Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics **MM05**

(Specification 6360)

Mechanics 5

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

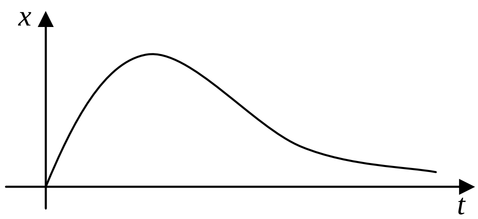
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM05

Q	Solution	Marks	Total	Comments
1(a)	$f = \frac{1}{3}$	B1	1	Accept 0.333
(b)	$3 = 2\pi\sqrt{\frac{L}{9.8}}$	M1		
	$L = 2.23$ metres	A1	2	
	Total		**3**	
2(a)	$v^2 = \omega^2\left(a^2 - x^2\right)$			
	$25 = \omega^2\left(a^2 - 9\right)$	M1		
	$\frac{25}{4} = \omega^2\left(a^2 - 36\right)$	M1		
	$4 = \frac{a^2 - 9}{a^2 - 36}$	m1		
	$4a^2 - 144 = a^2 - 9$			
	$3a^2 = 135$			
	$a^2 = 45$			
	$a = 3\sqrt{5}$ metres	A1	4	AG
(b)	max speed $= \omega a$			
	$\omega^2 = \frac{25}{45-9} = \frac{25}{36}$	M1		
	$\omega = \frac{5}{6}$	A1		
	max speed $= a\omega = 3\sqrt{5} \times \frac{5}{6}$	M1		
	$= \frac{5\sqrt{5}}{2}$	A1	4	Accept 5.59; ft slip in ω
	Total		**8**	

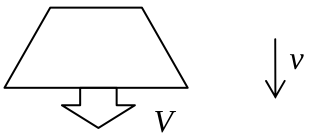
MM05 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$m\ddot{x} = -\dfrac{\lambda x}{a}$	M1		
	$= -amn^2 \dfrac{x}{a}$			
	$\ddot{x} = -n^2 x$ SHM	A1	2	
(ii)	$T = \dfrac{2\pi}{n}$	B1	1	
(b)(i)	$m\ddot{x} = -amn^2 \dfrac{x}{a} - mkv$	M1 A1		3 appropriate terms attempted Signs consistent
	$\ddot{x} + k\dot{x} + n^2 x = 0$	A1	3	AG
(ii)	$p^2 + \dfrac{5n}{2} p + n^2 = 0$			$2p^2 + 5np + 2n^2 = 0$
	$\left(p + \dfrac{5n}{4}\right)^2 - \dfrac{9n^2}{16} = 0$	M1		$(2p+n)(p+2n) = 0$
	$p + \dfrac{5n}{4} = \pm\dfrac{3n}{4}$, $p = -2n,\ p = -\dfrac{n}{2}$	A1		$p = -\dfrac{n}{2},\ p = -2n$
	$x = Ae^{-2nt} + Be^{-\frac{n}{2}t}$	M1		
	$t = 0,\ x = 0:\quad A + B = 0$ $t = 0,\ \dot{x} = U$			
	$\dot{x} = -2nAe^{-2nt} - \dfrac{n}{2}Be^{-\frac{n}{2}t}$	m1		
	$U = -2nA - \dfrac{n}{2}B$			
	$A = -\dfrac{2U}{3n} \quad B = \dfrac{2U}{3n}$	A1,A1	6	
	$x = \dfrac{2U}{3n}\left(e^{\frac{-nt}{2}} - e^{-2nt}\right)$			
(iii)		B1		Accept sketch with correct shape not reaching origin but not crossing *x*-axis elsewhere Accept reference to real distinct roots of auxiliary equation
	Heavy damping	B1	2	Independent of previous mark
	Total		**14**	

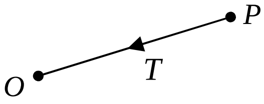
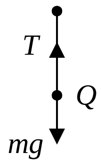
MM05 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$h = b\cot\theta$	B1		OE
	$y = a\cos\theta$	B1		OE
	$V = mgh + 2mg(h - y)$	M1		
		A1		Top rod
		A1		Other rods
	$V = mg(h + 2h - 2y)$			
	$V = mg(3b\cot\theta - 2a\cos\theta)$	A1	6	AG
(b)	$\frac{\mathrm{d}v}{\mathrm{d}\theta} = mg\left(3b\left(-\operatorname{cosec}^2\theta\right) + 2a\sin\theta\right)$	M1A2		
	$0 = -3b\operatorname{cosec}^2\theta + 2a\sin\theta$	m1		
	$\sin^3\theta = \frac{3b}{2a}$	A1	5	AG
(c)(i)	$b = \frac{a}{3}$ $\quad \sin^3\theta = \frac{1}{2}$			
	$\sin\theta = 0.7937$	M1		
	$\theta = 0.917$ or $\theta = 2.22$	A1,A1	3	-1 if degrees
(ii)	$\frac{\mathrm{d}^2v}{\mathrm{d}\theta^2} = mg(2a\cos\theta + 2a\operatorname{cosec}\theta\operatorname{cosec}\theta\cot\theta)$	M1A1		
	$= mg\left(2a\cos\theta + 2a\frac{\cos\theta}{\sin^3\theta}\right)$			
	$= mg(2a\cos\theta + 4a\cos\theta)$			
	$= 6mga\cos\theta$	A1	3	AG
(iii)	$\theta = 0.917$, $\ddot{\theta} = 3.65mga$, stable	B1		
	$\theta = 2.22$, $\ddot{\theta} = -3.65mga$, unstable	B1	2	
	Total		**19**	

MM05 (cont)

Q	Solution	Marks	Total	Comments
5(a)				
	$Mg_1\delta t = (M+\delta M)(v+\delta v) - Mv - \delta M(v+V)$	M1		
	$Mg_1\delta t = Mw + M\delta v + v\delta M - Mv - v\delta M - V\delta M$	A2		
	$Mg_1\delta t = M\delta v - V\delta M$			
	$Mg_1 + \frac{V\mathrm{d}M}{\mathrm{d}t} = \frac{M\mathrm{d}v}{\mathrm{d}t}$	M1		
	$\frac{\mathrm{d}M}{\mathrm{d}t} = -\lambda$	B1		
	$M\frac{\mathrm{d}v}{\mathrm{d}t} = Mg_1 - \lambda V$	A1	6	AG
(b)(i)	$m = 1800 - 50t$	B1		
	$(1800-50t)\frac{\mathrm{d}v}{\mathrm{d}t} = (1800-50t)g_1 - 50\times 360$	M1		Substitute
	$(36-t)\frac{\mathrm{d}v}{\mathrm{d}t} = (36-t)g_1 - 360$	A1		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 1.62 - \frac{360}{36-t}$	A1	4	AG
(ii)	$\int_{75}^{v}\mathrm{d}v = \int_0^t\left(g_1 - \frac{360}{36-t}\right)\mathrm{d}t$			
	$[v]_{75}^{v} = \left[g_1 t + 360\ln(36-t)\right]_0^t$	M1A1		For A1, require constant or presence of limits
	$v - 75 = g_1 t + 360\ln\frac{36-t}{36}$			
	$v = 75 + 1.62t + 360\ln\frac{36-t}{36}$	A1	3	AG
(c)	$t = 7.5,\quad v = 3.05$	B1		
	$v^2 = u^2 + 2as:\ v^2 = 3.05^2 + 2\times 5\times 1.62$	M1		
	$v = 5.05\ \mathrm{m\,s^{-1}}$	A1	3	
	Total		**16**	

MM05 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)		B1	1	
(ii)		B1	1	
(b)	For Q, $T - mg = m\ddot{x}$	M1		
	$\ddot{x} = \ddot{r}$ $\therefore$ $T - mg = m\ddot{r}$	A1	2	AG
(c)	Consider P:			
	$-T = m\left(\ddot{r} - r\dot{\theta}^2\right)$	M1A1		
	$-mg = 2m\ddot{r} - mr\dot{\theta}^2$	m1		
	$2\ddot{r} = r\dot{\theta}^2 - g$	A1	4	AG
(d)	Transverse acceleration = 0 $\Rightarrow$			
	$\frac{1}{r}\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\theta\right)\right) = 0$	B1		
	$r^2\dot{\theta} = \text{constant}$	B1		
	Initially $r^2\dot{\theta} = a \times 2\sqrt{ag}$			
	$\therefore \dot{\theta} = \frac{2a\sqrt{ag}}{r^2}$	M1A1		
	$2\ddot{r} = r\left(\frac{2a\sqrt{ag}}{r^2}\right) - g = \frac{4a^3g}{r^3} - g$	A1	5	AG
(e)	Initially $\left.\begin{matrix} r = a \\ \dot{r} = 0 \end{matrix}\right\}$ $\therefore \frac{4a^3}{r^3} > 1$	M1		
	$\ddot{r} > 0$			
	$\therefore$ direction away from O	A1	2	
	Total		**15**	
	TOTAL		**75**	