Version 1.0

**General Certificate of Education (A-level)**
**June 2012**

# Mathematics MPC4

# (Specification 6360)

# Pure Core 4

# *Mark Scheme*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MPC4**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)(i)** | $5x-6=A(x-3)+Bx$ | M1 | | Multiply by denominator and use two values of *x*. |
| | $x=0 \quad x=3$ | | | |
| | $A=2 \quad B=3$ | A1 | 2 | |
| | **Alternative**: equate coefficients | | | |
| | $-6=-3A \quad 5=A+B$ | (M1) | | Set up and solve simultaneous equations for values of *A* and *B*. |
| | $A=2 \quad B=3$ | (A1) | | |
| **(ii)** | $\left(\int \frac{2}{x}+\frac{3}{x-3}\,dx=\right) 2\ln x$ | B1ft | | their *A* ln *x* |
| | $+3\ln(x-3) \ (+C)$ | B1ft | 2 | their *B* ln (*x* – 3) and no other terms; condone *B* ln *x* – 3 |
| **(b)(i)** | $2x+1\,\overline{)\,4x^3+5x-2}$ giving quotient $2x^2-x+3$:<br>$4x^3+2x^2$<br>$-2x^2+5x$<br>$-2x^2-x$<br>$6x-2$<br>$6x+3$<br>$-5$ | M1 | | Division as far as $2x^2+px+q$ with $p\neq 0,\ q\neq 0$, PI |
| | $p=-1$ | A1 | | PI by $2x^2-x+q$ seen |
| | $q=3$ | A1 | | PI by $2x^2-x+3$ seen |
| | $r=-5$ | A1 | 4 | and must state *p*= –1, *q* = 3, *r* = –5 explicitly or write out full correct RHS expression |
| | **Alternative 1:** | | | |
| | $4x^3+5x-2=$<br>$4x^3+(2+2p)x^2+(p+2q)x+q+r$ | | | |
| | $2+2p=0$<br>$p+2q=5$<br>$q+r=-2$ | (M1) | | Clear attempt to equate coefficients, PI by *p* = –1 |
| | $p=-1$ | (A1) | | |
| | $q=3 \quad r=-5$ | (A1A1) | | |
| | **Alternative 2:** | | | |
| | $4x^3+5x-2=(2x+1)(2x^2+px+q)+r$ | | | |
| | $x=-\frac{1}{2} \quad 4\times\left(-\frac{1}{2}\right)^3+5\left(-\frac{1}{2}\right)+2=r$ | (M1) | | $x=-\frac{1}{2}$ used to find a value for $r$ |
| | $r=-5$ | (A1) | | |
| | $p=-1$ , $q=3$ | (A1A1) | | |

**MPC4**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **(b)(ii)** | $\left(\dfrac{4x^3+5x-2}{2x+1}=\right)2x^2+px+q+\dfrac{r}{2x+1}$ | M1 | | |
| | $\dfrac{2}{3}x^3-\dfrac{1}{2}x^2+3x+k\ln(2x+1)\quad(+C)$ | A1ft | | ft on $p$ and $q$ |
| | $\dfrac{2}{3}x^3-\dfrac{1}{2}x^2+3x-\dfrac{5}{2}\ln(2x+1)\quad(+C)$ | A1 | 3 | CSO |
| | **Total** | | **11** | |
| **2(a)** | $R=\sqrt{10}$ | B1 | | Accept 3.2 or better. Can be earned in (b) |
| | $\tan\alpha=3$ | M1 | | OE; M0 if $\tan\alpha=-3$ seen |
| | $\alpha=71.6$ or better | A1 | 3 | $\alpha=71.56505...$ |
| **(b)** | $\sin(x\pm\alpha)=\dfrac{-2}{R}$ | M1 | | or their $R$ and/or their $\alpha$; PI |
| | $x(=-39.2+71.6)\quad=32(.333)$ | A1 | | 32 or better<br>Condone 32.4 |
| | or | | | |
| | $x-71.6=219.2$ | m1 | | must see 219 and 72 or better<br>PI by 291 or better as answer<br>Condone extra solutions |
| | $x=$ 291 | A1 | 4 | Condone 290.8 or better CSO<br>Withhold final A1 if more than two answers given within interval |
| | **Total** | | **7** | |

**MPC4**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)** | $(1+4x)^{\frac{1}{2}} = 1 + 4\times\frac{1}{2}x + kx^2$ | M1 | | |
| | $= 1 + 2x - 2x^2$ | A1 | 2 | |
| **(b)(i)** | $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ | B1 | | OE $\frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$ |
| | $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} =$ | | | |
| | $1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{x}{4}\right)^2$ | M1 | | Condone missing brackets and use of $\left(+\frac{x}{4}\right)$ instead of $\left(-\frac{x}{4}\right)$ |
| | $=1+\frac{1}{8}x+\frac{3}{128}x^2$ | | | |
| | $(4-x)^{-\frac{1}{2}} = \frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$ | A1 | 3 | CSO <br> $0.5+0.0625x+0.0117(1875)x^2$ |
| | **Alternative** using formula from FB | | | |
| | $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)\times 4^{-\frac{3}{2}}(-x)$ | (M1) | | Condone one error and missing brackets |
| | $+\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\times 4^{-\frac{5}{2}}(-x)^2$ | | | |
| | $=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$ | (A2) | | CSO <br> Must be fully correct |
| **(b)(ii)** | $-4 < x < 4$ <br> or $x < 4$ and $x > -4$ | B1 | 1 | Condone $\lvert x\rvert < 4$ <br> Must be **and**; not **or** not , (comma) |
| **(c)** | $\sqrt{\frac{1+4x}{4-x}} = (1+4x)^{\frac{1}{2}}(4-x)^{-\frac{1}{2}}$ | | | |
| | $=\left(1+2x-2x^2\right)\left(\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2\right)$ | M1 | | product of their expansions |
| | $=\frac{1}{2}+\frac{17}{16}x-\frac{221}{256}x^2$ | A1 | 2 | CSO <br> $0.5+1.0625x-0.8632(8...)x^2$ |
| | **Total** | | **8** | |

**MPC4**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)(i)** | $1000\times1.03^5 \approx (£)1160$ | B1 | 1 | Condone missing £ sign;1160 only. |
| **(ii)** | $2000 < 1000\left(1+\frac{3}{100}\right)^n$ | B1 | | Condone '=' or '<' used throughout |
| | $\ln 2 < n\ln 1.03$ | M1 | | Take logs, any base, of their initial expression **correctly** |
| | $(n > 23.449....)\quad (N =)24$ | A1 | 3 | Condone 23 |
| **(b)** | $1000\times\left(1+\frac{3}{100}\right)^n > 1500\times\left(1+\frac{1.5}{100}\right)^n$ | B1 | | Condone use of $T$ for $n$<br>Condone '=' or '<' used throughout |
| | $\ln 1000 + n\ln 1.03 > \ln 1500 + n\ln 1.015$ | M1 | | Take logs, any base, of their initial expression **correctly** |
| | $n > \dfrac{\ln(1.5)}{\ln\left(\dfrac{1.03}{1.015}\right)}$ | A1 | | Correct expression for $n$ or $T$ |
| | $(n > 27.63...)\quad (T =)28$ | A1 | 4 | Condone 27 |
| | **Total** | | **8** | |

**MPC4**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5** **(a)(i)** | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}\theta}}{\frac{\mathrm{d}x}{\mathrm{d}\theta}} = \frac{6\cos 2\theta}{-2\sin\theta}$ | M1<br>A1 | | condone coefficient errors |
| | $= \frac{6\left(1-2\sin^2\theta\right)}{-2\sin\theta}$ | m1 | | Use $\cos 2\theta = 1 - 2\sin^2\theta$ |
| | $= 6\sin\theta - 3\operatorname{cosec}\theta$ | A1 | 4 | $a = 6 \quad b = -3$ |
| **(a)(ii)** | $\theta = \frac{\pi}{6} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = 6 \times \frac{1}{2} - 3 \times 2 = -3$ | B1ft | | $\theta = \frac{\pi}{6}$ substituted into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and evaluated |
| | gradient normal $= \frac{1}{3}$ | B1ft | 2 | ft $\frac{\mathrm{d}y}{\mathrm{d}x}$, provided non-zero |
| **(b)** | $y = 6\sin\theta\cos\theta$<br>$= (\pm)6\sqrt{1-\cos^2\theta} \times \cos\theta$ | M1 | | Correct expansion of $\sin 2\theta$ and use $x = 2\cos\theta$ to eliminate $\theta$ |
| | $= (\pm)6\sqrt{1-\left(\frac{x}{2}\right)^2} \times \left(\frac{x}{2}\right)$ | A1 | | Correct elimination of $\theta$ |
| | $y^2 = \frac{9}{4}x^2\left(4-x^2\right)$ | A1 | 3 | $p = \frac{9}{4}$ OE and $\left(4-x^2\right)$ shown |
| | **Alternative** using verification<br>$y^2 = 9\sin^2 2\theta = 36\sin^2\theta\cos^2\theta$ | (M1) | | must be squared |
| | $x^2\left(4-x^2\right) = 4\cos^2\theta \times 4\sin^2\theta$ | (A1) | | |
| | $p = \frac{9}{4}$ OE | (A1) | | or $y^2 = \frac{9}{4}x^2\left(4-x^2\right)$ |
| | **Total** | | **9** | |

**MPC4**

| Q | Solution | Marks | Total | Comments |
| --- | --- | --- | --- | --- |
| **6** | $9x^2-6xy+4y^2 \quad =3$ | | | |
| | $18x \quad =0$ | B1 | | =0 PI |
| | $-6y-6x\dfrac{\mathrm{d}y}{\mathrm{d}x}$ | B1 | | or $\dfrac{\mathrm{d}(6xy)}{\mathrm{d}x}=6y+6x\dfrac{\mathrm{d}y}{\mathrm{d}x}$ seen separately |
| | $+8y\dfrac{\mathrm{d}y}{\mathrm{d}x}$ | B1 | | $\dfrac{\mathrm{d}y}{\mathrm{d}x}(-6x+8y)=6y-18x$ |
| | Use $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ | M1 | | |
| | $\Rightarrow y=3x$ or $x=\dfrac{y}{3}$ | A1 | | CSO |
| | $y=3x \Rightarrow 9x^2-6x\times3x+4(3x)^2=3$ | m1 | | Substitute $y=ax$ into equation and solve for a value of $x$ or $y$. Condone missing brackets. |
| | $27x^2=3 \Rightarrow x=\pm\dfrac{1}{3}$ OE | A1ft | | Both values of $x$ or $y$ required. ft on their $y=3x$ |
| | $\left(\dfrac{1}{3},1\right) \quad \left(-\dfrac{1}{3},-1\right)$ | A1 | 8 | CSO Correct corresponding simplified values of $x$ and $y$. Withhold if additional answers given |
| | **Total** | | **8** | |

**MPC4**

| Q | Solution | Marks | Total | Comments |
| --- | --- | --- | --- | --- |
| **7(a)** | $2\lambda = 8 + 2\mu$<br>$-2 \quad = 3 + 5\mu$<br>$\lambda = 3 \ , \ \mu = -1$ | M1 | | Use the first two equations to set up and attempt to solve simultaneous equations for $\lambda$ or $\mu$. Must not assume $q = 4$. |
| | $q - \lambda = 5 + 4\mu$<br>$q = 5 + 3 - 4 = 4$ | A1 | | Use 3rd equation to show $q = 4$ **AG**. |
| | $P$ is at $(6, -2, 1)$ | B1 | 3 | Condone as a column vector |
| **(b)** | $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \bullet \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} = 4 - 4 = 0 \Rightarrow$ perpendicular | B1 | 1 | or $2 \times 2 + -1 \times 4 = 0$ seen **and** conclusion (condone $\theta = 90$) |
| **(c)(i)** | $A$ is at (2, –2, 3)<br>$AP^2 = (6-2)^2 + (-2--2)^2 + (1-3)^2$ | M1 | | Candidate's $\left\lvert \overrightarrow{AP} \right\rvert^2$ |
| | $= 20$ | A1 | 2 | CAO<br>NMS $AP = \sqrt{20}$ M1A0 |
| **(ii)** | $\left(\overrightarrow{PB} =\right) \begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \\ 1 \end{bmatrix} \quad \left( = \begin{bmatrix} 2+2\mu \\ 5+5\mu \\ 4+4\mu \end{bmatrix} \right)$ | M1 | | Clear attempt to find $\overrightarrow{BP}$ or $\overrightarrow{PB}$ in terms of $\mu$ |
| | $\left(PB^2 =\right)(2+2\mu)^2 + (5+5\mu)^2 + (4+4\mu)^2$ | m1 | | Find distance $BP$ in terms of $\mu$ |
| | $45\mu^2 + 90\mu + 45 = 20$<br>$(5)(9\mu^2 + 18\mu + 5) = 0$ | m1 | | Attempt to set up three-term quadratic in $\mu$ and equate to their $AP^2$ |
| | $(3\mu + 1)(3\mu + 5) = 0$ | m1 | | Solve quadratic equation to obtain **two** values of $\mu$ |
| | $\mu = -\frac{1}{3}$ and $\mu = -\frac{5}{3}$ | A1 | | Both values correct. |
| | $B$ is at $\left(\frac{22}{3}, \frac{4}{3}, \frac{11}{3}\right)$ and $\left(\frac{14}{3}, -\frac{16}{3}, -\frac{5}{3}\right)$ | A1 | 6 | Both sets of coordinates required.<br>Condone column vectors.<br>SC one value of $\mu$ correct and corresponding coordinates of $B$ correct scores A1 A0. |

**MPC4**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| | **Alternative 1** | | | |
| | $\left(\overrightarrow{AB}=\right)\begin{bmatrix}8\\3\\5\end{bmatrix}+\mu\begin{bmatrix}2\\5\\4\end{bmatrix}-\begin{bmatrix}2\\-2\\3\end{bmatrix}\quad\left(=\begin{bmatrix}6+2\mu\\5+5\mu\\2+4\mu\end{bmatrix}\right)$ | (M1) | | Clear attempt to find $\overrightarrow{AB}$ or $\overrightarrow{BA}$ in terms of $\mu$ |
| | $\left(AB^2=\right)(6+2\mu)^2+(5+5\mu)^2+(2+4\mu)^2$ | (m1) | | Find distance $AB$ in terms of $\mu$ |
| | $45\mu^2+90\mu+65=40$<br>$(5)\left(9\mu^2+18\mu+5\right)=0$ | (m1) | | Attempt to set up three-term quadratic in $\mu$ and equate to their $2\times$ their $AP^2$ |
| | *As before* | | | |
| | **Alternative 2** | | | |
| | $\overrightarrow{PB}=k\begin{bmatrix}2\\5\\4\end{bmatrix}$ | (M1) | | |
| | $k^2\left(2^2+5^2+4^2\right)=20$ | (m1)<br>(m1) | | m1 for LHS<br>m1 for equating to 'their 20'<br>May score M1m0m1 |
| | $k=\pm\frac{2}{3}$ | (A1) | | |
| | $\overrightarrow{OB}=\overrightarrow{OP}+(\pm)(\text{their value of } k)\begin{bmatrix}2\\5\\4\end{bmatrix}$ | (m1) | | |
| | $B$ is at $\left(\frac{22}{3},\frac{4}{3},\frac{11}{3}\right)$ and $\left(\frac{14}{3},-\frac{16}{3},-\frac{5}{3}\right)$ | (A1) | | |
| | **Total** | | **12** | |

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** | $\frac{\mathrm{d}h}{\mathrm{d}t}$ | B1 | | |
| | $derivative = * \times (2-h)$ | M1 | | Use of $2-h$ or $h-2$; $*$ is a constant or expression in $h$ and/or $t$. |
| | $\frac{\mathrm{d}h}{\mathrm{d}t} = k(2-h)$ | A1 | 3 | All correct; must be $(2-h)$ |
| **(b)(i)** | $\int x\sqrt{2x-1}\ \mathrm{d}x = \int \frac{1}{15}\mathrm{d}t$ | B1 | | Correct separation and notation; condone missing integral signs. |
| | $= \frac{1}{15}t$ | B1 | | |
| | Substitute $u = 2x-1$ | | | |
| | $\int x\sqrt{2x-1}\ \mathrm{d}x = \int \frac{1}{2}(u+1)\sqrt{u}\ \frac{1}{2}\mathrm{d}u$ | M1 | | Suitable substitution and attempt to write integral in terms of $u$ including d$x$ replaced by $\frac{1}{2}\mathrm{d}u$ or 2 d$u$. |
| | $= \left(\frac{1}{4}\right)\int u^{\frac{3}{2}} + u^{\frac{1}{2}}\ \mathrm{d}u$ | A1 | | $\frac{1}{4}$ need not be seen |
| | $= \frac{1}{4}\left(\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}}\right)\ (+C)$ | A1 | | Integration correct including $\frac{1}{4}$ |
| | $x = 1$, $t = 0$ | | | |
| | $u = 1$, $t = 0$ $\quad \frac{1}{4}\left(\frac{2}{5} + \frac{2}{3}\right) + C = 0$ | M1 | | Use $x = 1$, $t = 0$ to find a value for constant $C$ from equation in $x$ and $t$. |
| | $C = -\frac{4}{15}$ | A1 | | $C = -0.2666...$<br>$C = -0.267$ or better |
| | $t = \frac{1}{2}\left(3(2x-1)^{\frac{5}{2}} + 5(2x-1)^{\frac{3}{2}}\right) - 4$ | A1 | 8 | ISW $\quad t = (2x-1)^{\frac{3}{2}}(3x+1) - 4$ |
| | **Alternative** (Parts) | | | |
| | As before | (B1B1) | | |
| | $u = x$ , $\frac{\mathrm{d}v}{\mathrm{d}x} = (2x-1)^{\frac{1}{2}}$ | (M1) | | Attempt to use parts |
| | $\mathrm{d}u = 1 \quad v = k(2x-1)^{\frac{3}{2}}$ | | | |
| | $\int x\sqrt{2x-1}\ \mathrm{d}x = x\frac{1}{3}(2x-1)^{\frac{3}{2}} - \int \frac{1}{3}(2x-1)^{\frac{3}{2}}\ \mathrm{d}x$ | (A1) | | Condone missing d$x$ |
| | $= x\frac{1}{3}(2x-1)^{\frac{3}{2}} - \frac{1}{15}(2x-1)^{\frac{5}{2}}\ (+C)$ | (A1) | | |
| | $x = 1$, $t = 0$ $\quad \frac{1}{3} - \frac{1}{15} + C = 0$ | (M1) | | Use $x = 1$, $t = 0$ to find a value for constant $C$ from equation in $x$ and $t$ |
| | $C = -\frac{4}{15}$ | (A1) | | $C = -0.2666...$<br>$C = -0.267$ or better |
| | $t = 5x(2x-1)^{\frac{3}{2}} - (2x-1)^{\frac{5}{2}} - 4$ | (A1) | | ISW $\quad t = (2x-1)^{\frac{3}{2}}(3x+1) - 4$ |
| **(ii)** | $x = 2 \quad t = 32.4 \quad$ (minutes) | B1 | 1 | 32.4 or better (32.373…) |
| | **Total** | | **12** | |
| | **TOTAL** | | **75** | |