FURTHER MATHEMATICS **9231/02**

Paper 2 **May/June 2007**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m s}^{-2}$.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

[Turn over

1 Particles of masses m_1 and m_2 lie at rest on a smooth horizontal plane. Each particle is given a horizontal impulse of magnitude I towards the other particle so that the particles collide directly. Given that the collision is perfectly elastic, find the speeds of the particles after the collision. [6]

2
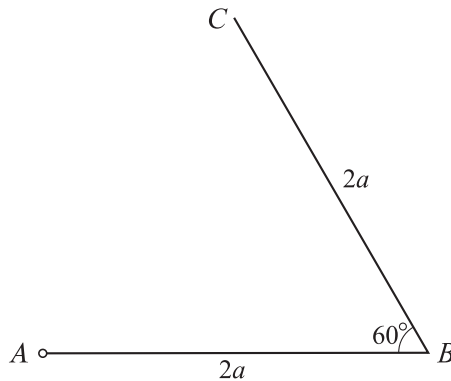
Two uniform rods *AB* and *BC*, each of length $2a$ and mass m, are rigidly joined at *B* so that angle $ABC = 60°$ (see diagram). The system is free to rotate in a vertical plane about a horizontal axis though *A*. Show that the moment of inertia of the system about this axis is $\frac{14}{3}ma^2$. [3]

The system is released from rest with *AB* horizontal and *C* above *AB*. Find the angular speed when *BC* is first vertical. [4]

3
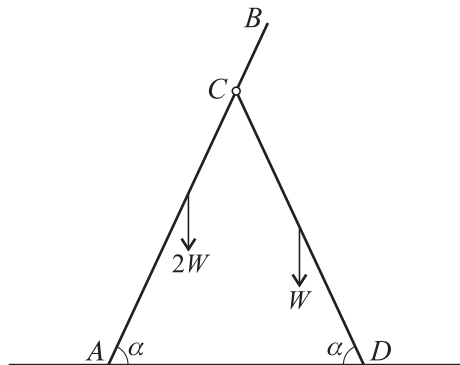
The diagram shows a pair of stepladders modelled by two uniform rods, *AB* of length $5a$ and weight $2W$ and *CD* of length $4a$ and weight W, smoothly hinged at *C*. The stepladders rest on a rough horizontal floor with *A* fixed so that it cannot slide. The angles *CAD* and *CDA* are both equal to α. By taking moments about *A* for the system, find the normal contact force at *D*. [3]

Show that the frictional force at *D* is $\frac{7}{8}W \cot \alpha$. [3]

Given that the coefficient of friction between *CD* and the floor is 0.7, find the set of values of α for which slipping will not take place at *D*. [3]

4
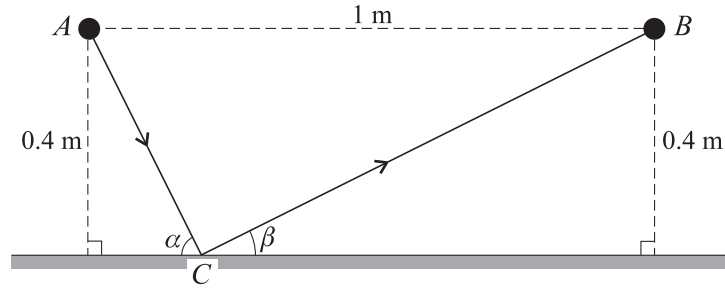
Two identical snooker balls are at rest at the points A and B of a snooker table, where $AB = 1$ m. The line AB is parallel to a smooth side of the table and at a distance 0.4 m from that side. The ball at A is projected along the table with speed $2\,\text{m s}^{-1}$ to hit the side of the table at C. It rebounds and subsequently hits the ball at B. The angles that AC and CB make with the side of the table are α and β respectively (see diagram). The coefficient of restitution between the side and a ball is $\frac{1}{4}$. Resistances may be ignored and the balls may be considered to be particles.

  (i) Show that $\cot\alpha + \cot\beta = 2.5$ and that $\tan\alpha = 2$. [5]

  (ii) Show that the direction of motion of the ball turns through $90°$ on impact at C. [2]

  (iii) Find the speed of the ball after the impact at C. [2]

5 A small ring R of mass m is threaded onto a smooth straight wire fixed at an angle of $30°$ to the horizontal. A light elastic string of natural length l has one end attached to a fixed point O of the wire. The other end of the string is attached to R. In the equilibrium position R is below the level of O and $OR = \frac{5}{4}l$. Find the modulus of elasticity of the string. [2]

The ring is moved to the position where $OR = l$, with R below the level of O. It is projected away from O along the wire with speed $\sqrt{(gl)}$. At a general position in the subsequent motion $OR = \frac{5}{4}l + x$. Show that, while the string is taut, and neglecting air resistance,

$$\ddot{x} = -\frac{2g}{l}x.$$ [3]

Show that the maximum speed of the ring is $\frac{3}{4}\sqrt{(2gl)}$, and find the time after projection at which the maximum speed first occurs. [7]

6 Levels of mercury in fish are monitored regularly. The levels in a random sample of five sea bass caught in a certain area were measured with the following results, in parts per million.

<div align="center">

0.093   0.102   0.171   0.124   0.274

</div>

Assuming that mercury level has a normal distribution, calculate a 95% confidence interval for the mean mercury level in sea bass caught in that area. [6]

7 Laboratory measurements were made of the speed of sound, $v\,\text{cm}\,\text{s}^{-1}$, in air at 9 different controlled temperatures, $\theta\,°\text{C}$. The results are given in the following table.

θ	−10	−5	0	5	10	15	20	25	30
v	3.24	3.29	3.31	3.34	3.38	3.41	3.44	3.47	3.51

$[\Sigma\theta = 90,\ \Sigma\theta^2 = 2400,\ \Sigma v = 30.39,\ \Sigma v^2 = 102.6805,\ \Sigma\theta v = 313.65.]$

Determine whether a linear model is appropriate for the data. [3]

Obtain the equation of a suitable regression line. [3]

Estimate the temperature of the air when the speed of sound is $4\,\text{cm}\,\text{s}^{-1}$ and comment on the reliability of this estimate. [2]

8 On average 1 in 8 people in a particular community is left-handed and the rest are right-handed. A sample of people is chosen at random, one by one, until a left-handed person is obtained. Find the probability that the number of people in the sample is less than 12. [3]

Another sample is chosen at random, one by one, until at least one left-handed and at least one right-handed person have been obtained. The number of people in this sample is denoted by N, so that the sample consists of

either : $(N-1)$ left-handed people followed by one right-handed person

or : $(N-1)$ right-handed people followed by one left-handed person.

Obtain $\text{P}(N = r)$, for $r \geqslant 2$, and show that $\text{E}(N) = 8\frac{1}{7}$. [5]

9 Part of a study carried out on professional Sumo wrestlers involved measurement of body fat. Random samples of Sekitori and Makushita wrestlers were selected and the body fat index, x, of each wrestler was measured. The results are summarised in the following table.

	Sekitori	Makushita
Sample size	7	12
$\bar{x}$	28.6	22.2
s^2	26.01	29.16

In the table, $\bar{x}$ denotes the sample mean and s^2 the unbiased estimate of the relevant population variance. Assuming that these samples are from normal distributions with equal variances, test at the 1% significance level whether the mean body fat index of Sekitori wrestlers, μ_S, exceeds the mean body fat index of Makushita wrestlers, μ_M. [7]

Find the greatest value of a for which it could be claimed at the 5% significance level that μ_S exceeds μ_M by more than a. [3]

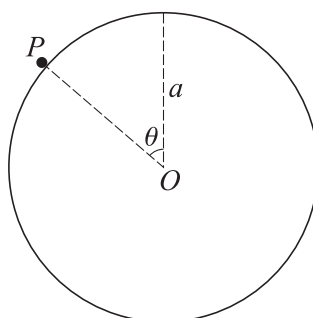**10** The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} 0 & x < 3, \\ 2e^{-(2x-6)} & x \geqslant 3. \end{cases}$$

Obtain the distribution function of Y, where $Y = e^X$. [6]

Find the probability density function of Y and hence find the value of $\text{E}(\sqrt{Y})$. [5]

11 Answer only **one** of the following two alternatives.

EITHER

A uniform disc has centre O, radius a and mass $2m$. It is free to rotate in a vertical plane about a horizontal axis through O. A particle P of mass m is placed on the highest point of the rough edge of the disc and the system is slightly disturbed so that OP begins to rotate with the particle in contact with the edge. In the subsequent motion OP makes an angle θ with the upward vertical (see diagram). For the motion while the particle remains in contact with the disc without slipping, find $a\ddot{\theta}$ and $a\dot{\theta}^2$ in terms of g and θ. [6]

Show that if the particle remains in contact with the disc then it begins to slip when

$$4\mu \cos \theta - \sin \theta = 2\mu,$$

where μ is the coefficient of friction between the particle and the edge of the disc. [5]

Show that, however large the value of μ, the particle cannot lose contact with the disc before it starts to slip. [3]

OR

A large population consists of four species, A, B, C and D, which are claimed to be in the proportions p_1, p_2, p_3 and p_4 respectively, where $\sum_{i=1}^{4} p_i = 1$. In order to test this claim a random sample of size n is selected and the numbers of each species are found to be a, b, c and d respectively. Show that the χ^2 statistic for a goodness of fit test simplifies to

$$\chi^2 = \frac{a^2}{np_1} + \frac{b^2}{np_2} + \frac{c^2}{np_3} + \frac{d^2}{np_4} - n.$$ [4]

It is given that $p_1 = p_2$, $p_3 = p_4$ and the values of a, b, c and d are 15, 25, 39 and 21 respectively. Denoting the common value of p_1 and p_2 by p, show that

$$\chi^2 = \frac{8.5}{p} + \frac{19.62}{0.5 - p} - 100.$$ [2]

Determine the value p_0 of p for which this value of χ^2 is a minimum. [3]

Carry out the goodness of fit test at the 5% significance level, with $p = p_0$. [4]

State, giving your reason, the conclusion of the test for values of p other than p_0. [1]

BLANK PAGE

BLANK PAGE

8

BLANK PAGE