AQA

A-LEVEL

Mathematics

MM04 – Mechanics 4

Mark scheme

6360

June 2016

Version 1.0: Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	Angular momentum before = $I\omega = 10I$	B1		Correct initial angular momentum
	MI of particle after collision = $0.6(0.25)^2$	M1		MI for particle after collision - seen
	Angular momentum after = $8(I + 0.6(0.25)^2)$	A1		Correct total angular momentum after collision
	Conservation of angular momentum gives $10I = 8(I + 0.6(0.25)^2)$	M1 A1		**M1** Forming equation using their conservation of momentum expressions. **A1** Fully correct equation
	$I = 0.15$	A1	6	Correct value of I found - **CSO**
	Total		**6**	

Q2	Solution	Mark	Total	Comment
(a)(i)	$\overrightarrow{PQ} = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$ $\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$	B1	1	**CAO**
(ii)	Moment = $\mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & 1 & 4 \\ \mathbf{j} & 2 & -1 \\ \mathbf{k} & -3 & 3 \end{vmatrix} = \begin{bmatrix} 3 \\ -15 \\ -9 \end{bmatrix}$	M1 A1F A1F		**M1** Use of $\mathbf{r} \times \mathbf{F}$ or $\mathbf{F} \times \mathbf{r}$ **A1F** at least two of their components correct All three of their components correct
	Magnitude = $\sqrt{3^2 + 15^2 + 9^2}$ $= \sqrt{315} = \sqrt{9 \times 35} = 3\sqrt{35}$	m1 A1	5	Finding the magnitude of their vector – dependent on first **M1** Fully correct – must have used $\mathbf{r} \times \mathbf{F}$ **CSO**
(b)	Using $\sin\theta = \frac{\lvert\mathbf{r} \times \mathbf{F}\rvert}{\lvert\mathbf{r}\rvert\lvert\mathbf{F}\rvert}$			
	$\lvert\mathbf{r}\rvert = \sqrt{14}$ $\lvert\mathbf{F}\rvert = \sqrt{26}$	B1		$\lvert\mathbf{r}\rvert = \sqrt{14}$ or $\lvert\mathbf{F}\rvert = \sqrt{26}$ seen
	$\theta = \sin^{-1}\left[\frac{3\sqrt{35}}{\sqrt{14}\sqrt{26}}\right] = 68.475......$	M1 A1		**M1** Use of vector product with correct vector pair. All correct values seen and used
	$68°$ to nearest degree	A1	4	Must be to the nearest degree – **CAO**

	ALTERNATIVE $\cos\theta = \frac{\lvert \mathbf{r}.\mathbf{F} \rvert}{\lvert \mathbf{r} \rvert \lvert \mathbf{F} \rvert}$ $\mathbf{r}.\mathbf{F} = -14$ $\theta = \cos^{-1}\left\lvert \frac{-14}{\sqrt{56}\sqrt{26}} \right\rvert = 68.475\ldots\ldots$ $68°$ to nearest degree	**(B1)** **(M1)** **(A1)** **(A1)**	**(4)**	**r**.**F** correct (could also use **r**.**F** = -7) **M1** Use of scalar product with correct vector pair. **A1** All correct values seen and used (could also use $\lvert \mathbf{r} \rvert = \sqrt{14}$ $\lvert \mathbf{F} \rvert = \sqrt{26}$) Must be to the nearest degree – **CAO**
	Total		**10**	

Q3	Solution	Mark	Total	Comment
(a)	Take moments about B $20(1.5) = P(1.5\cos 50^0)$ $P = \dfrac{20}{\cos 50^0} = 31.1 \text{ N}$	**M1A1** **A1**	 **3**	**M1** for forming a moment equation about B, with one side correct. **A1** both sides correct **CAO**
(b)	Tension = 51.1 N	**B1F**	**1**	Resolve vertically, T = 20 + their answer to **(a)**
(c)	Resolve vertically at A $20 + T_{AC}\cos 65^0 = 0$ $T_{AC} = \dfrac{-20}{\cos 65^0}$ Hence magnitude of T_{AC} = 47.3 N Resolve horizontally at A $T_{AB} + T_{AC}\cos 25^0 = 0$ $T_{AB} = \dfrac{20}{\cos 65^0}\cos 25^0 = 42.9\text{ N}$ Resolve horizontally at B $T_{BC}\cos 50^0 = T_{AB}$ $T_{BC} = \dfrac{20\cos 25^0}{\cos 65^0 \cos 50^0} = 66.7\text{ N}$ *AC* in compression *AB* and *BC* in tension	**M1** **A1** **M1** **A1** **M1** **A1** **B1**	 **7**	Resolving to obtain an equation to find the force in *AC* **CAO –** must be positive Resolving to obtain an equation to find the force in *AB* **CAO** Resolving to obtain an equation to find the force in *BC* **CAO** All three correctly stated
	Total		**11**	

Q4	Solution	Mark	Total	Comment
(a)	Volume = $\pi\int_0^2\left(1-\frac{x^2}{4}\right)^2\mathrm{d}x$	M1		Correct formula used – limits not needed
	$\pi\int_0^2\left(1-\frac{x^2}{2}+\frac{x^4}{16}\right)\mathrm{d}x = \pi\left[x-\frac{x^3}{6}+\frac{x^5}{80}\right]_0^2$	A1 B1		Correct expansion and integration Correct limits found/used
	$= \pi\left[2-\frac{4}{3}+\frac{2}{5}\right] = \frac{16}{15}\pi$	A1	4	Fully correct – **CSO** – answer given
(b)	$\pi\int_0^2 xy^2\mathrm{d}x=\pi\int_0^2 x\left(1-\frac{x^2}{2}+\frac{x^4}{16}\right)\mathrm{d}x$	M1		Correct formula used – limits not needed
	$= \pi\left[\frac{x^2}{2}-\frac{x^4}{8}+\frac{x^6}{96}\right]_0^2$	A1		Correct integration – all terms
	$= \frac{2}{3}\pi$	A1		Correct limits used and evaluated
	$\bar{x}\times\frac{16}{15}\pi=\frac{2}{3}\pi$	M1		**M1** Forming a valid equation to find the x coordinate
	Hence $\bar{x} = \frac{5}{8}$ (= 0.625)	A1F	5	**A1** Follow through their value for integral – must be less than 2
(c)	$\tan\theta=\frac{1}{\bar{x}}$	M1		$\tan\theta$ seen
	$\tan\theta=\frac{8}{5}$	A1F	2	Follow through part **(b)**
	Total		**11**	

Q5	Solution	Mark	Total	Comment
(a)	**F**= **i** + 4**j**	**B1**	**1**	**B1** both components correct
(b)	Take moments about the origin			
	Moment of **F** acting at $-4\mathbf{i}+6\mathbf{j}$ = -22**k**	**M1A1**		**M1** For use of **r**x**F** or **Fxr** **A1** correct magnitude
	Moment of $\mathbf{F}_1 = 2\mathbf{i}+\mathbf{j}$ acting at $\mathbf{i}-\mathbf{j}$ = 3**k** Moment $\mathbf{F}_2 = 3\mathbf{i}-2\mathbf{j}$ acting at $4\mathbf{i}-2\mathbf{j}$ = – 2**k** Moment $\mathbf{F}_3 = -4\mathbf{i}+5\mathbf{j}$ acting at $-3\mathbf{i}+\mathbf{j}$ = -11**k**	**M1** **A(2,1,0)**		Use of **r**x**F** or **Fxr** to find moments of three individual forces **A2** all magnitudes correct **A1** two magnitudes correct
	Hence –22**k** + **G** = 3**k** – 2**k** – 11**k**	**M1** **A1**		**M1** forming moment equation **A1** fully correct equation – including consistent signs
	Hence |**G**| = 12	**A1**	8	**CSO**
	ALTERNATIVE 1			
	Take moments about the origin			
	Moment of single force = -1(6) -4(4) = -22	**(M1A1)**		**M1** For correct pairings – force x distance **A1** correct magnitude
	Total moment of individual forces = 1(1) + 2(1) +3(2) - 2(4) + 4(1) - 5(3) = - 10	**(M1 A1)** **(A1)**		**M1** For correct pairings – force x distance **A1** all signs consistent **A1** correct magnitude
	–22 + **G** = -10	**(M1A1)**		**M1** forming moment equation **A1** correct equation - consistent signs
	Hence |**G**| = 12	**(A1)**	(8)	**CSO**
	ALTERNATIVE 2			
	Take moments about (-4 , 6) for system			
	= -4(5) + 5(1) +1(5) + 2(7) + 3(8) - 2(8)	**(M1A1)** **(M1A1)**		**M1A1** For three correct vertical force and distance pairings **M1A1** For three correct horizontal force and distance pairings
	= - 20 +5 + 5 + 14 + 24 -16 = 12	**(A1)** **(A1)**		**A1** all signs consistent **A1** correct evaluation
	0 + **G** = 12 Hence |**G**| = 12	**(M1)** **(A1)**	(8)	**M1** forming moment equation **CSO**
	Total		**9**	

Q6	Solution	Mark	Total	Comment
(a)	MI of hoop about centre = $6m(2a)^2$ MI of one rod about end = $\frac{4}{3}ma^2$ Total MI of system =	B1		Correct MI of hoop
	$6m(2a)^2 + 6\times\frac{4}{3}ma^2$	M1		Summing MI of hoop and 6 rods
	$= 32ma^2$	A1	3	**CSO** – printed answer AG
(b)(i)	Equation of motion for wheel = $T(2a) = 32ma^2\ddot{\theta}$ Hence $T = 16ma\ddot{\theta}$ **(1)**	M1 A1		Use of $C = I\ddot{\theta}$ **M1 –** LHS side correct **A1** both sides correct
	Equation of motion for particle is $4mg - T = 4m(2a\ddot{\theta})$	M1 A1		**M1** – LHS side correct **A1** both sides correct
	Hence $4mg - T = 8ma\ddot{\theta}$ **(2)** Substitute **(1)** into **(2)**			
	$4mg - 16ma\ddot{\theta} = 8ma\ddot{\theta}$ $4mg = 24ma\ddot{\theta}$	m1		Solving a pair of simultaneous equations – dependent on both **M1**s above
	$\ddot{\theta} = \frac{g}{6a}$	A1	6	**CAO**
(b)(ii)	Using $T = 16ma\ddot{\theta}$ and part **(b)(i)**	M1		Use of their equation for T and part **(b)(i)**
	$T = 16ma\left(\frac{g}{6a}\right) = \frac{8mg}{3}$	A1F	2	Follow through their part **(b)(i)**
	ALTERNATIVE FOR 6(b)(i) Conservation of energy using Loss in PE = Gain in KE	(M1)		Must consider PE **and** two distinct KE terms and form an equation
	Loss in PE = $4mg(2a\theta) = 8mga\theta$	(A1)		PE correct
	Gain in KE = $\frac{1}{2}(32ma^2\dot{\theta}^2) + \frac{1}{2}(4m)(2a\dot{\theta})^2 = 24ma\dot{\theta}^2$	(A1,A1)		**A1**each component on LHS correct
	Gives $\dot{\theta}^2 = \frac{g\theta}{3a}$			

	Differentiating both sides $2\dot{\theta}\ddot{\theta} = \frac{g\theta\dot{\theta}}{3a}$ Hence cancelling gives $\ddot{\theta} = \frac{g}{6a}$	**m1** **A1**	 **(6)**	Differentiating to obtain angular acceleration **CAO**
	Total		**11**	

Q7	Solution	Mark	Total	Comment
(a)	MI about G for rod = $\frac{2m(4a)^2}{3} = \frac{32ma^2}{3}$	B1		Correct MI for rod – can be unsimplified
	MI about B = $\frac{32ma^2}{3} + 2ma^2 = \frac{38ma^2}{3}$	M1 A1	3	Correct application of parallel axis theorem Fully correct – **CAO**
(b)(i)	KE gained = $\frac{1}{2}\left(\frac{38ma^2}{3}\right)\dot{\theta}^2$	B1F		Correct KE – follow through part **(a)**
	PE lost = $2mga\sin\theta$	B1		Correct use of *mgh*
	Use conservation of energy			
	$\frac{1}{2}\left(\frac{38ma^2}{3}\right)\dot{\theta}^2 = 2mga\sin\theta$	M1		Form an equation using conservation of energy
	$\dot{\theta}^2 = \frac{12mga\sin\theta}{38ma^2} = \frac{6g\sin\theta}{19a}$			
	Hence angular speed is $\sqrt{\frac{6g\sin\theta}{19a}}$	A1	4	**CSO** – printed answer **- AG**
(b)(ii)	Differentiating $\dot{\theta}^2$ gives			
	$2\dot{\theta}\ddot{\theta} = \frac{6g\cos\theta}{19a}\dot{\theta}$	M1		One side correct
	Hence $\ddot{\theta} = \frac{3g\cos\theta}{19a}$	A1	2	**CAO**
	ALTERNATIVE For 7(a)			
	Using integration $\rho = \frac{m}{4a}$ and MI elemental piece = $\rho x^2 dx$	(B1)		Both needed
	MI of rod = $\int_{-3a}^{5a} \rho x^2 dx = \left[\frac{\rho x^3}{3}\right]_{-3a}^{5a} = \frac{152\rho a^3}{3}$	(M1)		Integration completed and limits used
	= $\frac{38ma^2}{3}$	(A1)	(3)	**CAO**

	ALTERNATIVE For 7(b)(ii) Use of C = I$\ddot{\theta}$ $2mg\cos\theta = \frac{38}{3}ma^2\ddot{\theta}$ Hence $\ddot{\theta} = \frac{3g\cos\theta}{19a}$	**(M1)** **(A1)**	**(2)**	Use of C = I$\ddot{\theta}$ with one side correct **CAO**
(c)	R = normal reaction F = friction Law of motion perpendicular to the rod $2mg\cos\theta - R = 2ma\ddot{\theta}$ Using **(b)(ii)** $R = \frac{32mg\cos\theta}{19}$ Law of motion parallel to the rod $F - 2mg\sin\theta = 2ma\dot{\theta}^2$ Using **(b)(i)** $F = \frac{50mg\sin\theta}{19}$ At point of slipping $F = \mu R$ hence $\frac{50mg\sin\theta}{19} = \mu\frac{32mg\cos\theta}{19}$ giving $\tan\theta = \frac{16}{25}\mu$	**M1A1** **A1F** **M1A1** **A1F** **m1** **A1**	**8**	**M1 –** one side correct. **A1** both correct Their R –correct substitution of **(b)(ii)** **M1 –** one side correct. **A1** both correct Their F –correct substitution of **(b)(i)** Correct use of $F = \mu R$ dependent on first two **M1**s **CSO** – printed answer - **AG**
	Total		**17**	