Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MPC3

Unit Pure Core 3

Friday 5 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_{1.5}^{5.5} e^{2-x} \ln(3x-2)\,dx$, giving your answer to three decimal places.

[4 marks]

(b) Find the exact value of the gradient of the curve $y = e^{2-x}\ln(3x-2)$ at the point on the curve where $x = 2$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 (a) Sketch, on the axes below, the curve with equation $y = 4 - |2x + 1|$, indicating the coordinates where the curve crosses the axes.

[4 marks]

(b) Solve the equation $x = 4 - |2x + 1|$.

[3 marks]

(c) Solve the inequality $x < 4 - |2x + 1|$.

[2 marks]

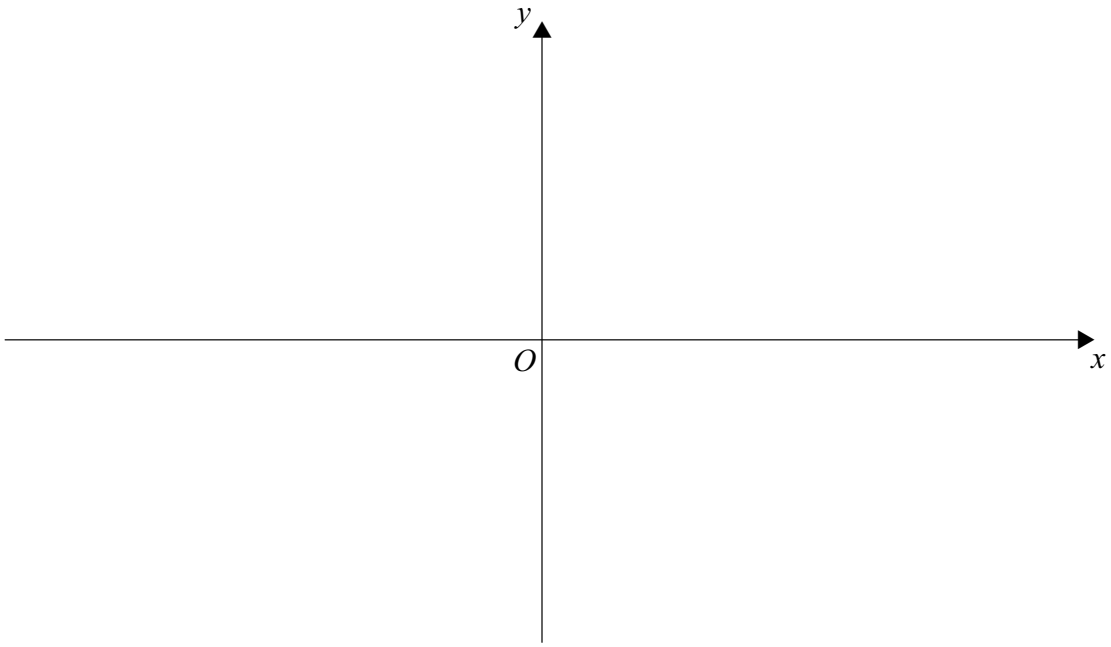
(d) Describe a sequence of two geometrical transformations that maps the graph of $y = |2x + 1|$ onto the graph of $y = 4 - |2x + 1|$.

[4 marks]

QUESTION PART REFERENCE

(a)

Answer space for question 2



QUESTION PART REFERENCE

Answer space for question 2

3 (a) It is given that the curves with equations $y = 6\ln x$ and $y = 8x - x^2 - 3$ intersect at a single point where $x = \alpha$.

(i) Show that α lies between 5 and 6.

[2 marks]

(ii) Show that the equation $x = 4 + \sqrt{13 - 6\ln x}$ can be rearranged into the form

$$6\ln x + x^2 - 8x + 3 = 0$$

[3 marks]

(iii) Use the iterative formula

$$x_{n+1} = 4 + \sqrt{13 - 6\ln x_n}$$

with $x_1 = 5$ to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

(b) A curve has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 6\ln x + x^2 - 8x + 3$.

(i) Find the exact values of the coordinates of the stationary points of the curve.

[5 marks]

(ii) Hence, or otherwise, find the exact values of the coordinates of the stationary points of the curve with equation

$$y = 2\mathrm{f}(x - 4)$$

[2 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 The functions f and g are defined by

$$\mathrm{f}(x) = 5 - \mathrm{e}^{3x}, \quad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{2x - 3}, \quad \text{for } x \neq 1.5$$

(a) Find the range of f.

[2 marks]

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$.

[3 marks]

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 0$.

[1 mark]

(c) Find an expression for $\mathrm{gg}(x)$, giving your answer in the form $\dfrac{ax + b}{cx + d}$, where a, b, c and d are integers.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

5 (a) By writing $\tan x$ as $\dfrac{\sin x}{\cos x}$, use the quotient rule to show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$.

[2 marks]

(b) Use integration by parts to find $\displaystyle\int x \sec^2 x \, \mathrm{d}x$.

[4 marks]

(c) The region bounded by the curve $y = (5\sqrt{x}) \sec x$, the x-axis from 0 to 1 and the line $x = 1$ is rotated through 2π radians about the x-axis to form a solid.

Find the value of the volume of the solid generated, giving your answer to two significant figures.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

Answer space for question 5

6 (a) Sketch, on the axes below, the curve with equation $y = \sin^{-1}(3x)$, where y is in radians.

State the exact values of the coordinates of the end points of the graph.

[3 marks]

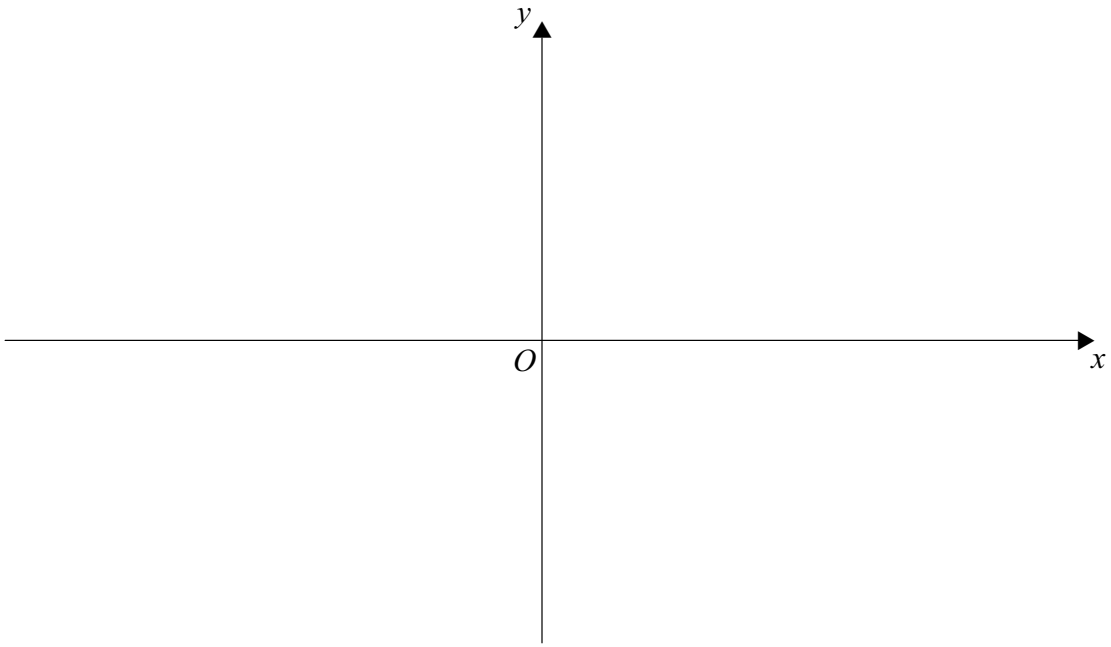
(b) Given that $x = \frac{1}{3}\sin y$, write down $\frac{\mathrm{d}x}{\mathrm{d}y}$ and hence find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y.

[2 marks]

QUESTION PART REFERENCE

(a)

Answer space for question 6



QUESTION PART REFERENCE

Answer space for question 6

7 Use the substitution $u = 6 - x^2$ to find the value of $\int_1^2 \frac{x^3}{\sqrt{6 - x^2}}\,\mathrm{d}x$, giving your answer in the form $p\sqrt{5} + q\sqrt{2}$, where p and q are rational numbers.

[7 marks]

QUESTION PART REFERENCE

Answer space for question 7

QUESTION PART REFERENCE

Answer space for question 7

8 (a) Show that the equation $4\,\text{cosec}^2\,\theta - \cot^2\,\theta = k$, where $k \neq 4$, can be written in the form

$$\sec^2\,\theta = \frac{k-1}{k-4}$$

[5 marks]

(b) Hence, or otherwise, solve the equation

$$4\,\text{cosec}^2(2x + 75°) - \cot^2(2x + 75°) = 5$$

giving all values of x in the interval $0° < x < 180°$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE

Answer space for question 8

END OF QUESTIONS