General Certificate of Education
January 2006
Advanced Level Examination

**MATHEMATICS** **MFP3**
**Unit Further Pure 3**

Friday 27 January 2006 1.30 pm to 3.00 pm

**For this paper you must have:**
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

**Information**
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

**1** (a) Find the roots of the equation $m^2 + 2m + 2 = 0$ in the form $a + ib$. *(2 marks)*

(b) (i) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 4x$$

*(6 marks)*

(ii) Hence express $y$ in terms of $x$, given that $y = 1$ and $\frac{dy}{dx} = 2$ when $x = 0$. *(4 marks)*

**2** (a) Find $\int_0^a xe^{-2x}\,dx$, where $a > 0$. *(5 marks)*

(b) Write down the value of $\lim_{a\to\infty} a^k e^{-2a}$, where $k$ is a positive constant. *(1 mark)*

(c) Hence find $\int_0^\infty xe^{-2x}\,dx$. *(2 marks)*

**3** (a) Show that $y = x^3 - x$ is a particular integral of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$

*(3 marks)*

(b) By differentiating $(x^2 - 1)y = c$ implicitly, where $y$ is a function of $x$ and $c$ is a constant, show that $y = \frac{c}{x^2 - 1}$ is a solution of the differential equation

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 0$$

*(3 marks)*

(c) Hence find the general solution of

$$\frac{dy}{dx} + \frac{2xy}{x^2 - 1} = 5x^2 - 1$$

*(2 marks)*

**4** (a) Use the series expansion

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$$

to write down the first four terms in the expansion, in ascending powers of $x$, of $\ln(1-x)$. *(1 mark)*

(b) The function f is defined by

$$f(x) = e^{\sin x}$$

Use Maclaurin's theorem to show that when $f(x)$ is expanded in ascending powers of $x$:

(i) the first three terms are

$$1 + x + \frac{1}{2}x^2$$ *(6 marks)*

(ii) the coefficient of $x^3$ is zero. *(3 marks)*

(c) Find

$$\lim_{x \to 0} \frac{e^{\sin x} - 1 + \ln(1-x)}{x^2 \sin x}$$ *(4 marks)*

**Turn over for the next question**

**5** (a) The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = x \ln x + \frac{y}{x}$$

and $$y(1) = 1$$

(i) Use the Euler formula

$$y_{r+1} = y_r + h\,\mathrm{f}(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(1.1)$. *(3 marks)*

(ii) Use the formula

$$y_{r+1} = y_{r-1} + 2h\,\mathrm{f}(x_r, y_r)$$

with your answer to part (a)(i) to obtain an approximation to $y(1.2)$, giving your answer to three decimal places. *(4 marks)*

(b) (i) Show that $\frac{1}{x}$ is an integrating factor for the first-order differential equation

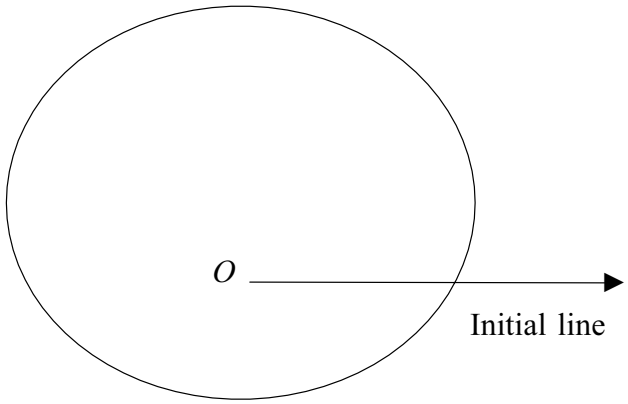
$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{1}{x}y = x \ln x$$ *(3 marks)*

(ii) Solve this differential equation, given that $y = 1$ when $x = 1$. *(6 marks)*

(iii) Calculate the value of $y$ when $x = 1.2$, giving your answer to three decimal places. *(1 mark)*

**6** (a) A circle $C_1$ has cartesian equation $x^2 + (y - 6)^2 = 36$. Show that the polar equation of $C_1$ is $r = 12 \sin \theta$. *(4 marks)*

(b) A curve $C_2$ with polar equation $r = 2 \sin \theta + 5,\ 0 \leqslant \theta \leqslant 2\pi$ is shown in the diagram.




Calculate the area bounded by $C_2$. *(6 marks)*

(c) The circle $C_1$ intersects the curve $C_2$ at the points $P$ and $Q$. Find, in surd form, the area of the quadrilateral $OPMQ$, where $M$ is the centre of the circle and $O$ is the pole. *(6 marks)*

**END OF QUESTIONS**

**There are no questions printed on this page**

**There are no questions printed on this page**

**There are no questions printed on this page**