

**MARK SCHEME for the May/June 2011 question paper  
for the guidance of teachers**

**9231 FURTHER MATHEMATICS**

9231/23

Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

<b>Page 2</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE A LEVEL – May/June 2011</b>	<b>9231</b>	<b>23</b>

## **Mark Scheme Notes**

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\checkmark$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

<b>Page 3</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE A LEVEL – May/June 2011</b>	<b>9231</b>	<b>23</b>

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A LEVEL – May/June 2011	9231	23

1	Use conservation of momentum for 1 <sup>st</sup> collision: $km u_B = mu$	B1	3	8
	Use Newton's law of restitution: $u_B = \frac{1}{2}u$	B1		
	Eliminate $u_B$ to find $k$ : $k = 2$ <b>A.G.</b>	B1		
	Use conservation of momentum for 2 <sup>nd</sup> collision: $km v_B + 6m v_C = km u_B$	M1		
	Use Newton's law of restitution: $v_B - v_C = -e u_B$	M1		
	Substitute and solve for $v_B$ : $2v_B + 6v_C = u, v_B - v_C = -\frac{1}{2}eu$ $v_B = (1 - 3e)u/8$ [ $v_C = (1+e)u/8$ ]	M1 A1		
	Use $v_B \geq 0$ if no further collisions: $1 - 3e \geq 0, e \leq \frac{1}{3}$ <b>A.G.</b>	B1		
<b>S.R.</b> Taking $v_B = 0$ throughout: $e = \frac{1}{3}$	(M1 A1)	(2)		
2	Find MI of large disc about $O$ : $\frac{1}{2}M(3a)^2 + M(5a)^2$ [= $59Ma^2/2$ ]	M1 A1	6	11
	Find MI of small disc about $O$ : $\frac{1}{2}(M/9)a^2 + (M/9)a^2$ [= $Ma^2/6$ ]	M1 A1		
	Find MI of particle about $O$ : $(M/3)(8a)^2$ [= $64Ma^2/3$ ]	B1		
	Sum to find MI of system about $O$ : <b>A.G.</b> $I = (177+1+128)Ma^2/6 = 51Ma^2$	A1		
	State or imply that speed is max when $OP$ vertical	M1		
	Use energy when $OP$ vertical (or at general point): $\frac{1}{2}I\omega^2 = (5+1/9+1/3)8Mga = 70Mga/9$	M1 A1		
Substitute for $a, I$ and find max speed $8a\omega$ : $\omega = \sqrt{6 \cdot 10} = 2.47, 8a\omega = 9.9$ [ $\text{ms}^{-1}$ ]	M1 A1	5		
3	Moments for system about $C$ , denoting $ACB$ by $\theta$ : $N_B \times BC = 2W \times 3a \cos \theta$ $+ W(BC - 4a \sin \theta)$ (A.E.F.)	M1 A1	4	12
	Substitute for $BC, \theta$ : $N_B \times 10a = 2W \times 9a/5 + W \times 34a/5$	A1		
	Simplify to give $N_B$ : $N_B = (26/25)W$ <b>A.G.</b>	A1		
	Find $N_C$ by vertical resolution or moments: $N_C = 3W - N_B = (49/25)W$	M1 A1		
	Find $F_B$ (or $F_C$ ) by moments about $A$ : $F_B \times 24a/5 = N_B \times 32a/5$ $- W \times 16a/5$	M1		
	$F_B = (18/25)W$ or $0.72W$	A1		
	Find limiting value for $\mu$ at $B$ [or $C$ ] (A.E.F.): $18/26$ [= $0.692$ or $18/49 = 0.367$ ]	M1 A1		
	Relate $F_B, F_C$ by e.g. horizontal resolution: $F_C = F_B$ [= $(18/25)W$ ]	B1		
Deduce least possible value of $\mu$ for system: $\mu_{min} = 9/13$ or $0.692$	B1	8		

Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A LEVEL – May/June 2011	9231	23

4	Use conservation of energy at general point:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 - \cos \theta)$	B1	4	12
	Equate radial forces to find tension $T$ :	$T = mg \cos \theta + mv^2/a$	B1		
	Eliminate $v^2$ , replace $u^2$ by $3ag$ and simplify:	$T = mg(1 + 3 \cos \theta)$ <b>A.G.</b>	M1 A1		
	Use energy to find speed $v$ when $PQ$ horizontal:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga, v^2 = ga$	M1 A1		
	Use energy to find speed $w$ when $P$ above $Q$ : (note that $v$ need not be found)	$\frac{1}{2}mw^2 = \frac{1}{2}mv^2 - mg(a-x)$ $[mw^2 = mg(2x - a)]$	M1 A1		
	<i>EITHER:</i>				
	Consider tension to find reqd. condition:	$T = mw^2/(a-x) - mg \geq (\text{or } >) 0$	M1 A1		
	Combine to find least value of $x$ :	$mg(3x - 2a)/(a-x) \geq 0$ $x \geq 2a/3, x_{\min} = 2a/3$	M1 A1		
	<i>OR:</i>				
	Find $x$ for which $T$ becomes zero:	$mw^2/(a-x) = mg, x = 2a/3$	(M1 A1)		
Show this is least possible value of $x$ , e.g.:	$T = mg(3x-2a)/(a-x) \geq 0$ if $x \geq 2a/3$	(M1 A1)	8		
5	(i) State or find $E(X)$ :	$E(X) = 1/0.01$ or 100	B1	1	6
	(ii) Integrate $f(x)$ to find median $m$ : Solve for $m$ :	$\int_0^m f(x) dx = 1 - e^{-0.01m} = \frac{1}{2}$ $m = 100 \ln 2$ or 69.3	M1 A1 A1	3	
	(iii) Integrate $f(x)$ to find probability:	$\int_m^{100} f(x) dx = \frac{1}{2} - e^{-1} = 0.132$	M1 A1	2	
6	Find pooled estimate:	$(15.05 - 5.5^2/5 + 36.4 - 8^2/n)/(3+n)$	M1 A1	7	7
	Equate to 3 and rearrange:	$45.4 - 64/n = 9 + 3n$ $3n^2 - 36.4n + 64 = 0$	M1 A1 A1		
	Solve for $n$ :	$n = (36.4 \pm 23.6) / 6 = 10$	M1 A1		
7	(i) Find probability for needing 5 throws:	$p(1-p)^4$ with $p = 1/6$ ; = 0.0804	M1 A1; A1	3	8
	(ii) Find probability for needing < 8 throws:	$1 - (1-p)^7 = 0.721$	M1 A1	2	
	(iii) Relate prob. to 0.99 (allow > but not =): Find least integer $n$ :	$1 - (1-p)^{n-1} \geq 0.99$ $(n-1) \log 5/6 \leq \log 0.01$	B1 M1		
	(Allow M1 A1 even if equality used)	$n-1 \geq 25.3, n_{\min} = 27$	A1	3	

<b>Page 6</b>	<b>Mark Scheme: Teachers' version</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE A LEVEL – May/June 2011</b>	<b>9231</b>	<b>23</b>

<b>8</b>	Consider differences e.g.:	5.1 2.1 2.2 0.6 5.1 3.1 3.9 3.7	M1	6	
	Calculate sample mean:	$d = 25.8 / 8 = 3.225$	M1		
	Estimate population variance: (allow biased here: 2.117 or 1.455 <sup>2</sup> )	$s^2 = (100 \cdot 14 - 25 \cdot 8^2 / 8) / 7$ [= 2.419 or 1.555 <sup>2</sup> ]	M1		
	Find confidence interval (allow z in place of t) e.g.:	$3.225 \pm t \sqrt{(2.419/8)}$	M1		
	(inconsistent use of 7 or 8 loses M1)				
	Use of correct tabular value:	$t_{7, 0.975} = 2.36[5]$	A1		
	Evaluate C.I. correct to 3 s.f. (in kg):	$3.225 \pm 1.301$ or [1.92, 4.53]	A1		
	State hypotheses:	$H_0: \mu_b - \mu_a = 2.5, H_1: \mu_b - \mu_a > 2.5$	B1		
	Calculate value of t (to 2 dp):	$t = (\bar{d} - 2.5) / (s / \sqrt{8}) = 1.32$	M1 *A1		
	Compare with correct tabular t value:	$t_{7, 0.95} = 1.89[5]$	*B1		
Correct conclusion (AEF, dep *A1, *B1):	Reduction not more than 2.5	B1	5	<b>11</b>	

Page 7	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A LEVEL – May/June 2011	9231	23

9	(i)	Find mean values:	$\bar{x} = 50.1[3], \bar{y} = 51.5[3]$	B1	1	
	(ii)	Calculate gradient $b'$ in $x - \bar{x} = b'(y - \bar{y})$ :	$b' = (40236 - 752 \times 773/15) / (45351 - 773^2/15)$	M1		
			$= 1482.9 / 5515.7 = 0.268[9]$	A1		
		Use regression line to estimate $x$ at $y = 56$ :	$x = 50.13 + 0.2689(56 - 51.53)$	M1		
			$[x = 36.28 + 0.2689y] = 51[.3]$	A1	4	
		OR Calculate gradient $b$ in $y - \bar{y} = b(x - \bar{x})$ :				
			$b = (40236 - 752 \times 773/15) / (38814 - 752^2/15)$	(M1)		
			$= 1482.9 / 1113.7 = 1.33[15]$	(A1)		
		Use regression line to estimate $x$ at $y = 56$ :	$x = 50.13 + (56 - 51.53) / 1.332$	(M1)		
			$[y = -15.22 + 1.332x] = 53[.49]$	(A1)		
	(iii)	Find correlation coefficient $r$ :				
			$r = (40236 - 752 \times 773/15) / \sqrt{(38814 - 752^2/15)(45351 - 773^2/15)}$	M1		
			$= 1482.9 / \sqrt{(1113.7 \times 5515.7)}$			
			$= 1483 / (33.37 \times 74.27)$			
			or $98.86 / \sqrt{(74.25 \times 367.7)}$			
			$= 98.86 / (8.617 \times 19.18)$			
			$= 0.598$	*A1	2	
	(iv)	State both hypotheses:	$H_0: \rho = 0, H_1: \rho \neq 0$	B1		
		Use correct tabular 2-tail $r$ value:	$r_{15, 5\%} = 0.514$ (to 2 dp)	*B1		
		Valid method for reaching conclusion:	Reject $H_0$ if $ r  >$ tabular value	M1		
		Correct conclusion (AEF, dep *A1, *B1):	There is a non-zero coefficient	A1	4	11

Page 8	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE A LEVEL – May/June 2011	9231	23

<b>10 (a)</b>	Resolve vertically at equilibrium:	$\lambda d/a = mg$ [ $\lambda/a = mg/d$ ]	B1		
	Use Newton's Law at general point:	$m d^2x/dt^2 = mg - \lambda(d+x)/a$ [or $-mg + \lambda(d-x)/a$ ]	M1 A1		
	Simplify:	$d^2x/dt^2 = -(\lambda/ma)x$ or $-(g/d)x$	A1		
	S.R.: Stating this without derivation (max 3/5):		(B1)		
	Find period $T$ using SHM with $\omega = \sqrt{g/d}$ :	$T [= 2\pi\sqrt{(ma/\lambda)}] = 2\pi\sqrt{(d/g)}$ <b>A.G.</b>	B1	5	
	Use SHM formula for $x$ with amplitude $2d$ :	$x = 2d \cos(\omega t)$ [or $\sin$ ]	M1		
	Find time $t_1$ to string becoming slack:	$t_1 = (1/\omega) \cos^{-1}(-1/2)$ or $T/4 + (1/\omega) \sin^{-1}(1/2)$	M1 A1		
	Evaluate:	<b>A.G.</b> $t_1 = (1/\omega) 2\pi/3 = (2\pi/3)\sqrt{(d/g)}$	A1		
	Find speed $v$ when string becomes slack:	$v = \omega\sqrt{(4d^2 - d^2)} = \omega d\sqrt{3}$ or $\sqrt{(3dg)}$	M1 A1		
	Find further time $t_2$ to instantaneous rest:	$t_2 = v/g$	B1		
Substitute and simplify:	<b>A.G.</b> $t_2 = \sqrt{(3dg)}/g = \sqrt{3}\sqrt{(d/g)}$	M1 A1	9	<b>14</b>	
<b>(b)</b>	Find mean and variance of sample:	$262/200 = 1.31$ and $(586 - 262^2/200) / 200 = 1.21$ [39]	M1 A1		
	Valid comment (AEF, needs values approx correct):				
	Values close, so distn. appropriate		B1	3	
	<b>(i)</b> State and evaluate expression for $p$	<b>A.G.:</b> $p = 200 (1.31^2/2)e^{-1.31} = 46.304$	B1		
	Find $q$ (can use $\Sigma E_i = 200$ ):	$q = 200 (1.31^3/6)e^{-1.31} = 20.2$ [19]	B1	2	
	<b>(ii)</b> State (at least) null hypothesis:	$H_0$ : Poisson fits data (A.E.F.)	B1		
	Combine last 3 cells since exp. value $< 5$ :	$O: \dots 5$ $E: \dots 8.82$	*M1 A1		
	Calculate $\chi^2$ (to 2 dp ; A1 dep *M1):	$\chi^2 = 5.54$	M1 A1		
	Compare consistent tabular value (to 2 dp): (A1 dep *M1)	$\chi_{3,0.9}^2 = 6.251$ $[\chi_{4,0.9}^2 = 7.779, \chi_{5,0.9}^2 = 9.236]$	M1 A1		
	Valid method for reaching conclusion:	Accept $H_0$ if $\chi^2 <$ tabular value	M1		
Conclusion (A.E.F., needs correct values):	$5.54 < 6.25$ so Poisson does fit	A1	9	<b>14</b>	