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General Certificate of Education

Mathematics 6360

MS03 Statistics 3

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS03

Q	Solution	Marks	Total	Comments
1(a)	$\hat{p}_1 = \dfrac{102}{150} = 0.68$ $\hat{p}_2 = \dfrac{36}{80} = 0.45$	B1		Both CAO
	99% (0.99) $\Rightarrow$ z = 2.57 to 2.58	B1		AWFW (2.5758)
	CI for $(p_1 - p_2)$ is			
	$(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	M1 m1		Use of $(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\text{attempted variance}}$ Use of correct expression for variance
	Thus $(0.68 - 0.45) \pm 2.5758 \times \sqrt{\dfrac{0.68 \times 0.32}{150} + \dfrac{0.45 \times 0.55}{80}}$	A1F		F on $\hat{p}_1$, $\hat{p}_2$ and z
	Hence 0.23 ± (0.173 to 0.174) **or** (0.056 to 0.057, 0.403 to 0.404)	A1	6	CAO & AWFW (accept 0.17) AWFW (accept 0.06 & 0.4) **Note:** Pooling of variances Maximum of B1 B1 M1
(b)	Whole of confidence interval is **above 0 or zero** so	B1F		F on (a) Or equivalent
	Disagree with claim / claim appears **doubtful**	B1F	2	F on (a) Or equivalent Dependent on previous B1F
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	P(B & B) = $(0.30 \times 0.80) + (0.55 \times 0.10) + (0.15 \times 0.30)$	M1		Use of **3 possibilities** each the product of **2 probabilities**
	$= 0.24 + 0.055 + 0.045 = 0.34$	A1	2	CAO; **AG**
(ii)	P(HB $\cap$ Coastal) = 0.55×0.65	M1		Can be implied by **correct** answer
	= 143/400 or 0.357 to 0.358	A1	2	CAO/AWFW (0.3575)
(iii)	P(Coastal | HB) = $\dfrac{P(\text{Coastal} \cap \text{HB})}{P(\text{HB})}$	M1 M1		$\dfrac{\text{answer to (ii)}}{\sum(3\times2)\text{ probabilities}}$
	$= \dfrac{0.3575}{(0.3\times0.15)+(0.3575)+(0.15\times0.5)}$	A1F		F on (ii)
	$= \dfrac{0.3575}{0.4775}$ = 143/191 or 0.747 to 0.75	A1	4	CAO/AWFW (0.74869)
(b)	P(City | HB) = $\dfrac{0.3\times0.15}{P(\text{HB})} = \dfrac{0.045}{0.4775} = \dfrac{90}{955}$	M1		
	P(Country | HB) = $\dfrac{0.15\times0.5}{P(\text{HB})} = \dfrac{0.075}{0.4775} = \dfrac{30}{191}$	M1		Or $\left(1 - (\text{a})(\text{iii}) - \dfrac{0.045}{0.4775}\right)$
	Thus Probability = $\dfrac{0.045}{P(\text{HB})}\times\dfrac{0.3575}{P(\text{HB})}\times\dfrac{0.075}{P(\text{HB})}$	M1		Multiplication of 3 different probabilities
	Multiplied by 3! = 6	B1		CAO
	$= 0.09424 \times 0.74869 \times 0.15707 \times 6$			
	= 0.063 to 0.068	A1	5	AWFW (0.06649)
	Total		**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
3	98% (0.98) CI $\Rightarrow$ $z = 2.32$ to 2.33	B1		AWFW (2.3263)
	CI width is $2 \times z \times \sqrt{\frac{p(1-p)}{n}}$	M1		Used; allow $z \times \sqrt{\frac{p(1-p)}{n}}$
	$p = 0.35$ or 0.50	B1		
	Thus $2 \times 2.3263 \times \sqrt{\frac{0.35 \times 0.65}{n}} = 0.1$	A1F		Or equivalent F on z; allow no multiplier of 2 and/or $p = 0.50$
	Thus $\sqrt{n} = \frac{2 \times 2.3263}{0.1} \times \sqrt{0.35 \times 0.65}$	m1		Solving for $\sqrt{n}$ or n
	Thus $n = 492.5$ $(p = 0.35)$ or $n = 541.2$ $(p = 0.50)$ Thus to nearest 10 $n = 500$ or 490	A1	6	Either
	Notes: No '$\times 2$' gives $n = 123.1$ No '$\times 2$' and $p = 0.50$ gives $n = 135.3$			
	Total		**6**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
4	H_0: $\mu_X - \mu_Y = 15$	B1		Or equivalent Accept H_0: $\mu_X - \mu_Y = 0$
	H_1: $\mu_X - \mu_Y > 15$	B1		Or equivalent
	SL α = 1% (0.01)			
	CV z = 2.32 to 2.33	B1		AWFW (2.3263) If H_1 involves '≠' then accept 2.57 to 2.58 (2.5758)
	CV t = 2.35 to 2.36	(B1)		AWFW If H_1 involves '≠' then accept 2.60 to 2.62
	$z = \dfrac{(\bar{x}-\bar{y})-15}{\sqrt{\dfrac{s_X^2}{n_X}+\dfrac{s_Y^2}{n_Y}}}$ **or** $z/t = \dfrac{(\bar{x}-\bar{y})-15}{\sqrt{s_P^2\left(\dfrac{1}{n_X}+\dfrac{1}{n_Y}\right)}}$	M1		Used Allow 'no –15'
	$s_P^2 = \dfrac{(64\times3.4^2)+(74\times2.8^2)}{65+75-2}$ $= \dfrac{1320}{138} = 9.56522$			$s_P = 3.09277$
	$z = \dfrac{(40.7-24.4)-15}{\sqrt{\dfrac{3.4^2}{65}+\dfrac{2.8^2}{75}}} = \dfrac{1.3}{\sqrt{0.28238}}$	A1 A1		Numerator; allow 'no –15' Denominator
	= 2.44 to 2.45	A1		AWFW (2.4464) 'no –15' gives z = 30.674
	OR			
	$z/t = \dfrac{(40.7-24.4)-15}{\sqrt{\dfrac{1320}{138}\left(\dfrac{1}{65}+\dfrac{1}{75}\right)}} = \dfrac{1.3}{\sqrt{0.27469}}$	(A1) (A1)		Numerator; allow 'no –15' Denominator
	= 2.48	(A1)		AWRT (2.4804) 'no –15' gives z = 31.100
	Thus evidence, at 1% level, to support Holly's belief	A1F	8	F on z and CV
	Total		**8**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
5	$\underline{X \sim \text{B}(n,\ p)}$			
(a)	$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$	M1		Used; may be implied
	$= \text{E}[X(X-1)] + \text{E}(X) - [\text{E}(X)]^2$ $= n(n-1)p^2 + np - n^2p^2$	M1		Rearranging & substitution
	$= np - np^2 = np(1-p)$	A1		Or equivalent
	OR			
	$\text{E}[X(X-1)] = \text{E}(X^2) - \text{E}(X)$ $= n(n-1)p^2 = n^2p^2 - np^2$	(M1)		Expansion & substitution
	$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$	(M1)		Used; may be implied
	$= \{n^2p^2 - np^2 + \text{E}(X)\} - n^2p^2$			
	$= np - np^2 = np(1-p)$	(A1)	3	Or equivalent
(b)(i)	Mean $= np = 36$ SD $= \sqrt{np(1-p)} = 4.8$	B1		Both CAO
	Thus $36(1-p) = 4.8^2$	M1		Attempt to solve for p or n
	Thus $n = 100$ & $p = 0.36$	A1	3	Both CAO
(ii)	$\text{P}(30 < X < 40) =$			
	$\text{P}\left(Z < \dfrac{39.5-36}{4.8}\right) - \text{P}\left(Z < \dfrac{30.5-36}{4.8}\right) =$	M1 B1		Standardising (39.5, 40 or 40.5) or (29.5, 30 or 30.5) with 36 and 4.8 and/or $(36-x)$ Use of 39.5 & 30.5
	$\text{P}(Z < 0.73) - \text{P}(Z < -1.15) =$			
	$\text{P}(Z < 0.73) - [1 - \text{P}(Z < 1.15)] =$	m1		Area change
	$0.76730 - [1 - (0.87286 \text{ to } 0.87493)] =$			
	0.64 to 0.643	A1	4	AWFW (0.64112)
	Total		**10**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$E(X) = \underline{\mathbf{2.2}}$	B1		CAO
	$Var(X) = E(X^2) - 2.2^2 =$	M1		Used; or equivalent
	$6.8 - 4.84 = 1.96$	A1	3	CAO
(b)(i)	$E(S) = E(X) + 2.0 = 4.2$	B1F		F on (a)
	$Var(S) = Var(X) + 1.5 + 2 \times (-0.43)$	M1		Used for S or D
	$= 2.6$	A1F		F on (a)
(ii)	$E(D) = E(X) - 2.0 = 0.2$	B1F		F on (a)
	$Var(D) = Var(X) + 1.5 - 2 \times (-0.43)$			
	$= 4.32$	A1F	5	F on (a)
(c)	$\underline{L \sim N(2.31,\ 0.89^2) \quad M \sim N(2.04,\ 0.43^2)}$			
	$T = L + M \sim N(\mathbf{4.35},\ \mathbf{0.977})$	B1 B1		Both CAO; SD = 0.98843
	$P(T > 5) = P\left(Z > \frac{5 - 4.35}{\sqrt{0.977}}\right)$	M1		Standardising 5 or 5.01 using C's mean & SD
	$= P(Z > 0.66) = 1 - P(Z < 0.66)$	m1		Area change
	0.25 to 0.26	A1	5	AWFW (0.25540)
	Total		**13**	

MS03 (cont)

Q	Solution	Marks	Total	Comments
7	$X_D \sim \text{Po}(24)$			
(a)	$T = X_{\Sigma D} \sim \text{Po}(\mathbf{144})$	B1		CAO
	Thus $T \sim$ approx N(144, 144)	M1		Normal with $\mu = \sigma^2$
	$P(T_{Po} \le 150) \approx P(T_N < \mathbf{150.5})$	B1		CAO
	$= P\left(Z < \dfrac{150.5 - 144}{12}\right)$	M1		Standardising (149.5, 150 or 150.5) with $\mu > 24$ and $\sqrt{\mu}$
	$= P(Z < 0.54) = 0.705$ to 0.71	A1	5	AWFW (0.70598)
(b)(i)	H_0: λ(or mean) = 2 (or 10) H_1: λ(or mean) > 2 (or 10)	B1		Both; or equivalent
	$P(Y \ge 17) = 1 - P(Y \le 16)$	M1		Accept $1 - P(Y \le 17)$
	$= 1 - 0.0.9730 = 0.027$	A1		AWRT
	< 0.10 (10%) [$z = 2.05$ to $2.38 > 1.2816$]	M1		Comparison of probability with 0.1 Comparison of z with 1.2816 or 1.6449
	Thus evidence, at 10% level, of increase in mean daily number of requests	A1F	5	F on probability or on z
(ii)	CV of Y is such that $P(Y \ge \text{CV}) \le 0.10$ (10%)	M1		Can be implied by 13, 14 or 15 Accept $P(Y = \text{CV}) = 0.10$
	Thus $P(Y \le \text{CV} - 1) \ge 0.90$	M1		Can be implied by 13, 14 or 15 Accept $P(Y = \text{CV}) = 0.90$
	Thus CV = 15	A1	3	CAO
(iii)	Power = 1 − P(Type II error) = 1 − P(accept H_0 | H_0 false) = P(accept H_1 | H_1 true)	B1		Or equivalent Stated or implied use
	$\lambda = 5 \times 3 = 15$	B1		Stated or implied use of Po(15)
	Thus power $= P(Y \ge \text{CV})$	M1		Attempt at a probability based on C's CV from (ii) and Po(15)
	$= P(Y \ge 15) = 1 - P(Y \le 14)$ $= 1 - 0.4657 = 0.53$ to 0.54	A1	4	AWFW (0.5343)
	Total		**17**	
	TOTAL		**75**	