Version : 1.0: 1105

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MFP4 Further Pure 4

Mark Scheme

2005 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MFP4

Q	Solution	Marks	Totals	Comments
1	E.g. **(1)** + **(2)** $\Rightarrow$ $5x + 8y = 15$ and **(3)**: $8x + 6y = 7$	M1		Eliminating first variable
	E.g. $40x + 64y = 120$, $40x + 30y = 35$	dM1		Solving 2×2 system
	$x = -1,\ y = 2\frac{1}{2}$	A1		Ft
	$z = 3\frac{1}{2}$	A1	4	All 3 correct
	Alt. I (Cramer's Rule) $\Delta = \begin{vmatrix} 2 & 7 & -3 \\ 3 & 1 & 3 \\ 8 & 6 & 0 \end{vmatrix}$, $\Delta_x = \begin{vmatrix} 5 & 7 & -3 \\ 10 & 1 & 3 \\ 7 & 6 & 0 \end{vmatrix}$, $\Delta_y = \begin{vmatrix} 2 & 5 & -3 \\ 3 & 10 & 3 \\ 8 & 7 & 0 \end{vmatrix}$ and $\Delta_z = \begin{vmatrix} 2 & 7 & 5 \\ 3 & 1 & 10 \\ 8 & 6 & 7 \end{vmatrix}$			
	102, – 102 , 255 and 357 respectively	B1		Any one correct
	$x = \frac{\Delta_x}{\Delta}$, $y = \frac{\Delta_y}{\Delta}$, $z = \frac{\Delta_z}{\Delta}$	M1		At least one attempted numerically
	$x = -1$, $y = 2\frac{1}{2}$, $z = 3\frac{1}{2}$	A1A1	4	Any 2 correct ft; all 3 correct
	Alt. II (Inverse matrix method) $C^{-1} = \frac{1}{102}\begin{bmatrix} -18 & -18 & 24 \\ 24 & 24 & -15 \\ 10 & 44 & -19 \end{bmatrix}$	M1 A1		M0 here if no inverse matrix is given
	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C^{-1}\begin{bmatrix} 5 \\ 10 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 \\ 2.5 \\ 3.5 \end{bmatrix}$	M1 A1	4	
			4	
2(a)	***Use of*** $r = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3-2\lambda \\ 1+6\lambda \\ -1+3\lambda \end{bmatrix}$	B1		
	Equating for λ : $\frac{x-3}{-2} = \frac{y-1}{6} = \frac{z+1}{3}$	M1 A1	3	
(b)	$\sqrt{2^2 + 6^2 + 3^2} = 7$	B1		
	d.c.'s are $\frac{-2}{7}, \frac{6}{7}$ and $\frac{3}{7}$	B1	2	Ft
	Total		**5**	

MFP4 (cont)

Q	Solution	Marks	Totals	Comments
3 (a)	Det **M** = – 15 + 12 + 0 – (– 12 + 0 – 30) = 39	M1 A1	 2	
(b)(i)	$V(S_1) = 12 \times 39 = 468$ cm^3	M1 A1	2	Ft
(ii)	$V(S_2) = 12 \times 39 \times \left(\frac{1}{3}\right)^2 = 52$ cm^3	M1 A1	2	Ft
			6	
4(a)	**A**: Reflection in $y = z$ **B**: Reflection in $y = 0$ (x–z plane)	M1 A1 M1 A1	 4	
(b)(i)	$\mathbf{AB} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$	B1 B1	 2	≥ 5 entries correct All correct
(ii)	About the x-axis; through 90°	B1 B1	2	+ /–; or 270°; or in radians
			8	
5(a)	$\vec{AB} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$, $\vec{AC} = 4\mathbf{i} - \mathbf{j} + \mathbf{k}$	B1 B1	2	Give one B1 if both $-_{ve}$ correct
(b)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 7 \\ 4 & -1 & 1 \end{vmatrix} = 10\mathbf{i} + 26\mathbf{j} - 14\mathbf{k}$	M1 A1		Ft (a)'s answers
	$d = \begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix} \bullet \begin{bmatrix} 10 \\ 26 \\ -14 \end{bmatrix} = 14$ (e.g.)	M1 A1	 4	Or divided throughout by 2 (etc.) Ft **n**
(c)	$\sin\theta / \cos\theta = \dfrac{\text{scalar product}}{\text{product of moduli}}$	M1		5**i** + **j** + **k** and 5**i** + 13**j** – 7**k** ft **n**
	Num^r. = 25 + 13 – 7 = 31	B1		Ft correct unsimplified
	Denom^r. = $\sqrt{27}.\sqrt{243} = 81$	B1		Ft both correct, unsimplified surds
	θ = 22.5°	A1	4	CAO
	Total		**10**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathbf{b}\times\mathbf{a}$ is perp^r. to both **a** and **b** Sc. prod. of two perp^r. vectors, **a** and ($\mathbf{b}\times\mathbf{a}$), is zero	B1 B1	2	Allow full co-planarity or zero volume arguments
(ii)	$\mathbf{a}\bullet(\mathbf{b}\times(\mathbf{c}+\mathbf{a}))=\mathbf{a}\bullet[\mathbf{b}\times\mathbf{c}+\mathbf{b}\times\mathbf{a}]$ $=\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})+\mathbf{a}\bullet(\mathbf{b}\times\mathbf{a})$ $=\mathbf{a}\bullet(\mathbf{b}\times\mathbf{c})$	M1 A1	2	Both brackets expanded Use of (i)'s result
(b)(i)	$\mathbf{p}\bullet(\mathbf{r}\times\mathbf{s})=\begin{vmatrix}3&4&1\\2&-5&2\\7&2&-3\end{vmatrix}=152$	M1 A1	2	Or longer alt. method; e.g. via $\mathbf{r}\times\mathbf{s}=11\mathbf{i}+20\mathbf{j}+39\mathbf{k}$
(ii)	**p**, **r**, **s** lin. indt. since $\mathbf{p}\bullet(\mathbf{r}\times\mathbf{s})\neq 0$	B1	1	
(iii)	$V = 152$	B1	1	ft (i)'s answer
(iv)	$\mathbf{t}=\mathbf{s}+\mathbf{p}\Rightarrow$ $\mathbf{p}\bullet(\mathbf{r}\times\mathbf{t})=\mathbf{p}\bullet(\mathbf{r}\times[\mathbf{s}+\mathbf{p}])$ $=\mathbf{p}\bullet(\mathbf{r}\times\mathbf{s})+\mathbf{p}\bullet(\mathbf{r}\times\mathbf{p})$ $=\mathbf{p}\bullet(\mathbf{r}\times\mathbf{s})$ since $\mathbf{p}\bullet(\mathbf{r}\times\mathbf{p})=0$ from (a)	M1 A1	2	Must expand, or identify with (a)
			10	
7(a)	$\begin{vmatrix}6.4&-7.2\\-7.2&10.6\end{vmatrix}=67.84-51.84=16$	M1 A1	2	Attempt at det.
(b)	Inv. pts. g.b. $x'=x$, $y'=y$ Subst^g. in eqns. $6.4x-7.2y=x$ $-7.2x+10.6y=y$ $y=\frac{3}{4}x$	B1 M2 A1 A1	5	
	Alt. I Char. Eqn. is $\lambda^2-17\lambda+16=0$ $\lambda=1$ or 16 $\lambda=1$ for l.o.i.p.s $\Rightarrow$ $5.4x-7.2y=0$	M1 A1 A1 M1 A1	(5)	i.e. $y=\frac{3}{4}x$ ignore $\lambda=16$ work
	Alt. II $y=mx$ a l.o.i.p.s $\begin{bmatrix}6.4&-7.2\\-7.2&10.6\end{bmatrix}\begin{bmatrix}x\\mx\end{bmatrix}=\begin{bmatrix}6.4x-7.2mx\\-7.2x+10.6mx\end{bmatrix}$ $-7.2x+10.6mx=m(6.4x-7.2mx)$ also $\Rightarrow 7.2m^2+4.2m-7.2=0$ $\Rightarrow (4m-3)(3m+4)=0$ $\Rightarrow y=\frac{3}{4}x$ or $y=-\frac{4}{3}x$ Checking which one works	B1 M1 A1 A1 B1	(5)	
	Total		**7**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b+c & c+a & a+b \\ b-c & c-a & a-b \end{vmatrix}$	M1 A1		By $R_1' = R_1 + R_2$
	$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b-c & c-a & a-b \end{vmatrix}$	A1		Factoring out correctly
	or $\begin{vmatrix} a+b+c & b & c \\ 2(a+b+c) & c+a & a+b \\ 0 & c-a & a-b \end{vmatrix}$			By $C_1' = C_1 + C_2 + C_3$ Etc.
	$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 2 & c+a & a+b \\ 0 & c-a & a-b \end{vmatrix}$			
	Method for obtaining remaining factor $\Delta = 2(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$	M1 A1	 5	 2 can be anywhere
(b)	Noting $a = a,\ b = 3,\ c = 1$ Setting $\Delta = 0$ i.e. $0 = 2(a+4)(a^2-4a+7) \Rightarrow a = -4$	B1 M1 A1		
	Showing $(a-2)^2 + 3 \neq 0$ so only one value of a	M1 A1	 5	Ignore incorrect quadratic factors until here CSO
	Alt. $\begin{vmatrix} a & 3 & 1 \\ 4 & 1+a & a+3 \\ 2 & 1-a & a-3 \end{vmatrix} = 2a^3 - 18a + 56$	B1		
	Equating to zero + solving attempt $2(a+4)(a^2-4a+7) = 0 \Rightarrow a = -4$ $(a-2)^2 + 3 > 0 \Rightarrow$ no other real solutions.	M1 A1 M1 A1	 (5)	 Or discriminant < 0
	Total		**10**	

MFP4 (cont)

Q	Solution	Marks	Total	Comments
9(a)	$\begin{bmatrix}2 & 7\\4 & k\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}9\\4+k\end{bmatrix}=\lambda\begin{bmatrix}1\\1\end{bmatrix}$ when $k=5$, $\lambda = 9$	M1 A1 A1	3	**M** × given evec. Ft
(b)	Char. Eqn. is $\lambda^2 - 7\lambda - 18 = 0$ $(\lambda - 9)(\lambda + 2) = 0$ and 2nd eval. is -2	M1 A1 A1		
	Or det **M** $= \lambda_1 \lambda_2 \Rightarrow -18 = 9\lambda_2 \Rightarrow \lambda_2 = -2$			Or via trace **M** $= \lambda_1 + \lambda_2$
	Subst[g]. $\lambda = -2 \Rightarrow 4x + 7y = 0$ $\Rightarrow$ evec. $\begin{bmatrix}7\\-4\end{bmatrix}$	M1 A1	5	
(c)	**D** = $\begin{bmatrix}-2 & 0\\0 & 9\end{bmatrix}$, **U** = $\begin{bmatrix}7 & 1\\-4 & 1\end{bmatrix}$	B1 B1	2	Ft (alternatives possible)
(d)	$\mathbf{U}^{-1} = \frac{1}{11}\begin{bmatrix}1 & -1\\4 & 7\end{bmatrix}$	B1	1	Ft non-trivial U's
(e)	$\mathbf{M}^{2n} =$ $\begin{bmatrix}7 & 1\\-4 & 1\end{bmatrix}\begin{bmatrix}(-2)^{2n} & 0\\0 & (9)^{2n}\end{bmatrix}.\frac{1}{11}\begin{bmatrix}1 & -1\\4 & 7\end{bmatrix}$	B1		For $\mathbf{D}^{2n}$
	$= \begin{bmatrix}7\times 4^n & 81^n\\-4^{n+1} & 81^n\end{bmatrix}.\frac{1}{11}\begin{bmatrix}1 & -1\\4 & 7\end{bmatrix}$ or $\begin{bmatrix}7 & 1\\-4 & 1\end{bmatrix}.\frac{1}{11}\begin{bmatrix}4^n & -4^n\\4\times 81^n & 7\times 81^n\end{bmatrix}$	M1 A1		CAO any correct form
	Thus $a = \frac{1}{11}\left\{7\times 4^n + 4\times 81^n\right\}$	A1	4	i.e. $p = \frac{7}{11}$, $q = \frac{4}{11}$
	In its original form, the question asked for the following conclusion to be made: Since a is an integer, and hcf (4, 11) = 1, $7\times 4^{n-1} + 81^n$ is a multiple of 11			
	Total		**15**	
	Total		**75**	