General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS
Unit Mechanics 4

MM04

Wednesday 21 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM04.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Three forces $2\mathbf{i}$, $3\mathbf{i} - 5\mathbf{j} + a\mathbf{k}$ and $b\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$ act at the points with coordinates $(1, 1, 0)$, $(0, 0, 0)$ and $(-1, 2, 1)$ respectively, where a and b are constants.

Given that the three forces form a couple, find:

(a) the values of a and b; *(2 marks)*

(b) the moment of the couple. *(4 marks)*

2 Forces $\begin{bmatrix} 8 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 6 \\ 5 \end{bmatrix}$, $\begin{bmatrix} -2 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$ act at the points with coordinates $(0, 0)$, $(0, 3)$, $(3, 4)$ and $(4, 0)$ respectively.

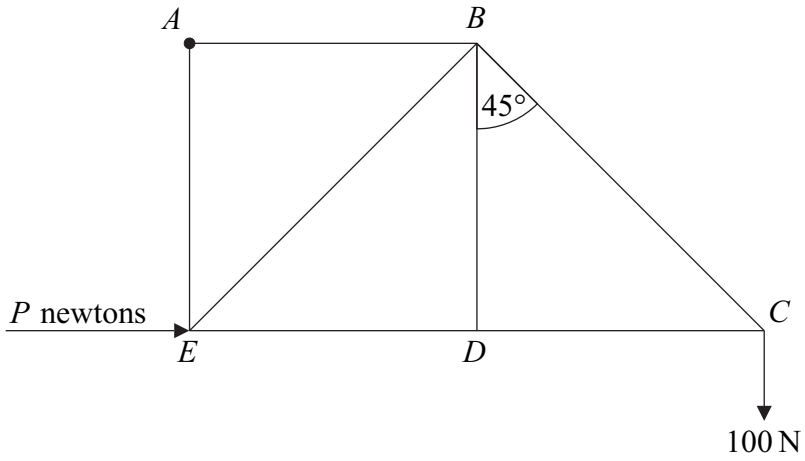
(a) (i) Find the magnitude of the resultant $\mathbf{F}$ of this system of forces. *(3 marks)*

(ii) Show that the line of action of $\mathbf{F}$ cuts the y-axis at the point $(0, 2)$. *(4 marks)*

(b) The system of forces is equivalent to a force acting at the origin together with a couple C. Write down the magnitude of C and indicate its sense on a diagram. *(2 marks)*

3 A framework is composed of seven light smoothly-jointed rods AB, AE, BE, BD, ED, BC and DC, so that $ABDE$ is a square and BDC is a right-angled triangle. The rod AB has length l and angle $CBD = 45°$.

The framework is in a vertical plane and is freely hinged at A to a fixed support. A vertical force of 100 N acts at C. The rod AE is kept vertical by a horizontal force of magnitude P newtons applied at E, as shown in the diagram.



(a) Show that $P = 200$. *(2 marks)*

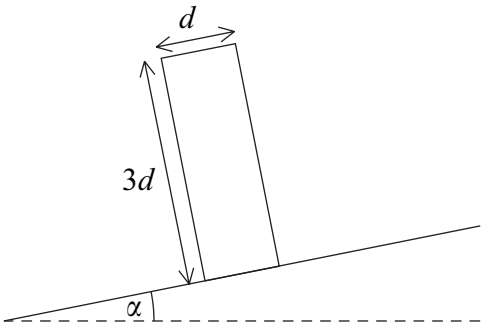
(b) (i) Find the magnitude of the reaction force on the framework at A. *(2 marks)*

(ii) Find the angle between this reaction force and the horizontal, giving your answer to the nearest degree. *(1 mark)*

(c) Find the magnitudes of the forces in each of the rods AB, AE and BE, stating whether they are in tension or compression. *(5 marks)*

Turn over for the next question

4 A uniform solid circular cylinder is in equilibrium with one plane face on a rough inclined plane. The plane is inclined to the horizontal at an angle α degrees, which can be varied. The cylinder has weight W, diameter d and height $3d$.



(a) Draw a diagram showing the forces acting on the cylinder. *(2 marks)*

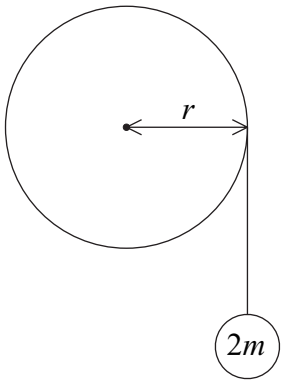
(b) If the plane is sufficiently rough to prevent sliding, find the maximum value of α for the cylinder to remain in equilibrium. *(3 marks)*

(c) The coefficient of friction between the cylinder and the plane is $\frac{2}{9}$. If the value of α is gradually increased from zero, show that the cylinder will slide before it topples. *(5 marks)*

5 A light inextensible string is wrapped several times around a uniform cylinder and a particle of mass $2m$ is attached to the free end of the string.

The cylinder, of radius r, is free to rotate about a smooth fixed horizontal axis through its centre, perpendicular to its plane face. The moment of inertia of the cylinder about this axis is $4mr^2$.

The system is released from rest with the particle hanging freely. After time t, the cylinder has turned through an angle θ radians. Assume that during this subsequent motion no slipping of the string occurs.

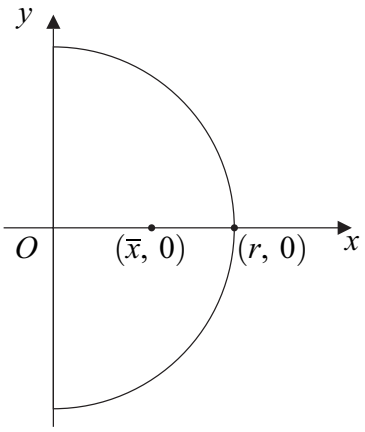


(a) Show that $\ddot{\theta} = \dfrac{g}{3r}$. *(6 marks)*

(b) Hence find an expression for the tension in the string in terms of m and g. *(1 mark)*

Turn over for the next question

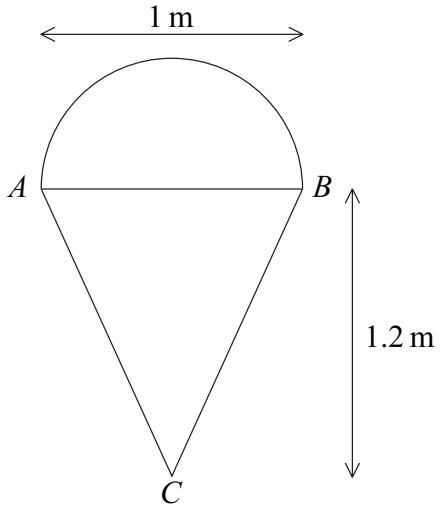
6 (a) A uniform semicircular lamina of radius r has its centre at the origin and its axis of symmetry along Ox. The position of its centre of mass has coordinates $(\overline{x}, 0)$.



(i) Show that $\frac{1}{2}\pi r^2\overline{x} = \int_0^r 2x\sqrt{r^2 - x^2}\,\mathrm{d}x$. *(4 marks)*

(ii) Hence prove that $\overline{x} = \dfrac{4r}{3\pi}$. *(3 marks)*

(b) The diagram below shows a uniform lamina, used as an advertising feature in a local supermarket to promote ice cream. It consists of a semicircle, of diameter $AB = 1$m, and an isosceles triangle ABC, where C is at a distance 1.2 m from AB.



(i) State the distance of the centre of mass of the **triangle** from C. *(1 mark)*

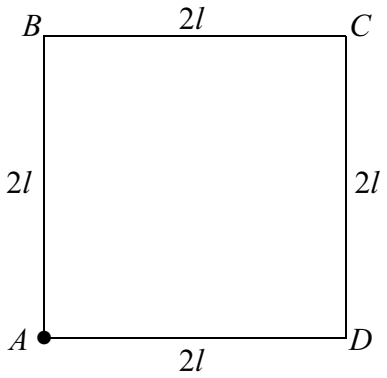
(ii) Show that the distance of the centre of mass of the **semicircle** from C is approximately 1.41 m. *(1 mark)*

(iii) Find the distance of the centre of mass of the complete lamina from C. *(4 marks)*

(c) The lamina is freely suspended from A. Find the angle that AB makes with the vertical through A, giving your answer to the nearest degree. *(3 marks)*

7 A rigid square framework $ABCD$ is formed from four identical uniform rods. Each rod has length $2l$ and mass m.

The framework can rotate freely in a vertical plane about a horizontal axis through A perpendicular to the plane of the square $ABCD$.



(a) Show that the moment of inertia of the **rod *BC*** about the axis is $\frac{16ml^2}{3}$. *(4 marks)*

(b) Particles of masses $4m$, $3m$, $2m$ and m are fixed at the vertices A, B, C and D respectively.

Show that the moment of inertia of the whole system about the axis through A is $\frac{136ml^2}{3}$. *(6 marks)*

(c) The system is released from rest with AD horizontal and B vertically **above** A. Find, in terms of g and l, the angular velocity of the system when B is vertically **below** A. *(7 marks)*

END OF QUESTIONS

There are no questions printed on this page