

General Certificate of Education June 2010

Mathematics

MPC1

Pure Core 1

Mark Scheme

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Key to mark scheme and abbreviations used in marking

M	mark is for method				
m or dM	mark is dependent on one or more M marks and is for method				
A	mark is dependent on M or m marks and is for accuracy				
В	mark is independent of M or m marks and is for method and accuracy				
Е	mark is for explanation				
$\sqrt{\text{or ft or F}}$	follow through from previous				
	incorrect result	MC	mis-copy		
CAO	correct answer only	MR	mis-read		
CSO	correct solution only	RA	required accuracy		
AWFW	anything which falls within	FW	further work		
AWRT	anything which rounds to	ISW	ignore subsequent work		
ACF	any correct form	FIW	from incorrect work		
AG	answer given	BOD	given benefit of doubt		
SC	special case	WR	work replaced by candidate		
OE	or equivalent	FB	formulae book		
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme		
–x EE	deduct x marks for each error	G	graph		
NMS	no method shown	c	candidate		
PI	possibly implied	sf	significant figure(s)		
SCA	substantially correct approach	dp	decimal place(s)		

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

MPC1 Q	Solution	Marks	Total	Comments
1(a)	$y = \frac{14}{3} - \frac{2}{3}x$	M1		Attempt at $y =$
1(a)		IVI I		Attempt at $y = \dots$
	Gradient $AB = -\frac{2}{3}$	A 1	2	Condone error in rearranging equation
	3			
(b)(i)	y-7 = "their grad AB"(x-3)	M1		or $2x + 3y = k$ and sub $x = 3, y = 7$
	, ,			or $y = mx + c$, $m = their grad AB$ and
	2			attempt to find c using $x = 3$, $y = 7$
	$y - 7 = -\frac{2}{3}(x - 3)$ OE	A1	2	$2x+3y = 27$, $y = -\frac{2}{3}x+9$ etc
(ii)	$m_1 m_2 = -1$	M1		or negative reciprocal (stated or used PI)
	$m_1 m_2 = -1$ $\Rightarrow \text{grad } AD = \frac{3}{2}$	A1√		FT their grad AB
	$y-7=\frac{3}{2}(x-3)$	A 1		Any correct equation unsimplified
	$\Rightarrow 3x - 2y + 5 = 0$			Integer coefficients; all terms on one side,
	$\Rightarrow 3x - 2y + 3 = 0$	A 1	4	condone different order or multiples. eg $0 = 4y - 6x - 10$
(c)	2x + 3y = 14 and $5y - x = 6$ used			
	with x or y eliminated (generous)	M1		2(5y-6)+3y=14 etc
	x = 4, $y = 2$	A1 A1	3	B(4,2) full marks NMS
	y - 2 Total	AI	11	<i>b</i> (4,2) full limits (4)
2(a)	$(3-\sqrt{5})^2 = 9-6\sqrt{5}+(\sqrt{5})^2$	M1		Allow one slip in one of these terms
	$(3 - \sqrt{5})^2 = 9 - 6\sqrt{5} + (\sqrt{5})^2$ $= 14 - 6\sqrt{5}$	A1	2	M0 if middle term is omitted
			_	
(h)	$\frac{\left(3 - \sqrt{5}\right)^2}{1 + \sqrt{5}} \times \frac{1 - \sqrt{5}}{1 - \sqrt{5}}$	M1		or $\times \frac{\sqrt{5}-1}{\sqrt{5}-1}$
(b)	$\frac{1+\sqrt{5}}{1-\sqrt{5}}$	M1		$\sqrt{5}-1$
	$14 + 6\sqrt{5}\sqrt{5} - 6\sqrt{5} - 14\sqrt{5}$	m1		Expanding <i>their</i> numerator
	$\frac{1+\sqrt{5}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$ $14+6\sqrt{5}\sqrt{5}-6\sqrt{5}-14\sqrt{5}$ $(=44-20\sqrt{5})$	1111		(condone one error or omission)
	(Denominator) = -4	B1		Must be seen as denominator
	$(Answer) = -11 + 5\sqrt{5}$	A1	4	Accept "answer = $5\sqrt{5} - 11$ "
	Total		6	
	1 otai		J	I .

O O	Solution	Marks	Total	Comments
	$p(-3) = (-3)^3 + 7(-3)^2 + 7(-3) - 15$		TULAI	p(-3) attempted; NOT long division
3(a)(i)	p(-3) = (-3) + 7(-3) + 7(-3) - 13 $= -27 + 63 - 21 - 15$	M1		- (
				This line alone implies M1
	$p(-3)=0 \Rightarrow (x+3 \text{ is}) \text{ factor}$	A1	2	p(-3) shown = 0 plus statement
(ii)	$p(x) = (x+3)(x^2 + px + q)$	M1		Full long division, comparing coefficients or by inspection either $p = 4$ or $q = -5$
	(Quadratic factor) $(x^2 + 4x - 5)$	A1		or M1 A1 for either <i>x</i> –1 or <i>x</i> +5 <i>clearly</i> found using Factor Theorem
	(p(x) =) (x+3)(x-1)(x+5)	A1	3	Must be seen as a product of 3 factors NMS full marks for correct product
				SC B2 for 3 correct factors listed NMS SC B1 for $(x + 3)(x - 1)()$ or $(x + 3)(x + 5)()$ or $(x + 3)(x + 1)(x - 5)$
(b)	$p(2) = 2^3 + 7 \times 2^2 + 7 \times 2 - 15$	M1		NOT long division; must be p(2)
(2)	or (2+3)(2-1)(2+5)	1,11		May use "their" product of factors
	(Remainder) = 35	A1cso	2	r r r r r r r r r r r r r r r r r r r
	()			
(c)(i)	p(-1) = -16; $p(0) = -15\Rightarrow p(-1) < p(0)$	B1	1	Values must be evaluated correctly
(ii)				
(11)	<i>y</i> ↑ ,	B1		y- intercept -15 marked or $(0,-15)$ stated
		M1		Cubic graph – 1 max, 1 min
	-5 -3 1 x	A1		✓ shape with −5, −3, 1 marked
		A1	4	Graph correct with minimum point to left of <i>y</i> -axis and going beyond both –5 and 1
	Cannot score M1A0A1 but can score			Previous A1 must be scored
	B0M1A1A1 Total		12	
	Total		14	

O O	Solution	Marks	Total	Comments
			Total	
	$x^5 + 8 = 2$	M1		One term correct
4(a)(i)	$\frac{1}{5} - \frac{1}{2}x^2 + 9x$	A1		Another term correct
	3 2	A1		All correct (may have $+ c$)
	$\frac{32}{5}$ - 16 + 18	m1		F(2) attempted
	$\frac{x^5}{5} - \frac{8}{2}x^2 + 9x$ $\frac{32}{5} - 16 + 18$ $= 8\frac{2}{5}$	A1	5	$\frac{42}{5}$, 8.4
(ii)	Shaded area = 18 – 'their integral'	M1		PI by 18 – (a)(i) NMS
	$=9\frac{3}{5}$	A1	2	$\frac{48}{5}$, 9.6 NMS full marks
a.v.a.	dv 4.3 0	M1		One term correct
(b)(i)	$\frac{3}{dx} = 4x^3 - 8$	A1		All correct (no $+ c$ etc)
	$\frac{dy}{dx} = 4x^3 - 8$ $x = 1 \Rightarrow \frac{dy}{dx} = 4 - 8$ (Gradient of curve) = -4	m1		$sub x = 1 into their \frac{dy}{dx}$
	(Gradient of curve $)=-4$	Alcso	4	No ISW
(ii)	y-2=-4(x-1); y=-4x+c, c=6	B1√	1	any correct form; FT <i>their</i> answer from (b)(i) but must use $x = 1$ and $y = 2$
	Total		12	

MPC1 (cont		Marilya	Total	Comments
Q	Solution	Marks	Total	Comments One term correct LUS
5(a)	$(x+5)^2 + (y-6)^2 = 5^2$	M1		One term correct LHS LHS all correct
	(x+3) + (y-6) = 3	A1 B1	3	RHS correct: condone = 25
		Di	3	KHS correct. condone – 25
(b)(i)	sub $x = -2$, $y = 2$ into circle equation			Circle equation must be correct
(6)(1)	-			Cheie equation must be correct
	$3^2 + (-4)^2 = 25$			
	\Rightarrow lies on circle	B1	1	Must have concluding statement
				Č
(::)	Cred DC - 4	D1		Condons 4
(ii)	Grad $PC = -\frac{4}{3}$	B1		Condone $\frac{4}{-3}$
	Normal to circle has equation			
	y-6 = 'their gradient PC'(x+5)	M1		M0 if tangent attempted or incorrect
	· · ·	IVII		coordinates used
	or $y-2 = 'their gradient PC'(x+2)$			
	$y-6=-\frac{4}{3}(x+5)$			Any correct form eg $4x+3y+2=0$
	3 (3 + 5)	Alcso	3	
	or $y-2=-\frac{4}{3}(x+2)$	AICSU	3	$y = -\frac{4}{3}x + c$, $c = -\frac{2}{3}$
	or $y-2=-\frac{1}{3}(x+2)$			3 3
				Alternative 1
(;;;)	DM 1 v madina	M1		Attempt at $M\left(-\frac{7}{2},4\right)$ with at least one
(iii)	$PM = \frac{1}{2} \times \text{radius}$	M1		Attempt at $M\left(-\frac{1}{2}, \frac{1}{2}\right)$ with at least one
				correct coordinate and PM ² attempted
	-25	A 1		
	= 2.5	Alcso		$PM^2 = \frac{9}{4} + 4 = \frac{25}{4}$
	$PO = \sqrt{8}$	B1		$PO^2 = 4 + 4 = 8$
	P is closer to the point M	E1cso	4	Statement following correct values
		21000	·	Survey Tone wang Control was a
				Alternative 2
				7 ()
		(M1)		Attempt at $M\left(-\frac{7}{2}, 4\right)$ with at least one
		(M1)		correct coordinate and attempt at vectors
				or difference of coordinates
		(Alcso		(1.5)
)		$\overline{PM} = \begin{pmatrix} -1.5\\2 \end{pmatrix}$ OE
		(E1aaa)		P is closer to the point M
		(Elcso)		·
		(E1)	(4)	Components of their PM and OP
	m . x			considered – <i>totally independent</i> of M1
	Total		11	

Q Q	Solution	Marks	Total	Comments
6(a)(i)	S.A. = $4xy + 5xy + 3xy + 6x^2 + 6x^2$ OE	M1		Condone one slip or omission
	$=12xy+12x^2$	A1		_
	$144 = 12xy + 12x^2$			Must see this line
	$\Rightarrow xy + x^2 = 12$	Alcso	3	AG
	1			
(ii)	(Volume =) $\frac{1}{2} \times 3x \times 4x \times y$ OE	M1		
	2			$(12-x^2)$
	$=6x^2 \times \frac{(12-x^2)}{x}$			Must see $(y =) \frac{(12 - x^2)}{x}$ or $xy = 12 - x^2$
				for A1
	$(V=)72x-6x^3$	A1	2	AG must be convinced not working back from answer
(b)(i)	$\frac{dV}{dx} = 72 - 18x^2$	M1 A1	2	One term correct All correct (no + c etc)
	$\mathrm{d}x$	Ai	2	All collect (no + c etc)
(ii)	$x=2 \Rightarrow \frac{dV}{dx} = 72 - 18 \times 2^2$	M1		Substitute $x = 2$ into their $\frac{dV}{dx}$
(11)	d.v	1711		Substitute $x=2$ into then $\frac{1}{dx}$
	$\Rightarrow \frac{dV}{dx} = 72 - 72 = 0$			
	$\Rightarrow \text{ stationary (value when } x = 2)$	A1	2	Shown = 0 plus statement
				Statement may appear first
	1277			117
(c)	$\frac{\mathrm{d}^2 V}{\mathrm{d}x^2} = -36x$	B1√		FT their $\frac{dV}{dx}$
				u.
	$\frac{d^2V}{dx^2} = -72$ or when $x = 2 \Rightarrow \frac{d^2V}{dx^2} < 0$			
	dx^2 dx^2 dx^2			12
	⇒maximum	E1√	2	FT their $\frac{d^2V}{dx^2}$ value when $x = 2$
				with appropriate conclusion
	Total		11	

MPC1 (cont				
Q	Solution	Marks	Total	Comments
7(a)(i)	$2(x-5)^2$	B1		p = 5
	+ 3	B1	2	q=3
(ii)	Stating both $(x-5)^2 \geqslant 0$ and $3 > 0$	M1		FT their $p \& q$, but must have $q > 0$
	$\Rightarrow 2x^2 - 20x + 53 > 0 \text{ or } 2(x-5)^2 + 3 > 0$			
	$\Rightarrow 2x^2 - 20x + 53 = 0$ has no real roots	Alcso	2	Must have statement and correct $p \& q$.
(b)(i)	$b^2 - 4ac = (k+1)^2 - 4k(2k-1)$	M1		Condone one slip (including <i>x</i> is one slip)
	$=-7k^2+6k+1$	A1		Condone recovery from missing brackets
	real roots $\Rightarrow b^2 - 4ac \geqslant 0$			Their discriminant ≥ 0 (in terms of k)
	$-7k^2 + 6k + 1 \geqslant 0$	B1√		Need not be simplified & may earn earlier
	$\Rightarrow 7k^2 - 6k - 1 \le 0$	Alcso	4	AG (must see sign change)
	$\rightarrow m$ on $1 \leqslant 0$	AICSO	-	AG (must see sign change)
(ii)	(7k+1)(k-1)	M1		Correct factors or correct use of formula
				May score M1, A1 for correct critical
	Critical values $k = 1, -\frac{1}{7}$	A 1		values seen as part of incorrect final
	,			answer with or without working.
	Use of sign diagram or sketch	3.61		If previous A1 earned, sign diagram or
		M1		sketch must be correct for M1
	$(+)$ $-\frac{1}{2}$ $(-)$ 1 $(-)$			
	,			Otherwise M1 may be earned for an
				attempt at the sketch or sign diagram
	$\frac{1}{-\frac{1}{7}}$			using <i>their</i> critical values.
	1			
	$-\frac{1}{7} \leqslant k \leqslant 1$	A1	4	$\left \left(-\frac{1}{7} < k < 1 \right), \left(k \geqslant -\frac{1}{7} \text{ OR } k \leqslant 1 \right) \right ,$
	,			
	Full marks for correct answer NMS			$\left(k \geqslant -\frac{1}{7}, \ k \leqslant 1\right) \text{ score M1A1M1A0}$
	2			Answer only of $k < -\frac{1}{7}$, $k < 1$ etc
	Condone $-\frac{2}{14}$ throughout			/
	Condens 1 > 1 AND 1 < 1 C CH			scores M1, A1, M0 since the critical values are evident.
	Condone $k \geqslant -\frac{1}{7}$ AND $k \leqslant 1$ for full			
	marks			Answer only of $\frac{1}{7} \leqslant k \leqslant 1$ etc
	Take their final line as their answer.			scores M0, M0 since the critical values
	<u> </u>			are not both correct.
	Total		12	
	TOTAL		75	