General Certificate of Education
June 2005
Advanced Level Examination

# MATHEMATICS
# Unit Mechanics 3

**MM03**

Tuesday 28 June 2005 Afternoon Session

**In addition to this paper you will require:**
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

**Information**
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

**Advice**
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

**1** An expression for calculating the volume per second of liquid flowing through a cylindrical pipe is of the form

$$k l^{-1} r^a p^b \eta^c$$

where $k$ is a dimensionless constant;
$l$ metres is the length of the pipe;
$r$ metres is the radius of the pipe;
$p$ is the excess pressure, measured in $\mathrm{N\,m^{-2}}$;
and $\eta$ is the viscosity of the liquid measured in $\mathrm{kg\,m^{-1}s^{-1}}$.

(a) (i) Write down the dimensions of $\eta$. *(1 mark)*

(ii) Determine the dimensions of $p$. *(2 marks)*

(b) By using dimensional analysis, find the values of $a$, $b$ and $c$. *(6 marks)*

**2** The unit vectors **i** and **j** are directed due east and due north respectively.

At 1:00 am, two ships, $A$ and $B$, have position vectors $(3\mathbf{i}+2\mathbf{j})$ km and $(-4\mathbf{i}+7\mathbf{j})$ km respectively, relative to an origin at a lighthouse $O$. The ships, $A$ and $B$, are moving with constant velocities of $(-5\mathbf{i}+8\mathbf{j})\,\mathrm{km\,h^{-1}}$ and $(2\mathbf{i}+3\mathbf{j})\,\mathrm{km\,h^{-1}}$ respectively.

(a) Write down the position vectors of $A$ and $B$, relative to $O$, at time $t$ hours after 1:00 am. *(2 marks)*

(b) Show that $A$ and $B$ will collide at 2:00 am. *(2 marks)*

(c) In order to prevent a collision, ship $B$ changes its velocity to $(2\mathbf{i}+10\mathbf{j})\,\mathrm{km\,h^{-1}}$ at 1:45 am. Ship $A$ does not change its velocity.

(i) Show that, at time $T$ hours after 1:45 am, the position vector of $B$ relative to $A$ is given by $(7T-1.75)\mathbf{i}+(2T+1.25)\mathbf{j}$ km. *(6 marks)*

(ii) Find the distance between $A$ and $B$ at 2:00 am. *(2 marks)*

**3** Three smooth spheres, $A$, $B$ and $C$, of equal radii lie at rest in a straight line on a smooth horizontal table. Sphere $A$ has mass $4m$ and is set in motion with velocity $u$. It collides directly with sphere $B$ which has mass $2m$. The coefficient of restitution between $A$ and $B$ is $e$.

(a) Find, in terms of $e$ and $u$, the velocities of $A$ and $B$ immediately after the collision. *(5 marks)*

(b) Given that the magnitude of the impulse exerted by $A$ on $B$ is $\frac{12mu}{5}$, show that

$$e = \frac{4}{5}$$ *(3 marks)*

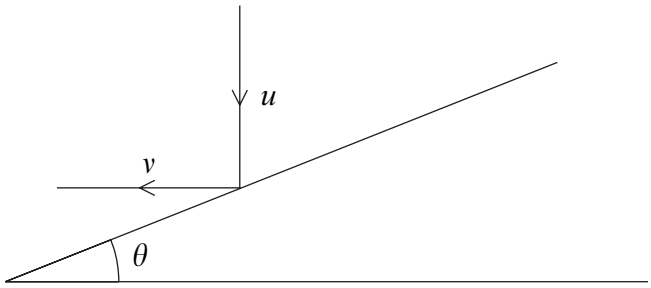
(c) After the collision between $A$ and $B$, the sphere $B$ collides directly with the sphere $C$ which has mass $m$. The coefficient of restitution between $B$ and $C$ is $\frac{4}{5}$.

Find, in terms of $u$, the velocity of $B$ after this collision. *(4 marks)*

(d) Determine whether or not $A$ and $B$ will collide again. Give a reason for your answer. *(2 marks)*

**4** A small smooth ball $B$, of mass $m$, falls vertically and collides with a fixed smooth plane inclined at an angle of $\theta$ to the horizontal. The speed of $B$ just before the collision is $u$. Immediately after the collision, $B$ moves horizontally with speed $v$.

The diagram shows the velocities of $B$ immediately before and after the collision.




(a) Briefly explain why the component of the velocity of $B$ parallel to the plane is not changed by the collision. *(2 marks)*

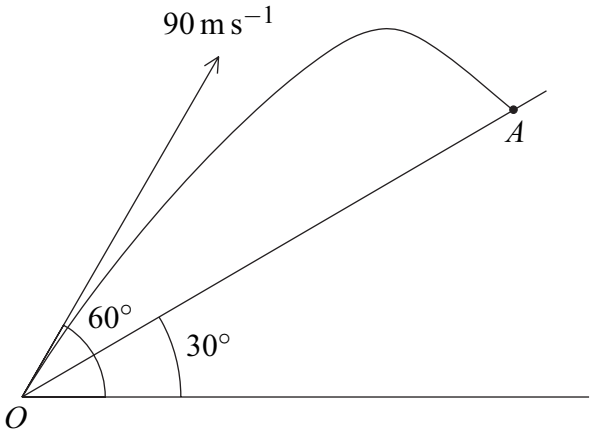
(b) Given that the coefficient of restitution between $B$ and the plane is $\frac{3}{4}$, show that

$$3u\cos\theta = 4v\sin\theta$$ *(2 marks)*

(c) Hence show that the value of $\theta$ is approximately 40.9°. *(4 marks)*

(d) Show that the magnitude of the impulse exerted by the plane on $B$ is approximately $1.32\,mu$. *(5 marks)*

**5** A projectile is fired from a point $O$ on the slope of a hill which is inclined at 30° to the horizontal. The projectile hits a target $A$. The projectile and the target are modelled as particles and the hill is modelled as a plane with $OA$ as a line of greatest slope. The projectile is launched with a velocity of $90\,\mathrm{m\,s^{-1}}$ at an inclination of 60° to the horizontal.




(a) Show that the time of the flight of the projectile is approximately 10.6 seconds. *(5 marks)*

(b) Find the distance $OA$. *(5 marks)*

(c) Find the maximum perpendicular distance of the projectile from the slope. *(4 marks)*

**6** A golf ball is driven with speed $40\,\mathrm{m\,s^{-1}}$ at an angle $\alpha$ to the horizontal from a point on a horizontal golf course. The ball travels in a vertical plane.

(a) Show that, during its flight, the horizontal and vertical displacements, $x$ metres and $y$ metres respectively, of the ball from the point of projection satisfy the equation

$$y = x\tan\alpha - \frac{gx^2}{3200}(1 + \tan^2\alpha)$$ *(5 marks)*

(b) The golf ball just clears a tree of vertical height 4 m at a horizontal distance of 100 m from its point of projection.

Find the two possible values of $\alpha$. *(6 marks)*

(c) State two modelling assumptions that you have made. *(2 marks)*

**END OF QUESTIONS**