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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MD02 Discrete 2

Mark Scheme

2005 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme and abbreviations used in marking

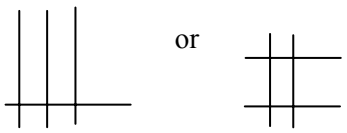
M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

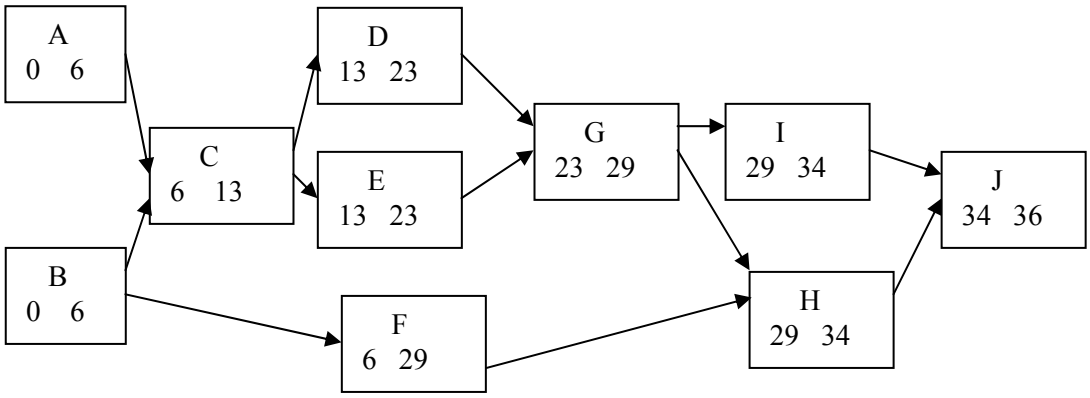
MD02

Q	Solution	Marks	Total	Comments
1(a)	Hungarian algorithm minimises	E1		
	20–x gives measure of questions not correct which needs minimising	E1	2	
(b)	3 1 2 5 4 0 2 5 1 3 7 3 3 4 6 8 4 2 5 6 6 4 5 4 5	B1		Array giving 20–x
	2 0 1 4 3 0 2 5 1 3 4 0 0 1 3 6 2 0 3 4 2 0 1 0 1	M1 A1√		Reduce rows ft their 20 – x matrix
	2 0 1 4 2 0 2 5 1 2 4 0 0 1 2 6 2 0 3 3 2 0 1 0 0	M1 A1		Reduce columns **CSO**
	Zeros can be covered with only 4 lines So adjustment by 1	M1		
	2 0 1 3 1 0 2 5 0 1 4 0 0 0 1 6 2 0 2 2 3 1 2 0 0	A1		or 1 0 1 3 1 0 3 6 1 2 3 0 0 0 1 5 2 0 2 2 2 1 2 0 0
	Matching on particular zeros	M1		If adjustment not done correctly and selection is made
	Les- Tennis Mel - Athletics Nick- Swimming Ollie- Football Pete- Golf	A1	9	3 correct matchings B1 rest correct B1 Award marks here in whichever way benefits candidate most.
	Total		**11**	

MD02 (cont)

Q	Solution	Marks	Total	Comments
2(a)	*SAET* has maximum day journey of 9 hrs whereas for *SADT* max day journey is 10 hrs	M1 A1	2	Reasonable understanding with 9 and 10 specifically mentioned
(b)				
	Stage Initial State Action Value	M1		General idea of stage and state
	1 *D* *DT* 5*			
	E *ET* 7*	A1		First stage correct (may be reversed)
	2 *A* *AD* max(10,5) =10	M1		Idea of minimax
	AE max(9,7) = 9*	A1		One pair of actions correct
	B *BD* max(9,5) = 9			
	AE max(8,7) = 8*			
	C *CD* max(10,5) =10	A1		All values in second stage correct
	CE max(9,7) = 9*			
	3 *S* *SA* max(7,9) = 9			
	SB max(8,8) = 8*			
	SC max(9,10) = 10	A1		
	Working back along * values to find Minimax route is *SBET*	M1 A1	8	All values correct at all 3 stages Complete/enumeration or network with each stage and state carefully described if no evidence of minimax Maximum mark M1, A1 Minimax route *SBET* marks may also be earned if not finding minimum time through the network. M1 A1
	Total		**10**	

MD02 (cont)

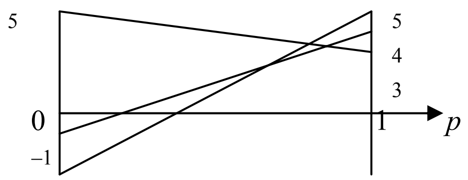
Q	Solution	Marks	Total	Comments
3				
(a)	Network	M1 A3	 4	**SCA** -1 ee
(b)	Forward pass	M1		
	All correct	A1	2	
(c)	Backward pass	M1		
	All correct	A1	2	
(d)(i)	Project completion time 36 hours	B1√	1	
(ii)	Critical path *BCEGHJ*	M1 A1		**SCA** All correct
	Earliest start + activity duration = latest finish time	E1	3	
(e)(i)	*I* now has new earliest time 29+3	M1		Extra 3 hours on edge *HI* or new activity between *H* and *I* of duration 3
	= 32	A1	2	
(ii)	*I* now becomes critical and increases *J* earliest start time to 35	M1		
	New completion time is 37 hours	A1	2	
	Total		**16**	

MD02 (Cont)

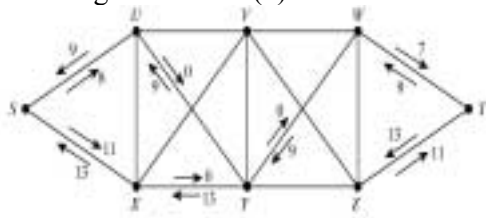
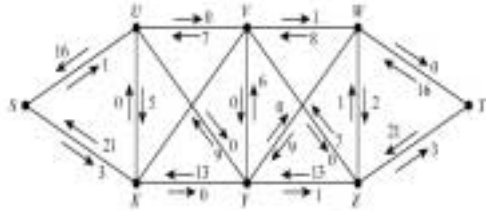
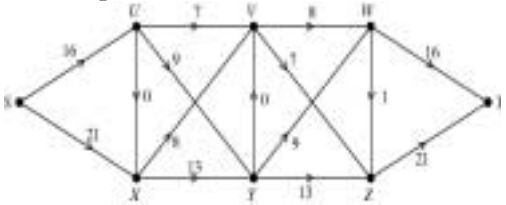
Q	Solution	Marks	Total	Comments
4(a)	$4x+5y \leqslant 36$ $2x+y \leqslant 12$ $5x+2y \leqslant 35$	M1 A1	 2	**SCA** at LHS and RHS All correct with correct inequalities
(b)(i)	P x y r s t *value* 1 -3 -2 0 0 0 0 0 4 5 1 0 0 36 0 1* $\frac{1}{2}$ 0 $\frac{1}{2}$ 0 6 0 5 2 0 0 1 35	M1		Identifying pivot and possibly dividing by 2
	1 0 $-\frac{1}{2}$ 0 $\frac{3}{2}$ 0 18 0 0 3* 1 -2 0 12 0 1 $\frac{1}{2}$ 0 $\frac{1}{2}$ 0 6 0 0 $-\frac{1}{2}$ 0 $-\frac{5}{2}$ 1 5	m1 A1		Row operations Correct tableau
	Next y pivot on 3	M1		
	1 0 0 $\frac{1}{6}$ $\frac{7}{6}$ 0 20 0 0 1 $\frac{1}{3}$ $-\frac{2}{3}$ 0 4 0 1 0 $-\frac{1}{6}$ $\frac{5}{6}$ 0 4 0 0 0 $\frac{1}{6}$ $-\frac{17}{6}$ 1 7	m1 A1		Row operations Correct tableau
	Optimal since no negative numbers in top row	B1	7	
(ii)	$P = 20$ $x = 4$, $y = 4$	B1√ B1√	 2	**FT** ONLY if no negs in top row
(iii)	$r = 0$, $s = 0$, $t = 7$ at optimum	B1√	1	
	Total		**12**	

MD02 (cont)

<table>
<tr><th>Q</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>5(a)</td><td>
<table>
<tr><td></td><td></td><td></td><td></td><td>Min</td></tr>
<tr><td></td><td>4</td><td>3</td><td>5</td><td>3*</td></tr>
<tr><td></td><td>–1</td><td>5</td><td>–2</td><td>–2</td></tr>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>1</td></tr>
<tr><td>Max</td><td>4*</td><td>5</td><td>5</td><td></td></tr>
</table>
</td><td>M1
A1</td><td></td><td>Either marginal row or column
all values correct</td></tr>
<tr><td></td><td>Since $3 \neq 4 \Rightarrow$ no stable solution</td><td>A1</td><td>3</td><td></td></tr>
<tr><td>(b)</td><td>P_1 dominates P_3 ; (1,2,3) < (4,3,5)

So it is unwise to play P_3</td><td>E1</td><td>1</td><td></td></tr>
<tr><td>(c)</td><td>P chooses P_1 with probability p
So chooses P_2 with probability $1 - p$</td><td></td><td></td><td></td></tr>
<tr><td></td><td>Expected gains when Q plays
$Q_1 : 4p - (1-p) = 5p - 1$
$Q_2 : 3p + 5(1-p) = 5 - 2p$</td><td>M1</td><td></td><td>Attempt at at least 2</td></tr>
<tr><td></td><td>$Q_3 : 5p - 2(1-p) = 7p - 2$</td><td>A1</td><td></td><td>All 3 correct (simplified)</td></tr>
<tr><td></td><td>Plot expected gains against p for
$0 \leqslant p \leqslant 1$</td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td> 
</td><td>A1</td><td></td><td></td></tr>
<tr><td></td><td>Choose highest point of region below lines</td><td></td><td></td><td></td></tr>
<tr><td></td><td>$5p - 1 = 5 - 2p$</td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td>leading to $p = \frac{6}{7}$</td><td>A1</td><td></td><td></td></tr>
<tr><td></td><td>Therefore P plays P_1 with probability $\frac{6}{7}$
and plays P_2 with probability $\frac{1}{7}$</td><td>B1√</td><td>7</td><td></td></tr>
<tr><td></td><td>**Total**</td><td></td><td>**11**</td><td></td></tr>
</table>

MD02 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	9+7+0+9+13=38	B1	1	
(ii)	Maximum flow is less than or equal to 38	M1 A1√	2	< their value of cut ⩽38 M0 for "equals" their cut
(b)	*SUYWT* flow of 9 *SXYZT* flow of 13	B1 B1	2	
(c)(i)	Indicating flows from (b) on network	M1		Preferably as backward flows
				
	with augmentation leading to something such as			
				
	Route / **Flow** *SUYWT* / 9 *SXYZT* / 13 *SUVWT* / 7 *SXVZT* / 7 *SXVWZT* / 1			
	Flow augmentation (many possibilities)			
	SUVWT 7 *SXVZT* 7 *SXVWZT* 1	M1A1 m1A1 A1	6	
(ii)	Network showing maximum flow Several possibilities	B1		
				Or {*S*, *U*, *X* / *V*, *W*, *Y*, *Z*, *T*}
	Maximum flow is 37	B1	2	
(iii)	Attempt to find cut through saturated arcs Cut through *UV, UY, XV, XY*	M1 A1	2	
	Total		**15**	
	Total		**75**	