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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

MATHEMATICS

GCE Advanced Subsidiary Level

Paper 8719/03

Paper 3

General comments

There was a considerable variety of standard of work by candidates on this Paper and a corresponding very wide spread of marks from zero to full marks. The Paper appeared to be accessible to candidates who were well prepared and no question seemed to be of undue difficulty. Moreover adequately prepared candidates seemed to have sufficient time to attempt all questions. However there were some very weak, often untidy, scripts from candidates who clearly lacked the preparation necessary for work at the level demanded by this Paper. All questions discriminated to some extent. The questions or parts of questions on which candidates generally scored highly were **Question 2** (integration by parts), **Question 8 (i)** (stationary point) and **(iii)** (iteration), and **Question 9 (i)** (vector geometry). Those on which scores were low were **Question 4 (ii)** (algebra), **Question 5** (complex numbers), **Question 6 (ii)** (series expansion) and **Question 10 (iii)** (trigonometrical integral).

The detailed comments that follow inevitably refer to common errors and can lead to a cumulative impression of poor work on a difficult Paper. In fact there were many scripts showing a good and sometimes excellent understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions, it should be understood that alternative forms are often possible and that the form given is not necessarily the sole 'correct answer'.

Comments on specific questions

Question 1

Errors of sign and in the values of cos 60°, sin 60°, cos 30°, sin 30°, prevented some candidates from reaching an equation in cos x only, but generally this question was well answered.

Answer: **(ii)** 125.3°.

Question 2

Most candidates approached the integration by parts correctly. Errors in integrating e^{2x}, and in simplification were quite frequent, but the main source of error was the failure to appreciate the correct meaning in this context of the adjective 'exact'.

Answer: $\frac{1}{4}(e^2 + 1)$.

Question 3

Very few candidates realised that $x = 1$ was the only critical value in relation to this inequality. Many answers involved a further value, usually $x = \frac{5}{3}$. Examiners also noted that, while some candidates investigated the related inequality obtained by squaring both sides of the given inequality, a substantial number dropped the modulus sign and mistakenly squared only one side. Fully correct solutions were rare and often obtained with the assistance of a sketch graph.

Answer: $x < 1$.

Question 4

Many candidates answered part **(i)** well, using either the factor theorem with $x = 2$, or long division.

There were very few completely satisfactory solutions to part **(ii)**. By contrast, there were many fallacious attempts at a proof, e.g. those based on a set of instances of non-negative values of f(x), or the claim that f(2) = 0 and 2 > 0 together implied that f(x) > 0 for all x. The three correct methods seen were arguments based on (a) an exhaustive discussion of the stationary points and graph of $y = \mathrm{f}(x)$, (b) a discussion of the nature of the zeros of $x^2 - 4x + 4$ and $x^2 + 2x + 2$ together with a proof that both expressions only took non-negative or positive values, and (c) completing the squares and writing f(x) as $(x - 2)^2\ ((x + 1)^2 + 1)$. Most attempts to use method (a) or (b) omitted some essential detail. Method (c) was usually successfully completed.

Answer: **(i)** 8.

Question 5

Though some candidates found this question quite straightforward, it was generally poorly answered. Given the modulus and argument of the complex number w, many candidates were unable to state it in the form $x + \mathrm{i}y$ immediately. Thus they embarked on a lengthy search based on $x^2 + y^2 = 1$ and $x : y = \cos\frac{2}{3}\pi : \sin\frac{2}{3}\pi$, and quite frequently arrived at a wrong answer. Whatever the outcome, multiplication of 2i by w was often incorrectly done and though most knew how to divide 2i by w, errors, particularly of sign, were common. The plotting of points on an Argand diagram was usually well done, but part **(iii)** was only accessible to those who had completed part **(i)** correctly. The most common method here was to show that $UA = AB = BU$.

Answers: **(i)** $\cos\frac{2}{3}\pi + \mathrm{i}\sin\frac{2}{3}\pi$, $-\sqrt{3} - \mathrm{i}$, $\sqrt{3} - \mathrm{i}$.

Question 6

In part **(i)**, most candidates started out with an appropriate form of partial fractions with three unknown constants. Errors in identifying the numerator of f(x) with that of the combined fractions proved costly. A thorough check of the algebraic work at this stage would have helped. Indeed since full marks in part **(ii)** were clearly dependent on accurate work earlier, regular checks during part **(i)** were desirable, for example when setting up simultaneous equations in the unknowns or when evaluating expressions. A fairly common error was to start with an inappropriate form of fractions.

Examiners were disappointed to see so many poor attempts at part **(ii)**. Whereas most candidates could expand $(1 + 2x)^{-1}$ correctly, very few could deal with $(x - 2)^{-1}$ or $(x - 2)^{-2}$ accurately.

Answer: **(i)** $\frac{1}{2x+1} + \frac{4}{x-2} + \frac{8}{(x-2)^2}$ or $\frac{1}{2x+1} + \frac{4x}{(x-2)^2}$.

Question 7

There were many sound solutions to part **(i)**. A minority of candidates merely showed that the given differential equation is satisfied when initially $x = 5$. This did not show that x satisfies the equation at all times. In part **(ii)** the work was generally quite good with many candidates reaching a solution involving $\ln(100 - x)$, or equivalent. The main errors were the omission of a constant of integration and failure to give $\ln(100 - x)$ the appropriate sign.

Answers: **(ii)** $x = 100 - 95\exp(-0.02t)$; **(iii)** x tends to 100.

Question 8

Part **(i)** was generally well answered. The work in part **(ii)** was disappointing. Few candidates realised that the solution involved replacing the iterative formula with an equation in α and showing this to be equivalent to $3 = \ln\alpha + \frac{2}{\alpha}$, or vice versa. However part **(iii)** was often correctly done, though some failed to carry out sufficient iterations to establish convergence to 0.56.

Answers: **(i)** (2, ln 2 + 1), minimum point; **(ii)** $\alpha = \frac{2}{3 - \ln\alpha}$; **(iii)** 0.56.

Question 9

The first part was generally very well answered. Some candidates seemed not to understand what the angle between the two planes really was, for having found 40.4° correctly from the normals they followed it with its complement 49.6°.

Clearly some candidates were unprepared for part **(ii)** and failed to make progress. However others tackled it by a variety of methods. Some found two points on the line e.g. one with $x = 0$ and one with $y = 0$, and obtained the vector equation of the line from them. Others used the normals to the planes to obtain a direction vector for the line and completed the solution by finding a point on the line. Another method was to develop a Cartesian equation for the line by eliminating variables from the plane equations, and deduce an equation in vector form. Examiners remarked that algebraic and numerical slips were frequent here.

Answers: **(i)** 40.4°; **(ii)** $3\mathbf{j} + \mathbf{2}\mathrm{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$.

Question 10

In part **(i)** some attempts broke down because of trivial slips in manipulation. The majority succeeded and solutions varied in length from five lines to two pages. Part **(ii)** was fairly well answered though some solutions failed to contain sufficient working to justify the given answer. Examiners felt that part **(iii)** was poorly done. The structure of the question led to the integration of cot x – cot 2x, yet many of those who had integrated cot x correctly in part **(ii)** could not produce a correct integral of cot 2x here. Most attempts at integrating cosec 2x directly were very poor indeed, though occasionally a correct integral was obtained.

Answer: **(iii)** $\frac{1}{4}\ln 3$.

Paper 8719/05
Paper 5

General comments

There was a good response to this Paper. In the majority of scripts the work was well presented and there was no evidence that candidates were pressed for time in completing the Paper.

The dynamics questions were tackled with confidence with many all correct solutions by candidates from a fairly wide spectrum of the ability range. However, this could not be said of the statics questions where a lot of uncertainty was displayed particularly with the idea of taking moments.

Yet again marks were carelessly thrown away through failing to work correct to three significant figures. For example, in **Question 3**, the tension in the string was 467.6537..., which was then correctly rounded to 468N. Many candidates then used the value 468 to obtain the vertical component of the force at A as 44.7N rather than using the best value retained in the calculator to obtain 45N.

Thankfully the number of candidates still using $g = 9.81 \text{ ms}^{-2}$, despite the instructions on the front page of the Paper, continues to decline. However these candidates should have been alerted in **Question 7 (ii)** when this value of g was used. This gave a mass of 3.06 kg which was not the 3 kg requested in the question.

Comments on specific questions

Question 1

Although this question posed few problems for the more able candidates many of the remainder fell into error for a variety of reasons. The main one, and most serious, was the fact that candidates who were obviously taking moments about the centre of the ring, more often than not, had the term 1.5 x 25 appearing in the equation. No credit could be obtained as equations derived from either taking moments or resolving must include only the relevant number of terms, i.e. 2 terms if the moments were taken about the centre of the ring, or 3 terms if taken about the y-axis where the origin of coordinates was such that the centre of the ring was (25, 25).

Another frequent error was to assume that the masses of the ring and rod were proportional to their length (i.e. 2π x 25 and 48), despite the fact that the question did not specify that the components of the frame were made of the same material. The Method mark was allowed in this case provided all else was correct.

Answer: 2 cm.

Question 2

The better candidates coped with part **(i)**, but there were frequent errors by the rest in either calculating *OQ* or by using the wrong formula to find the distance of the centre of mass from *O*. It was depressing to find candidates taking an Advanced Level Paper who took the complement of 70° to be 30°. With the calculation of the distance of the centre of mass from *O*, it would be true to state that each of the first five centres of mass given in the formula sheet MF9 appeared often, with the most frequent offender being $\frac{1}{2}r$ for the hemispherical shell.

In part **(ii)** apart from a correct deduction, few candidates could give a coherent reason why the hemisphere did not fall on its plane face. There were many ambiguous statements of the type "the centre of mass falls before *P*". Only rarely was there a succinct statement, based on statical ideas, that after release, the resultant moment of the system about *P* was due to the weight of the hemisphere only and resulted in a clockwise moment.

Answers: **(i)** Centre of mass not between *O* and *Q* as 1.5 cm > 1.46 cm.

Question 3

It was generally appreciated that the tension in the string could only be found by taking moments, preferably about *A*. Then angle *PAB* was usually found correctly to be 30° although it was not unusual to see 26.6° from the less able candidates.

A frequent careless error was to have the weight also acting at a distance 2.5 cm from *A*. An approach by some candidates was to consider the tension as the vertical and horizontal components $T\cos 30°$ and $T\sin 30°$. Unfortunately the resulting moments equation often did not contain the moment of the $T\sin 30°$ component. Hence no credit could be given as the derived equation must contain the moments of all the relevant forces.

Part **(ii)** presented a lot of difficulty for many candidates. One large group thought that the resultant force exerted by the wall at *A* was in the direction *AB.* Another group seemed to interpret "horizontal and vertical components" as being parallel and perpendicular to *AB*.

Answers: **(i)** 468 N; **(ii)** 234 N and 45 N.

Question 4

Candidates of all abilities scored well on this question. The fact that the differential equation was given in part **(i)** undoubtedly helped nearly all candidates to score maximum marks in part **(ii)**. Only very weak candidates failed to see that it was necessary to apply Newton's Second Law of Motion to answer part **(i)**. It was encouraging to see that the negative sign appeared in its proper place in the development of the equation as there was very little evidence of sign fiddling to get the required answer.

Answer: **(ii)** 3 ms^{-1}.

Question 5

There were many excellent all correct solutions to this question, even by candidates who only performed modestly in other parts of the Paper. Considering the improvement generally on this topic, it is to be hoped that circular motion is no longer one of the great mysteries of mechanics. Some of the infrequent errors in part **(i)** were (a) the tension in the string in the wrong direction, (b) the omission of 8N force when resolving vertically and (c) resolving in the direction of the string and equating the forces to zero. The latter case could not be correct as the acceleration of the aircraft has a component $\frac{v^2}{r}\cos 30^\circ$ in this direction.

Part **(ii)** was also very well answered and only the weakest candidates used $r = 9$ rather than 9sin60°. A number of solutions were laboured through using the acceleration in the form $r\omega^2$ to find ω and then using $v = r\omega$ to find the speed. Occasionally the premature approximation of taking the radius to be 7.8 m led to an answer which was not correct to three significant figures and thus led to the needless loss of the final mark.

Answers: **(i)** 6 N; **(ii)** 9 ms^{-1}.

Question 6

On the whole there was a high degree of success with parts **(i)** and **(ii)**, but many of the routes to the answers were somewhat lengthy. In part **(ii)** for example, many went to great lengths to first calculate the time taken to reach the highest point rather than merely state that it was 5 seconds. Others, who seem to think that all projectile problems are dependent on the use of the equation of the trajectory, first found the horizontal distance at the instance when the particle was at its highest point.

Although able candidates coped well with part **(iii)**, many of the rest failed to appreciate that it depended on recognising that, at time *T*, the vertical component of the velocity was equal to the horizontal component. Had some of them drawn a simple sketch it could have avoided the frequent error of assuming that at time *T*, the components of the velocity were 60cos45° and 60sin45°. Across the whole ability range there were many who unnecessarily found the speed of the particle (46.9 ms^{-1}) at time *T* during the course of their calculations.

Answers: **(i)** 56.4°; **(ii)** 125 m; **(iii)** 1.68 seconds.

Question 7

It was a very weak candidate indeed who failed to answer part **(i)** correctly.

The vertical resolution of the forces in part **(ii)** was good with a vast improvement in performance over similar situations occurring in problems in the past. Despite the mass of the stone being given, only in a minority of solutions was there any evidence of a late adjustment when the first attempt produced, for example, a mass of 1.5kg.

One of the most frequent failures in part **(iii)** was to assume that *AB* = 10m. Examiners got so used to seeing the incorrect answer 511 J that they knew instantly where the error lay. A more disturbing error was the assumption that the extension of the string was proportional to the depth of the stone below *AB*.

Apart from the best candidates, the usual mark obtained in part **(iv)** was 2. Although the G.P.E. (= 240 J) invariably appeared in the energy equation, the E.P.E. of the string as the stone passed through the mid-point of *AB* did not.

Answers: **(i)** 39 N; **(iii)** 650 J; **(iv)** 16 ms^{-1}.

Paper 8719/07

Paper 7

General comments

The performance of candidates in this Paper was varied. A number of candidates scored very highly with well presented, clear solutions. However there were, equally, some very poor attempts from candidates who were unprepared for this examination.

Candidates performed well on **Questions 4** and **6** in particular, and often found **Questions 1** to **3** more challenging. **Question 5** on Type I and Type II errors was slightly better answered, in general, than has been the case in the past.

Comments on working to the correct level of accuracy have been made in the past; the Question Paper requires three significant figures unless otherwise stated. Whilst in general fewer candidates are losing marks because of this it is still surprising at this level that some (often very good) candidates still lose marks due to premature approximation (i.e. working to three significant figures or less in earlier stages of working) or even confusing significant figure accuracy with decimal place accuracy. This was particularly seen on **Question 2** where the answer required was 0.0834 to three significant figures and many candidates gave an answer of 0.083, and in **Question 5** the answer of 0.0234 was often given as 0.023.

Candidates did not appear to be under any time pressure to complete the Paper (despite many using a lengthy method in their attempt at **Question 1**). On the whole candidates gave clear and full solutions.

Comments on specific questions

Question 1

Many candidates did not use the straightforward way of attempting this question (mean = *np*, variance = *npq*) and instead tried to set up a probability distribution table. This caused a time penalty and often errors were made, (including some very fundamental ones with probabilities in tables totaling more than one). The main error noted on part **(ii)** was to multiply the variance in part **(i)** by 2 rather than by 4.

Some non-numerical solutions to both parts were also seen.

Answers: **(i)** 2.5, 1.25; **(ii)** 5, 5.

Question 2

Some candidates correctly appreciated that this was a significance test using a Binomial Distribution. Common errors were to calculate $P(X > 10)$, $P(X < 10)$, or merely $P(X = 10)$ rather than $P(X \leqslant 10)$ and occasionally contradictory comments were seen in the conclusions (e.g. "Reject $H_0(p = 0.6)$, therefore the player had not improved"). The majority of candidates attempted this question using a Normal Distribution which was not strictly valid as *nq* was equal to 4.8; however, some credit was given. Many errors were noted including lack of, or incorrect, continuity correction and much confusion between different methods was seen. This was not a well attempted question.

Answer: Accept claim at 10% level.

Question 3

Several careless mistakes were seen in calculating the mean, but on the whole candidates were able to calculate a confidence interval. A particularly common error was to use a wrong *z*-value. Some candidates found their own standard deviation from the sample rather than using the given value, or even used $\frac{n}{n-1}$ x 3. In part **(ii)**, many candidates ignored the instructions to "use your answer to part **(i)**" and did unnecessary further calculations (often incorrect). Candidates who did use their confidence interval calculated in part **(i)** were often not clear in their explanation. A statement such as "30 was inside the interval and therefore the claim could be accepted" was required. "It is in the interval" was too vague and could not be accepted. Other incorrect comments such as "Accept the claim because 30 is close to 31", "Reject because $30 \neq 29.4$, and $30 \neq 32.6$" were seen showing a lack of understanding by the candidate. Most candidates scored some marks on this question, but not many gained full marks.

Answers: **(i)** (29.4, 32.6), 30% is inside the interval; **(ii)** Accept claim (at 2% level).

Question 4

This question was well attempted by the majority of candidates. Errors in part **(i)** included using wrong limits for the integration, in part **(ii)** an error often seen was $\int x \, dx = x^2$ and surprisingly in part **(iii)** many basic errors were seen in solving the quadratic equation. Poor algebra and errors such as $m(4 - m) = 2 \Rightarrow m = 2$ or $(4 - m) = 2$ were too often seen. Candidates often lost the final answer mark in part **(iii)** by not rejecting the solution to the quadratic equation which was inadmissible.

Answer: **(i)** 0.0625; **(ii)** $\frac{2}{3}$; **(iii)** 0.586 .

Question 5

Whilst attempts at this topic were better than in the past there were still a large number of candidates who did not attempt the question at all. Lack of clear numerical interpretation of Type I/Type II errors was still evident. Common errors noted were omission of $\sqrt{20}$ in the denominator when standardising or use of the wrong tail. In part **(ii)** 2.1 was often incorrectly used.

Answer: **(i)** 0.0234; **(ii)** 0.160 .

Question 6

Many candidates scored well on this question. In part **(i)** most candidates correctly used $\lambda = 1.25$ but errors such as finding P(0, 1, 2, 3, 4), P(1, 2, 3) or 1 – P(0, 1, 2, 3) were seen. In part **(ii)** some candidates used an incorrect variance and omitted or used a wrong continuity correction. Many candidates correctly found the final answer of 0.123 in part **(iii)**, though finding P(4) with $\lambda = 1.25$ and P(4) with $\lambda = 5$ and multiplying these together was a common error, as was merely finding P(4) with $\lambda = 5$.

Answers: **(i)** 0.962; **(ii)** 0.0915; **(iii)** 0.123 .

Question 7

This was a reasonably well attempted question. In part **(i)** a common error of $20^2 \times 0.15^2$ or $20^2 \times 0.27^2$ when calculating the variances was noted by Examiners along with rounding errors. Some candidates considered $2A > B$ or $A - B < -2$ rather than $A - B > 2$, and even attempts to include a continuity correction were seen so that $A - B > 2$ became $A - B > 1.5$. Of the candidates who found $\overline{A} \sim N\left(20.05, \frac{0.15^2}{20}\right)$ and $\overline{B} \sim N\left(20.05, \frac{0.27^2}{20}\right)$ very few went onto consider $\overline{A} - \overline{B} > 0.1$ $\left(\frac{2}{20}\right)$ and some incorrectly used $\overline{A} - \overline{B} > 2$. In part **(b)** weaker candidates worked with 0.975 and never found the z-value of 1.96 and some candidates formed an incorrect equation involving an 'n' on the numerator. Surprisingly many candidates incorrectly went from $\sqrt{n} = 14.7$ to $n = 3.8$, and a final answer of 217 or 216.09 was also common.

Answers: **(i)** 0.0738; **(ii)** 216.