Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MFP4

Unit Further Pure 4

Friday 22 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 Find the value of the constant p for which the vectors

$$\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + p\mathbf{k}, \quad \mathbf{v} = 7\mathbf{i} - \mathbf{j} + 6\mathbf{k} \quad \text{and} \quad \mathbf{w} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$$

are linearly dependent. *(3 marks)*

QUESTION PART REFERENCE

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2 A line has vector equation $\left(\mathbf{r} - \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}\right) \times \begin{bmatrix} 4 \\ 7 \\ -4 \end{bmatrix} = \mathbf{0}$.

(a) Determine the direction cosines of this line. *(3 marks)*

(b) Explain the geometrical significance of the direction cosines in relation to the line. *(1 mark)*

QUESTION PART REFERENCE

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3 Let $\Delta = \begin{vmatrix} yz & xz & xy \\ x & y & z \\ x^2 & -y^2 & z^2 \end{vmatrix}$.

(a) Show that $(y+z)$ is a factor of Δ. *(2 marks)*

(b) Factorise Δ as completely as possible. *(4 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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4 The lines L_1 and L_2 have equations

$$\mathbf{r} = \begin{bmatrix} 7 \\ -25 \\ 9 \end{bmatrix} + \alpha \begin{bmatrix} 3 \\ -4 \\ 7 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} 7 \\ 19 \\ -2 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

respectively.

(a) Determine a vector, $\mathbf{n}$, which is perpendicular to both lines. *(2 marks)*

(b) (i) The point A on L_1 and the point B on L_2 are such that $\overrightarrow{AB} = \lambda\mathbf{n}$ for some constant λ.

Show that

$$3\alpha - 2\beta + 2\lambda = 0$$
$$4\alpha - 2\beta - 5\lambda = -44$$
$$7\alpha - 3\beta + 2\lambda = -11$$

(3 marks)

(ii) Find the position vectors of A and B. *(3 marks)*

(iii) Deduce the shortest distance between L_1 and L_2. *(2 marks)*

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5 The matrix $\mathbf{M} = \begin{bmatrix} -11 & 9 \\ -16 & 13 \end{bmatrix}$ represents the plane transformation T.

(a) (i) Determine the eigenvalue, and a corresponding eigenvector, of **M**. *(4 marks)*

(ii) Hence write down the value of m for which $y = mx$ is the invariant line of T which passes through the origin, and explain why it is actually a line of invariant points. *(2 marks)*

(iii) Show that, for this value of m, all lines with equations $y = mx + c$ are invariant lines of T. *(3 marks)*

(b) Given that T is a shear, give a full geometrical description of this transformation. *(2 marks)*

(c) Give a full geometrical description of the plane transformation represented by the matrix $\mathbf{M}^{-1}$. *(2 marks)*

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6 The planes Π_1, Π_2 and Π_3 have cartesian equations

$$2x + y - z = 3$$
$$3x - 2y + z = 5$$
$$12x - y - z = 40$$

respectively.

(a) Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$, a vector equation for the line L which is the intersection of Π_1 and Π_2. *(5 marks)*

(b) (i) Determine whether L meets Π_3, and use your answer to decide whether the system given by the equations of these three planes is consistent or inconsistent. *(3 marks)*

(ii) Describe geometrically the arrangement of the three planes. *(1 mark)*

(c) (i) Find the coordinates of a common point of Π_2 and Π_3. *(3 marks)*

(ii) **Deduce** a vector equation for the line of intersection of Π_2 and Π_3. *(1 mark)*

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7 The matrix $\mathbf{A} = \begin{bmatrix} k & 1 & 2 \\ 2 & k & 1 \\ 1 & 2 & k \end{bmatrix}$, where k is a real constant.

(a) (i) Show that there is a value of k for which

$$\mathbf{A}\mathbf{A}^{\mathrm{T}} = m\mathbf{I}$$

where m is a rational number to be determined and $\mathbf{I}$ is the 3×3 identity matrix. *(6 marks)*

(ii) Deduce the inverse matrix, $\mathbf{A}^{-1}$, of $\mathbf{A}$ for this value of k. *(1 mark)*

(b) (i) Find $\det \mathbf{A}$ in terms of k. *(2 marks)*

(ii) In the case when $\mathbf{A}$ is singular, find the integer value of k and show that there are no other possible real values of k. *(3 marks)*

(iii) Find the value of k for which $\lambda = 7$ is a real eigenvalue of $\mathbf{A}$. *(2 marks)*

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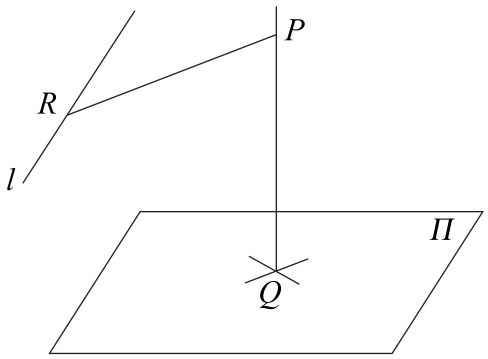
8 The point Q has position vector $\mathbf{q} = \begin{bmatrix} 7 \\ 4 \\ 6 \end{bmatrix}$, the plane Π has equation $\mathbf{r} \cdot \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = 36$,

and the line l has equation $\mathbf{r} = \begin{bmatrix} 20 \\ -8 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -7 \\ 5 \\ 3 \end{bmatrix}$.

(a) Show that Q lies in Π. *(1 mark)*

(b) Show also that l is parallel to Π. *(2 marks)*

(c) The diagram shows the point P, which lies on the normal to Π that passes through Q. The point R is the point on l which is closest to P, and $PQ = PR$.



Determine the coordinates of P. *(9 marks)*

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END OF QUESTIONS