FURTHER MATHEMATICS

Paper 9231/01
Paper 1

General comments

Some scripts of high quality and many of a good quality were received in response to this examination. There were very few poor scripts. Work was generally well presented by all, except the weakest candidates. Solutions were set out in a clear logical order and the standard of algebra and numerical accuracy was good.

There was no evidence to suggest that candidates had any difficulty completing the paper in the time allowed. The vast majority of scripts had substantial attempts at all questions. Once again there were very few misreads and this year there were few rubric infringements.

The Examiners generally felt that candidates had a sound knowledge of most topics on the syllabus. Induction and linear spaces still remain as areas of uncertainty, while improvement seems to have been made in curve sketching and vector work.

Comments on specific questions

Question 1

There were many complete and accurate answers to this question. Almost all candidates were able to establish the first result

$$\frac{1}{n^2+1}-\frac{1}{(n+1)^2+1}=\frac{2n+1}{(n^2+1)(n^2+2n+2)}.$$

Most were then able to use the method of difference to find

$$S_N=\frac{1}{2}-\frac{1}{(N+1)^2+1}.$$

It was then necessary to explain that since $(N+1)^2$ was positive for $N > 1$, then $\frac{1}{(N+1)^2+1}>0$ and $S_N<\frac{1}{2}$. Many candidates lost the mark for this piece of work, but, nevertheless, were able to state correctly that S_∞ was $\frac{1}{2}$.

Answer: $\frac{1}{2}$.

Question 2

Most candidates were able to differentiate correctly the given expressions for x and y with respect to t and then show that $\left(\frac{dx}{dt}\right)^2=\left(\frac{dy}{dt}\right)^2=\left(1+\frac{1}{t}\right)^2$.

Many knew the correct integral representation for the area of the surface generated and were able to successfully obtain the printed result.

Question 3

The vast majority of candidates produced a completely correct solution to this question. Almost all candidates knew how to derive the complementary function. There were only a few slips with the algebra involved and in a small number of cases the complementary function appeared as $e^{-2x}(A\cos5\theta + B\sin5\theta)$.

Almost all candidates knew how to obtain $2x+1$ as the particular integral and hence find the general solution correctly.

Answer: $y = e^{-2x}(A\cos5x + B\sin5x) + 2x + 1$

Question 4

The key to this question was the realisation that the given equation had to be differentiated implicitly with respect to x. A minority of candidates did not realise this.

The ability to differentiate correctly was somewhat variable and certainly many attempts, even successful ones, could have been far more concise. The Examiners hoped to see something along the lines of

$$y' = 1 - (y + xy')e^{-xy}$$

$$y'' = -(2y' + xy'')e^{-xy} + (y + xy')^2e^{-xy}.$$

Sadly few candidates were able to be this concise, and errors occurred, particularly when differentiating for the second time.

It was then required to find $y(0)$, $y'(0)$ and $y''(0)$. Numerically correct answers, with incorrect working, did not gain marks.

Occasionally, successful use was made of logarithms or a helpful substitution.

Answer: 1.

Question 5

Many candidates were unable to start the first part of this question. Here it was necessary to write x as $e^{\theta}\cos\theta$ and, using differentiation, find its maximum value in the range $0 \varnothing \theta \varnothing \frac{\pi}{2}$.

Candidates fared much better on the second part of the question and nearly all knew the correct integral representation and were successfully able to find the required area.

Pleasingly, most candidates followed the instruction to give *exact* answers.

Answers: $\frac{1}{\sqrt{2}}e^{\frac{\pi}{4}}$; $\frac{1}{4}\left(e^{\pi} - 1\right)$.

Question 6

This question was done well by many candidates. The eigenvectors for matrix **A** were usually found by solving a set of equations, or more elegantly by using a vector product. A considerable number of candidates abandoned the question at this point. A sizeable number of those candidates, who continued with the question, did not heed the word *hence* in the question and proceeded to find the eigenvalues of matrix **B** by setting up and solving the characteristic equation of matrix **B**. Candidates should have observed that **B** = **A** + 3**I** and hence the eigenvalues of **B** were 3 greater than those of **A**. Most candidates knew that the eigenvectors were unchanged.

Answers: $\begin{pmatrix}-1\\0\\1\end{pmatrix}, \begin{pmatrix}-4\\1\\4\end{pmatrix}, \begin{pmatrix}2\\3\\1\end{pmatrix}$; 4, 5, 7; $\begin{pmatrix}-1\\0\\1\end{pmatrix}, \begin{pmatrix}-4\\1\\4\end{pmatrix}, \begin{pmatrix}2\\3\\1\end{pmatrix}$.