General Certificate of Education
January 2006
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM2A/W**
Unit Mechanics 2A

Monday 16 January 2006 9.00 am to 10.15 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

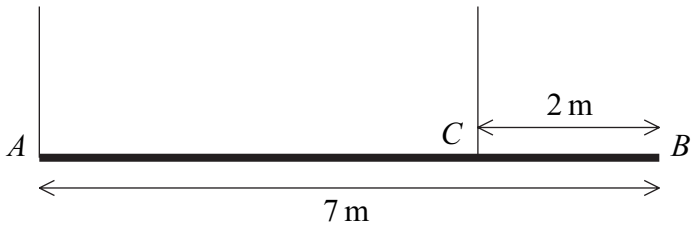
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2A/W.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- Unit Mechanics 2A has a **written paper and coursework**.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

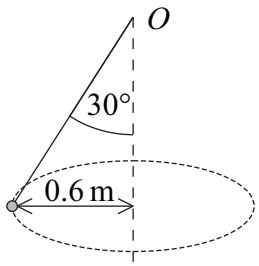
Answer **all** questions.

1 A uniform beam, AB, has mass 20 kg and length 7 metres. A rope is attached to the beam at A. A second rope is attached to the beam at the point C, which is 2 metres from B. Both of the ropes are vertical. The beam is in equilibrium in a horizontal position, as shown in the diagram.



Find the tensions in the two ropes. *(6 marks)*

2 A particle, of mass 2 kg, is attached to one end of a light inextensible string. The other end is fixed to the point O. The particle is set into motion, so that it describes a horizontal circle of radius 0.6 metres, with the string at an angle of 30° to the vertical. The centre of the circle is vertically below O.



(a) Show that the tension in the string is 22.6 N, correct to three significant figures. *(3 marks)*

(b) Find the speed of the particle. *(4 marks)*

3 A particle moves in a straight line and at time t has velocity v, where

$$v = 2t - 12e^{-t}, \quad t \geqslant 0$$

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) State the range of values of the acceleration of the particle. *(3 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

4 A car has a maximum speed of $42\,\text{m s}^{-1}$ when it is moving on a horizontal road. When the speed of the car is $v\,\text{m s}^{-1}$, it experiences a resistance force of magnitude $30v$ newtons.

(a) Show that the maximum power of the car is 52 920 W. *(2 marks)*

(b) The car has mass 1200 kg. It travels, from rest, up a slope inclined at 5° to the horizontal.

(i) Show that, when the car is travelling at its maximum speed $V\,\text{m s}^{-1}$ up the slope,

$$V^2 + 392 \sin 5° V - 1764 = 0$$ *(4 marks)*

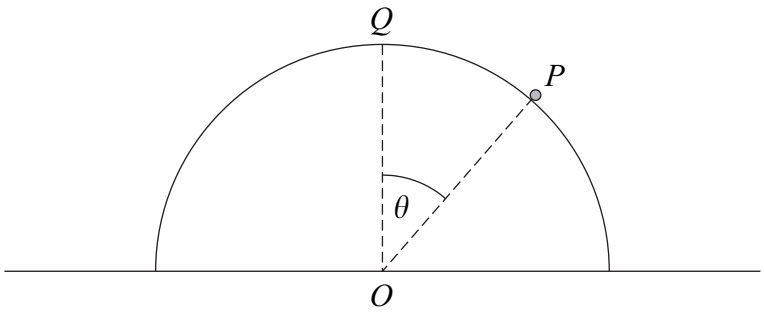
(ii) Hence find V. *(2 marks)*

5 A car, of mass 1600 kg, is travelling along a straight horizontal road at a speed of $20\,\text{m s}^{-1}$ when the driving force is removed. The car then freewheels and experiences a resistance force. The resistance force has magnitude $40v$ newtons, where $v\,\text{m s}^{-1}$ is the speed of the car after it has been freewheeling for t seconds.

Find an expression for v in terms of t. *(7 marks)*

Turn over for the next question

6 A particle P, of mass m kg, is placed at the point Q on the top of a smooth upturned hemisphere of radius 3 metres and centre O. The plane face of the hemisphere is fixed to a horizontal table. The particle is set into motion with an initial horizontal velocity of $2\,\mathrm{m\,s^{-1}}$. When the particle is on the surface of the hemisphere, the angle between OP and OQ is θ and the particle has speed $v\,\mathrm{m\,s^{-1}}$.



(a) Show that $v^2 = 4 + 6g(1 - \cos\theta)$. *(4 marks)*

(b) Find the value of θ when the particle leaves the hemisphere. *(5 marks)*

7 A particle, of mass 10 kg, is attached to one end of a light elastic string of natural length 0.4 metres and modulus of elasticity 100 N. The other end of the string is fixed to the point O.

(a) Find the length of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(b) The particle is pulled down and held at a point P, which is 1 metre vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 45 J. *(2 marks)*

(c) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\mathrm{m\,s^{-1}}$ when it is x metres **below *O***.

(i) Show that, while the string is taut,

$$v^2 = 39.6x - 25x^2 - 14.6$$ *(7 marks)*

(ii) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(3 marks)*

END OF QUESTIONS