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AQA

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)				
(i)	$f(-1)=0$	B1	1	
(ii)	$f(-\frac{1}{2})=4(-\frac{1}{8})-7(-\frac{1}{2})-3$	M1		Use of $\pm\frac{1}{2}$
	$=-\frac{1}{2}+\frac{7}{2}-3=0\Rightarrow$ factor	A1	2	Need to see simplification (at least $(-\frac{1}{2})^3=-\frac{1}{8}$), '=0' and conclusion
(iii)	Third factor is $(2x-3)$	B1		PI
	$\dfrac{(x+1)(2x+1)(2x-3)}{(x+1)(2x+1)}$	M1		$\dfrac{\text{3 linear factors}}{\text{2 linear factors}}$
	simplifies to $2x-3$	A1		Simplified result stated. Alternative; see end. Use remainder theorem.
	Alternative			
	Complete division to $2x+b$	(M1)		
	Complete division to $2x-3$	(A1)		
	Simplifies to $2x-3$	(A1)	3	Simplified result stated
(b)	$g(-\frac{1}{2})=-\frac{1}{2}+\frac{7}{2}+d=2$	M1		
	$d=-1$	A1		
	Alternative			
	Complete division leading to rem = 2	(M1)		Remainder $=d+p=2$
	$d=-1$	(A1)	2	
	Total		**8**	
2(a)	$R=\sqrt{10}$	B1		Accept $R=3.16$ or better.
	$\tan\alpha=3$	M1		OE (Can be implied by 71.57° seen)
	$\alpha=1.25$	A1	3	A0 if extra answers within given range SC 1 $\tan\alpha=\frac{1}{3}$ $\alpha=0.32$
(b)(i)	min value = $-\sqrt{10}$ (or $\geq\sqrt{-10}$)	B1F	1	ft on R
(ii)	$\sin(x-\alpha)=-1$	M1		or $\sin^{-1}\frac{3\pi}{2}$
	$x=5.96$	A1F	2	ft on their α (to 2 dp) $+\frac{3\pi}{2}$
	Total		**6**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)				
(i)	$\frac{2x+7}{x+2} = 2 + \frac{3}{x+2}$	B1 B1	2	
(ii)	$\int \frac{2x+7}{x+2} = 3\ln(x+2) + 2x + C$	B1F B1F	2	Either term correct Both correct; constant required; condone missing bracket ft on A, B
(b)(i)	$28 + 4x^2 =$ $P(5-x)^2 + Q(1+3x)(5-x)$ $+ R(1+3x)$	M1		
	$x = 5 \qquad x = -\frac{1}{3}$ $R = 8 \qquad P = 1$ $x = 0 \Rightarrow 28 = 25P + 5Q + R$ $Q = -1$	m1 A1 m1 A1		Two values of x used to find R and P. SC $R = 8$, $P = 1$ NMS can score B1,B1 Third value of x used to find Q .
	Alternative $28 + 4x^2 =$ $P(5-x)^2 + Q(1-3x)(5-x)$ $+ R(1+3x)$	(M1)		
	$= (25P + 5Q + R) +$ $(-10P + 14Q + 3R)x + (P - 3Q)x^2$	(m1)		Collect terms and form equations
	$P - 3Q = 4$ $14Q + 3R - 10P = 0$ $25P + 5Q + R = 28$	(A1)		Correct equations
	$P = 1 \quad Q = -1 \quad R = 8$	(m1) (A1)	5	Solve for P Q and R
(ii)	$\int \frac{1}{1+3x} - \frac{1}{5-x} + \frac{8}{(5-x)^2}\ \mathrm{d}x$	M1		Use partial fractions
	$= \frac{1}{3}\ln(1+3x) + \ln(5-x) + \frac{8}{5-x} + (C)$	m1 A1F A1F	4	$a\ln(1+3x) + b\ln(5-x)$ OE; both ln integrals correct; needs ( ) Other term correct ft on their P, Q, R SC: If no P,Q, R found in (b)(i), can gain method marks by inserting other values or retaining the letters (max 2/4)
	Total		**13**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a) (i)	$(1-x)^{\frac{1}{2}} = 1+\frac{1}{2}(-x)+px^2$	M1		
	$= 1-\frac{1}{2}x-\frac{1}{8}x^2$	A1	2	
(ii)	$\sqrt{4-x} = 2\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$	B1		or $(4)^{\frac{1}{2}}\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$
	$=(2)\left(1-\frac{1}{2}\left(\frac{x}{4}\right)-\frac{1}{8}\left(\frac{x}{4}\right)^2\right)$	M1		x replaced by $\frac{x}{4}$; condone missing ( )
				Or start again with $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$
	$=2-\frac{x}{4}-\frac{x^2}{64}$	A1		CAO or decimal equivalent
	Alternative			
	$(4-x)^{\frac{1}{2}} = 4^{\frac{1}{2}}+\frac{1}{2}\times 4^{-\frac{1}{2}}(-x)$	(M1)		Use of $(a+x)^n$ from formula book
	$+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}4^{-\frac{3}{2}}(-x)^2$	(A1)		Condone missing brackets and 1 error
	$=2-\frac{x}{4}-\frac{x^2}{64}$	(A1)	3	
(b)	$x=1 \qquad \sqrt{3}\approx 2-\frac{1}{4}-\frac{1}{64}$	M1		$x=1$ used in their expansion
	$=1.734\ (3\text{dp})$	A1	2	CSO
	Total		**7**	
5(a)	$\sin 2x = 2\sin x\cos x$	B1	1	OE, eg $\sin x\cos x+\sin x\cos x$ etc
(b)	$\cos x = 0 \qquad x = 90,\ 270$	B1		Both required
	$10\sin x+3=0$	M1		
	$x = 197.5 \quad 342.5$	A1A1	4	CAO if extra values in given range, max 1/2
(c)	$\cos 2x = \cos^2 x-\sin^2 x$	B1		$\cos 2x$ in any correct form
	$2\sin x\cos x+1-2\sin^2 x = 1+\sin x$	M1		sin $2x$ expanded and $\cos 2x$ in terms of $\sin x$ used
		A1		
	$2\sin x(\cos x-\sin x)=\sin x$			
	$2(\cos x-\sin x)=1$	A1	4	CSO; need to see $\sin x$ taken out as factor or cancelled
	Total		**9**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6 (a)	$x^2\frac{dy}{dx}+2xy$	M1 A1		Product rule used. Allow 1 error
	$+3y^2\frac{dy}{dx}$	B1		Chain rule
	$=2$	B1		RHS and equation with no spurious $\frac{dy}{dx}$ unless recovered.
	$(2,1)$, $4\frac{dy}{dx}+4+3\frac{dy}{dx}=2$	M1		Substitute $(2,1)$
	$\frac{dy}{dx}=-\frac{2}{7}$	A1	6	CSO
(b)	$\frac{dy}{dx}=0\Rightarrow$	M1		Derivative = 0 used
	$xy=1$	A1		OE
	$x^2\times\frac{1}{x}+\frac{1}{x^3}=2x+1$	m1		Use $xy=k$ to eliminate y on LHS
	$\frac{1}{x^3}=x+1$	A1	4	Answer given; CSO
	Total		**10**	
7(a) (i)	$\int\frac{dx}{e^{\frac{1}{2}x}}=\int -kt\ dt$	B1		Separate; condone missing integral signs
	$-2e^{-\frac{1}{2}x}=-k\frac{t^2}{2}\ \ (+C)$	B1B1	3	
(ii)	$-2e^{-\frac{1}{2}x}=-k\frac{t^2}{2}-2e^{-3}$	M1		Use $(6,0)$ to find constant
	$\ln\left(e^{-\frac{1}{2}x}\right)=\ln\left(k\frac{t^2}{4}+e^{-3}\right)$	M1		Take logarithms correctly; condone one side negative. Must have a constant.
	$-\frac{1}{2}x=\ln\left(k\frac{t^2}{4}+e^{-3}\right)$			
	$x=-2\ln\left(\frac{kt^2}{4}+e^{-3}\right)$	A1	3	Answer given; CSO
(b) (i)	$t=10$ $\quad x=-2\ln\left(\frac{0.004\times10^2}{4}+e^{-3}\right)$	M1		
	$=3.8\ \Rightarrow 3800$	A1	2	CAO
(ii)	$x=0$ $\quad \frac{0.004\times t^2}{4}+e^{-3}=1$	M1		
	$t=30.8$	A1	2	CAO Treat 0.04 or 0.0004 as misread (–1)
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a) (i)	$\overrightarrow{AB} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$	M1 A1	2	$\pm\left(\overrightarrow{OA} - \overrightarrow{OB} \right)$ A0 if answer as coordinates
(ii)	$\overrightarrow{OB} \bullet \overrightarrow{AB} = 3\times1 + 1\times0 + (-2)\times(-1) = 5$	M1 A1		Evaluate to single value
	$\cos\theta = \dfrac{\overrightarrow{OB} \bullet \overrightarrow{AB}}{\lvert\overrightarrow{OB}\rvert \times \lvert\overrightarrow{AB}\rvert}$	M1		Use formula for $\cos\theta$ with any 2 vectors and at least one of the corresponding modulii 'correct'
	$\lvert\overrightarrow{OB}\rvert = \sqrt{14}$ $\quad \lvert\overrightarrow{AB}\rvert = \sqrt{2}$			
	$\cos\theta = \dfrac{5}{\sqrt{7\times2}\sqrt{2}} = \dfrac{5}{2\sqrt{7}}$	A1		CSO; AG so need to see intermediate step eg $\dfrac{5}{\sqrt{7\times2}\sqrt{2}}$ or $\dfrac{5}{\sqrt{28}}$
	Alternative cos rule attempted with cos B cos rule correct with cos B derive correct given form	 (M1) (A1) (A2)	 4	
(b)	$\mathbf{r} = \begin{bmatrix} 6 \\ 2 \\ -4 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$	M1 A1F	 2	$\overrightarrow{OC} + \lambda\overrightarrow{AB}$. Allow one slip ft on $\overrightarrow{AB}$; needs $\mathbf{r}$ or $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$
(c)	$\overrightarrow{OD} \bullet \overrightarrow{AB} = \begin{bmatrix} 6+\lambda \\ 2 \\ -4-\lambda \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$	M1		
	$6+\lambda+4+\lambda = 0$	m1		
	$\lambda = -5$	A1F		ft on equation of line
	D is $(1,2,1)$	A1		CAO
	Alternative $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = a - c = 0$	 (M1)		Let D be (a,b,c) Scalar product evaluated and equated to 0
	$a = 6+\lambda, \quad b = 2, \quad c = -4-\lambda$	(m1) (A1)		Use equation of line
	$a + c = 2$ $a = 1 \quad b = 2 \quad c = 1$	 (A1)	 4	
	Total		**12**	
	TOTAL		**75**	