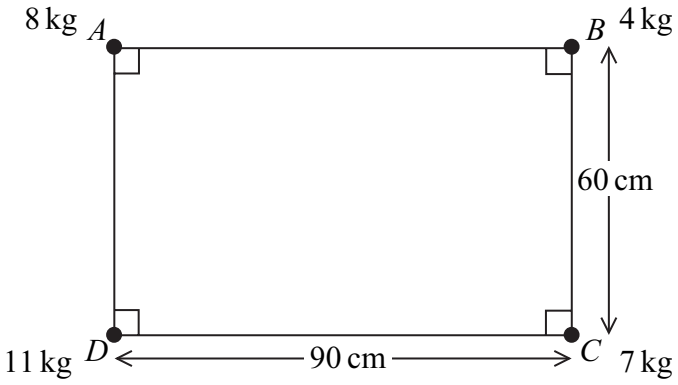
(ii) State the direction in which the particle is travelling at this time. *(1 mark)*

(c) Find the acceleration of the particle when $t = 4$. *(3 marks)*

(d) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 4$. *(3 marks)*

2 Particles of masses 8 kg, 4 kg, 7 kg and 11 kg are attached to the vertices A, B, C and D respectively of a light, rigid, rectangular framework $ABCD$.

In the framework, $CD = 90$ cm and $BC = 60$ cm, as shown in the diagram.



(a) Show that the centre of mass of the system of particles is 33 cm from AD. *(3 marks)*

(b) Find the distance of the centre of mass of the system of particles from AB. *(3 marks)*

(c) The framework is freely suspended from the corner A and hangs in equilibrium. Find the angle, in degrees, between the downward vertical and the side AB. *(3 marks)*

3 A block of mass 2 kg is placed on a horizontal surface. An elastic string has natural length 0.5 metres and modulus of elasticity 30 newtons. One end of the string is fixed to the surface at the point O and the other end is attached to the block. The block is pulled along the surface away from O until it is at the point P, where the length of OP is 1.8 metres. The block can be modelled as a particle.

(a) Calculate the elastic potential energy in the string when the block is at P. *(2 marks)*

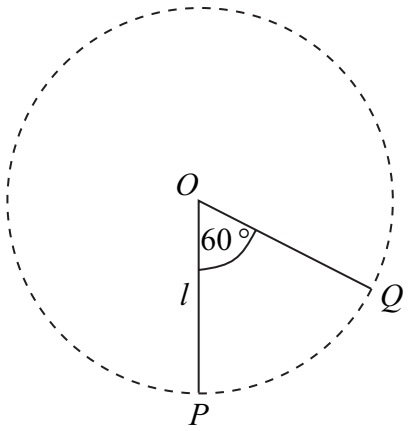
(b) Assume that the horizontal surface is smooth.

(i) The block is then released from P and moves towards O. Show that, when the block has moved 0.5 metres, its speed is $5.61\ \mathrm{m\,s^{-1}}$, correct to three significant figures. *(5 marks)*

(ii) Find the speed of the block when it reaches O. *(3 marks)*

(c) Assume that the horizontal surface is rough and that the coefficient of friction between the surface and the block is 0.1. Find the speed of the block when it reaches O. *(5 marks)*

4 A particle of mass m is suspended from a fixed point O by a light inextensible string of length l. The particle hangs in equilibrium at the point P vertically below O. The particle is then set into motion with a horizontal velocity U so that it moves in a complete vertical circle with centre O. The point Q on the circle is such that $\angle POQ = 60°$, as shown in the diagram.



(a) Find, in terms of g, l and U, the speed of the particle at Q. *(4 marks)*

(b) Find, in terms of g, l, m and U, the tension in the string when the particle is at Q. *(5 marks)*

Turn over for the next question

5 A car of mass 1200 kg travels round a roundabout on a horizontal, circular path at a constant speed of $14\,\mathrm{m\,s^{-1}}$. The radius of the circle is 50 metres. Assume that there is no resistance to the motion of the car and that the car can be modelled as a particle.

(a) A friction force, directed towards the centre of the roundabout, acts on the car as it moves. Show that the magnitude of this friction force is 4704 N. *(4 marks)*

(b) The coefficient of friction between the car and the road is μ. Show that $\mu \geqslant 0.4$. *(3 marks)*

6 A particle of mass 20 kg moves along a straight horizontal line. At time t seconds the velocity of the particle is $v\,\mathrm{m\,s^{-1}}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is $25\,\mathrm{m\,s^{-1}}$.

(a) Show that, at time t

$$v = \left(\frac{20 - t}{4}\right)^2$$

(7 marks)

(b) State the value of t when the particle comes to rest. *(1 mark)*

END OF QUESTIONS